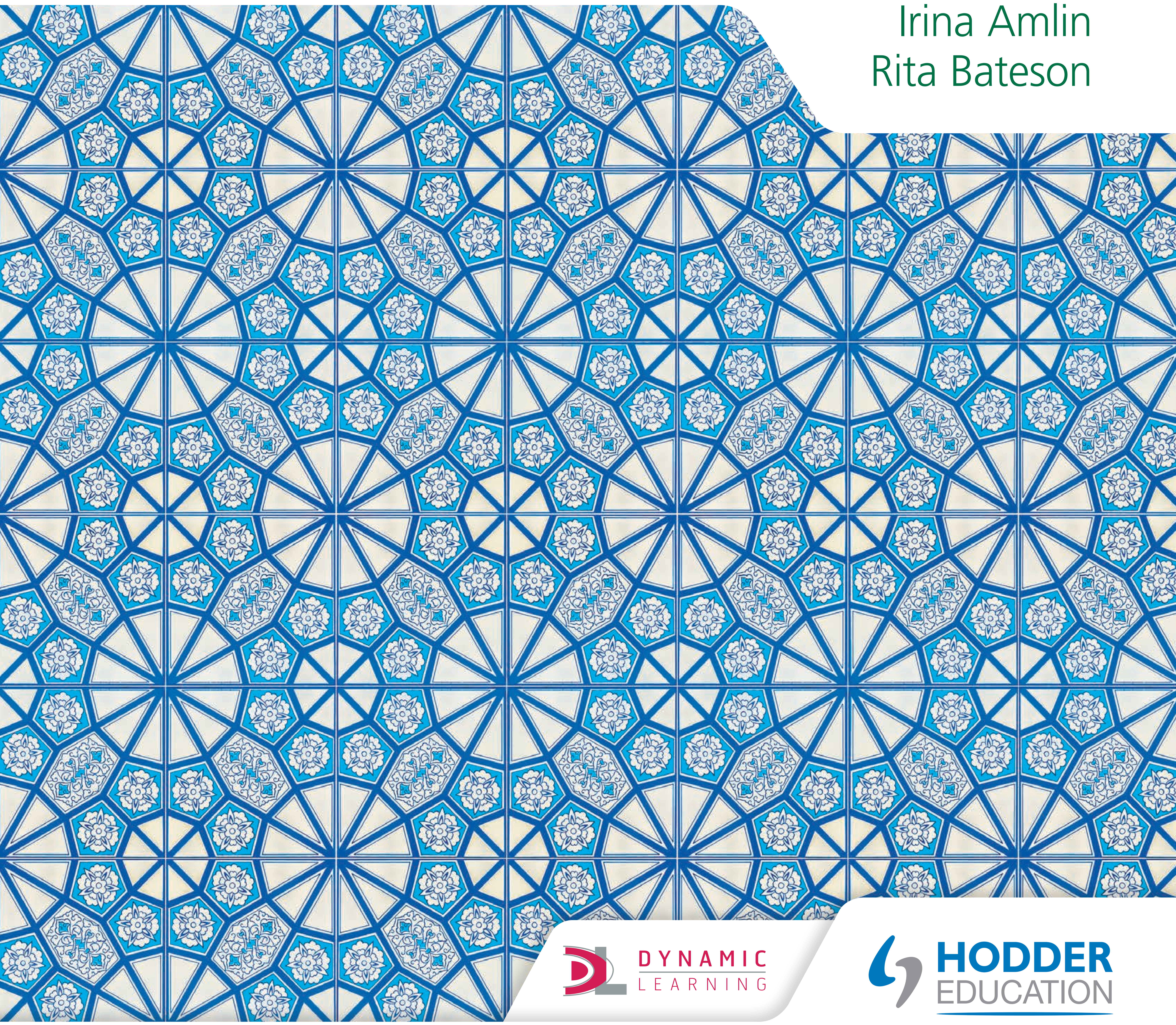
MYP by Concept

2

Mathematics

Irina Amlin
Rita Bateson



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MYP by Concept 2

Mathematics

Rita Bateson
Irina Amlin
Series editor: Paul Morris

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Acknowledgements

From Rita Bateson: With thanks to my first and most enthusiastic maths teacher, my dad, Sean, and to my ever-supportive mother, Eileen, as well as to my wonderful siblings and friends who made me laugh throughout. Special thanks to my proof-reading brother Sean and to one of the best mathematics teachers I ever had, Eoin O'Maoileoin.

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Most of all, this book is dedicated to my incredible daughter Ellie and amazing, patient and kind husband Andrew, without whom nothing would ever get done.

From Irina Amlin: This book is dedicated to my two remarkable daughters, Penelope and Cora, who I know will each achieve great things in their own right. Thank you for giving up so much playtime with mummy to allow her to work on this!

Thank you also to my husband Ryan, for his support and encouragement. My heartfelt gratitude goes to my mother Dara and brother Luka, who taught me how to learn, and to my father and chief advisor Janko, who taught me how to teach! To Fred Ferneyhough, who guided my decision to pursue teaching. To my many inspiring colleagues, especially Rita Bateson, Stephen Pulford and Graham Lewis – our invigorating discussions and joint projects have been my best PD. Thank you to Dr Jim Christopher, Charlie Judd and the late Julia Watson for encouraging creativity in mathematics. Finally, thank you to So-Shan, Megan, and all the patient, hard-working staff at Hodder Education, as well as to Dr Anna Clark for your keen eye!

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Contents

How to use this book

Welcome to Hodder Education's *MYP by Concept* series! Each chapter is designed to lead you through an *inquiry* into the concepts of mathematics, and how they interact in real-life global contexts.

Each chapter is framed with a *Key concept* and a *Related concept* and is set in a *Global context*.


Relationships | Change, Simplification | Globalization and sustainability

1 What really makes the world go round?

Financial, personal and economic **change** can be understood and **simplified** using proportional **relationships** like ratios and percentages.

CONSIDER THESE QUESTIONS:

Factual: How do we convert fractions to decimals, and vice versa? What is the difference between percentage *of* and percentage *off*? What is a ratio? How do we reason with ratios?

Conceptual: What is the relationship between fractions and decimals? How do we simplify a ratio? How do we share quantities in a given ratio? Can ratios shed light on problems?

Debatable: What is the best way to find the percentage of something? Can percentages go over 100%? Does money make the world go round? Is money good or bad, neither or both?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

IN THIS CHAPTER, WE WILL ...

- **Find out** how and why ratios and percentages help us when dealing with food or money.
- **Explore** the role of dividing and finding parts of a whole.
- **Take action** by analysing the contents and proportions to help us make better and more informed decisions.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how decimals and fractions are defined
- how to add, subtract, multiply, and divide decimals and fractions
- how to convert fractions into decimals
- how to convert decimals into fractions
- what a percentage is and how to find percentages of quantities.

2 Mathematics for the IB MYP 2: *by Concept*

The *Statement of Inquiry* provides the framework for this inquiry, and the *Inquiry questions* then lead you through the exploration as they are developed through each chapter.

KEY WORDS

Key words are included to give you access vocabulary for the topic. **Glossary** terms are highlighted and, where applicable, **search terms** are given to encourage independent learning and research skills.

As you explore, activities suggest ways to learn through *action*.

ATL

- Activities are designed to develop your *Approaches to Learning* (ATL) skills.

Assessment opportunities in this chapter:

- Some activities are *formative* as they allow you to practise certain of the MYP Mathematics *Assessment Criteria*. Other activities can be used by you or your teachers to assess your achievement against all strands of an Assessment Criterion.

Detailed information or explanation of certain points is given whenever necessary. Key Approaches to Learning skills for MYP Mathematics are highlighted whenever we encounter them.

Worked examples and Practice questions are given in colour-coded boxes to show the level of difficulty:

Problem	**Challenging**
Complex	**Unfamiliar**

Each chapter covers one of the four branches of mathematics identified in the MYP Mathematics skills framework.


Number

These Approaches to Learning (ATL) skills will be useful …

- Creative-thinking skills
- Communication skills
- Transfer skills
- Information literacy skills

We will reflect on these Learner Profile attributes …

- **Knowledgeable** – We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.
- **Communicator** – We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

Assessment opportunities in this chapter:

- **Criterion A:** Knowing and understanding
- **Criterion B:** Investigating patterns
- **Criterion C:** Communicating
- **Criterion D:** Applying mathematics in real-world contexts

THINK-PAIR-SHARE

Recipe

Muffins
2 medium eggs
25 mL oil
250 mL milk
200 g sugar
400 g self-raising flour (or same quantity plain flour and 3 tsp baking powder)
tsp salt
00 g chocolate chips, nuts or dried fruit

Makes approximately 20 muffins.

1. Beat eggs lightly and add the oil and milk. Mix sugar in and add the flour and salt. Mix until smooth. Stir in the chocolate chips or dried fruit, if required.
2. Fill muffin cases two-thirds full. Place muffins into a pre-heated oven (200 °C) for 20 to 25 minutes approximately. Cool and enjoy.

The image shows a recipe for simple muffins. The recipe makes approximately 20 muffins but doesn't say for how many people. How many muffins would make a suitable serving for each person? Is it a healthy recipe? How inclusive is this recipe? What if we wanted to make enough for the whole class? Or for your family? What other mathematical ideas are used in this recipe? How would your prior knowledge help you to solve these questions?

KEY WORDS

decrease, growth, increase, interest, portion, precise, proportion, scale, whole

1 What really makes the world go round? 3

We have incorporated Visible Thinking – ideas, framework, protocol and thinking routines – from Project Zero at the Harvard Graduate School of Education into many of our activities. You are prompted to consider your conceptual understanding in a variety of activities throughout each chapter.

Finally, at the end of each chapter, you are asked to reflect back on what you have learned with our Reflection table, maybe to think of new questions brought to light by your learning.

Use this table to reflect on your own learning in this chapter.					
Questions we asked	Answers we found	Any further questions now?			
Factual					
Conceptual					
Debatable					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Learner Profile attribute(s)	Reflect on the importance of this attribute for your learning in this chapter.				

Hint

In some of the Activities, we provide Hints to help you work on the assignment. This also introduces you to the new Hint feature in the on-screen assessment in MYP5.

! Take action

! While the book provides opportunities for action and plenty of content to enrich the conceptual relationships, you must be an active part of this process. Guidance is given to help you with your own research, including how to carry out research, how to make changes in the world informed by mathematics, and how to link and develop your study of mathematics to the global issues in our twenty-first century world.

Links to:

Like any other subject, mathematics is just one part of our bigger picture of the world. Links to other subjects are discussed.

We will reflect on this Learner Profile attribute …

- Each chapter has an *IB Learner Profile* attribute as its theme, and you are encouraged to reflect on these too. We have explored the Learner Profile further with our feature, Meet a mathematician.

1 What really makes the world go round?

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CONSIDER THESE QUESTIONS:

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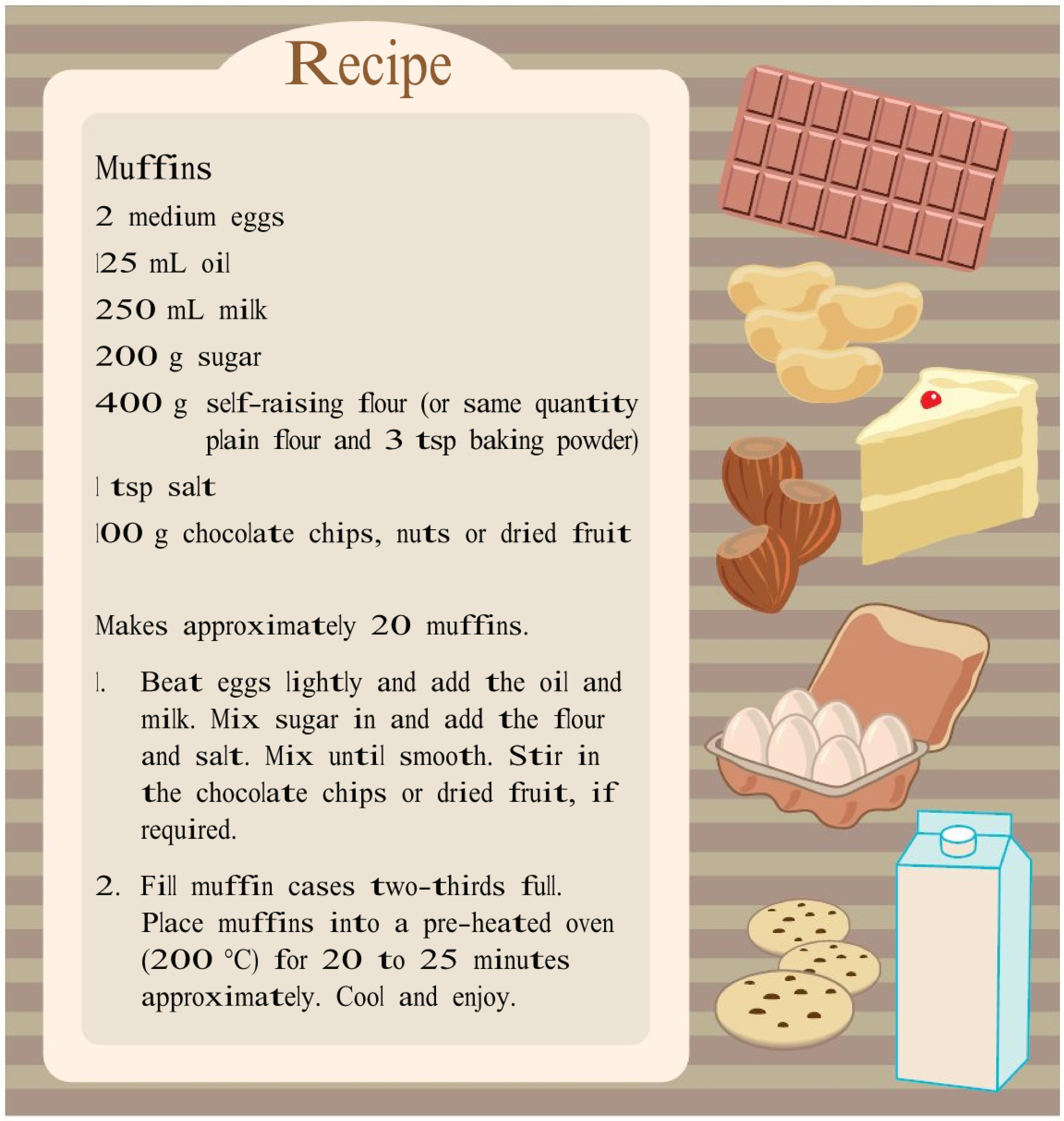
- **Criterion D:** Applying mathematics in real-world contexts

THINK-PAIR-SHARE



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KEY WORDS

decrease	interest	proportion
growth	portion	scale
increase	precise	whole

What is the relationship between fractions and decimals?

WHAT DO YOU REMEMBER ABOUT FRACTIONS AND DECIMALS?

To begin the year, let's start with an internationally minded activity, unless of course you come *from* the country we are thinking about!

Did you know that the Dutch celebrate a very specific anniversary? After **$12\frac{1}{2}$ years** of marriage, they usually celebrate with a party or a trip. It is the halfway point to 25 years of marriage, and cards are specially printed for the occasion. These images show cards for such an occasion. Can you identify how they are different?

Notice that some cultures use a comma (,) instead of a decimal point (.)

As we can see from the cards, the numbers 12.5 and $12\frac{1}{2}$ are interchangeable (for this purpose at least). They are different **representations** of the same number of years.

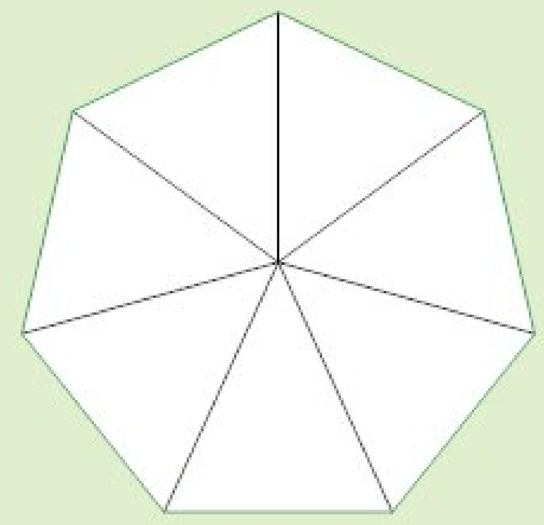
If we think back to *Mathematics for the IB MYP 1*, we can recall that a fraction is a numerical quantity (which is not an **integer** or whole number) expressed as a **quotient**. A decimal is also a numerical quantity that is not an integer or whole number but it is expressed using a decimal point to indicate place value of powers of 10.

Let's do some quick revision on how to handle fractions and decimals.

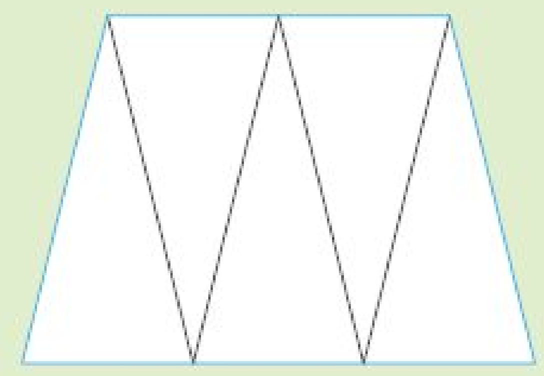
PRACTICE EXERCISE

Shade in the following shapes according to the fractions given.

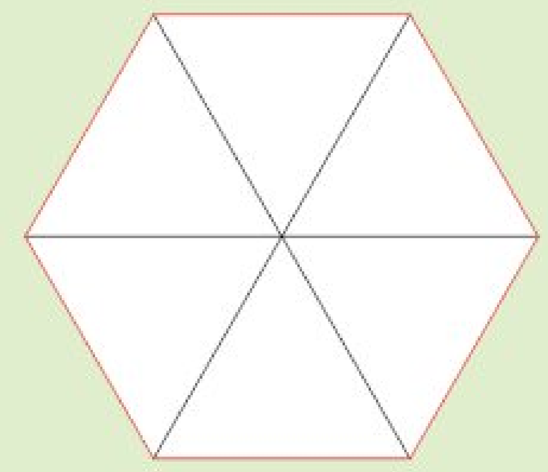
1 $\frac{1}{7}$



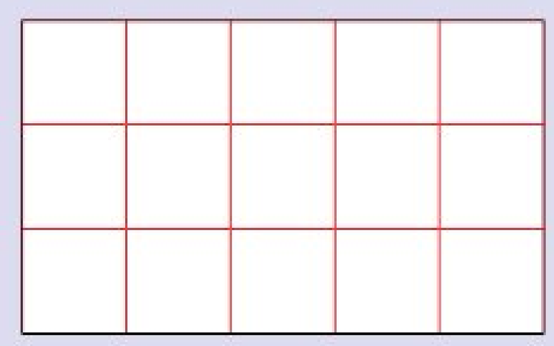
2 $\frac{1}{5}$



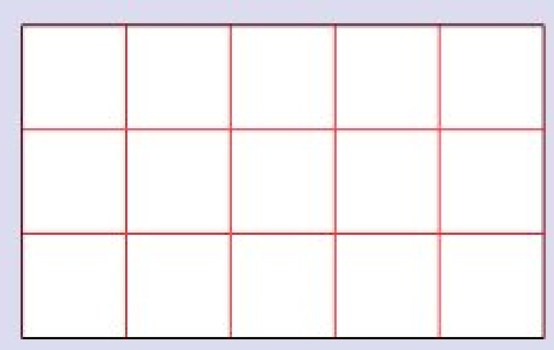
3 $\frac{1}{3}$



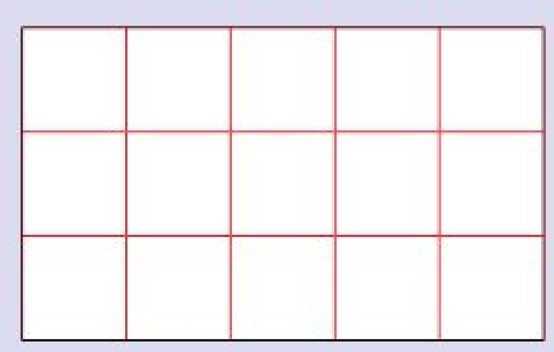
4 $\frac{3}{15}$



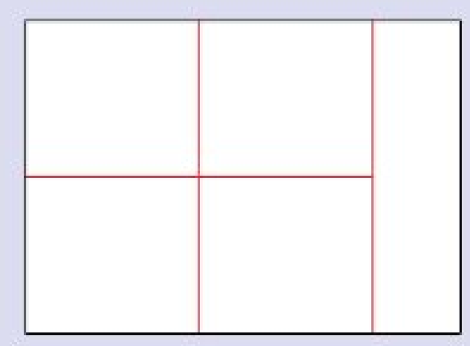
5 $\frac{5}{15}$



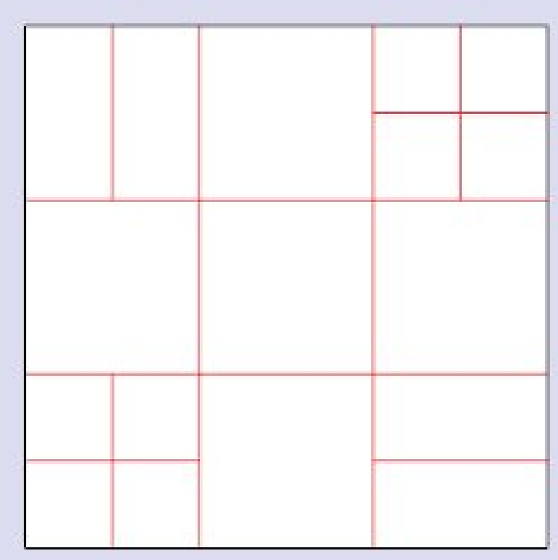
6 $\frac{13}{15}$



7 $\frac{1}{5}$

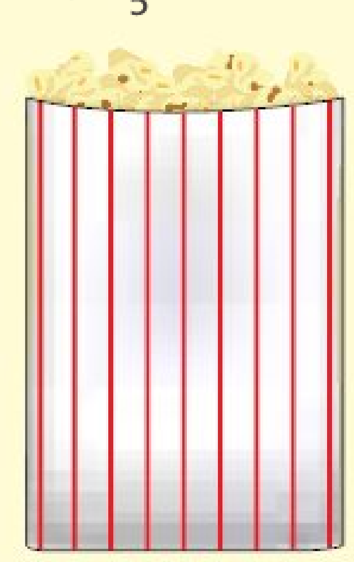


8 $\frac{3}{18}$

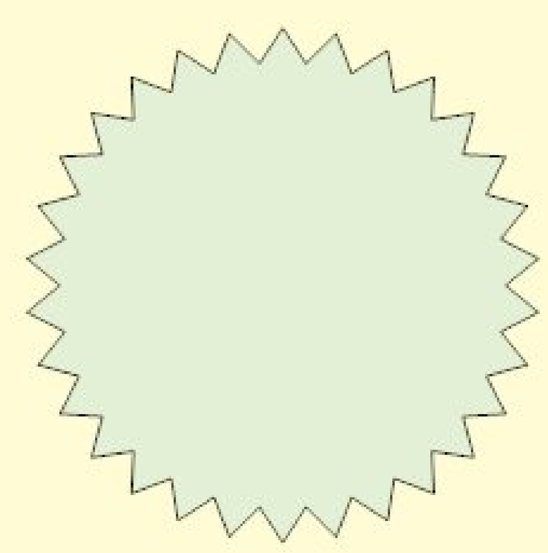


9 $\frac{11}{65}$

10 $\frac{3}{5}$



11 $\frac{6}{10}$



Before we move on, make sure that you are very comfortable handling fractions! Some students confuse the rules of operations for fractions, getting in a muddle about when they need to find common denominators or 'flip' a fraction.

Use this Practice exercise to see which rules you might need to revise.

PRACTICE EXERCISE

Calculate the following.

1 $\frac{1}{7} + \frac{3}{7}$	**6** $\frac{1}{7} \times \frac{3}{7}$	**11** $\frac{1}{7} \div \frac{3}{7}$
2 $\frac{1}{9} + \frac{1}{7}$	**7** $\frac{1}{45} \times \frac{3}{2}$	**12** $\frac{1}{45} \div \frac{3}{2}$
3 $\frac{6}{7} - \frac{3}{21}$	**8** $\frac{91}{88} \times \frac{22}{910}$	**13** $\frac{91}{88} \div \frac{910}{22}$
4 $\frac{11}{17} + \frac{3}{17}$	**9** $7\frac{1}{7} \times \frac{3}{10}$	**14** $7\frac{1}{7} \div \frac{3}{10}$
5 $\frac{21}{70} - \frac{3}{70}$	**10** $6\frac{1}{9} \times 5\frac{3}{9}$	**15** $6\frac{1}{9} \div 5\frac{3}{9}$

Hint

Don't forget that you can cancel common factors before you multiply fractions.

Ingredients for Great Rice Pudding

$\frac{1}{2}$ **cup** rice
$\frac{1}{2}$ **cup** water
1 L milk
$\frac{1}{4}$ **cup** sugar
1 tsp vanilla essence

Reviewer: NiceRice **Date:** Yesterday **Rating:** ★★★★★

We loved this rice pudding – so creamy. I only added $\boxed{\frac{1}{3}}$cup of sugar as we don't like things too sweet. It was perfect for us.

Explain how this reviewer has misunderstood a basic idea about fractions (and explain why this is funny, given the fact that they don't like things too sweet).

Before we move on, make sure you are very comfortable handling decimals! Many students confuse the rules of operations for decimals, getting in a muddle about where to put the decimal point or how numbers become smaller (or bigger) when dividing.

Use this Practice exercise to see which rules you might need to revise.

PRACTICE EXERCISE

Try operating with decimals.

1 0.2 + 0.5 =	**4** 0.2 × 0.5 =
2 0.08 – 0.03 =	**5** 1.6 × 0.2 =
3 1.7 + 0.2 + 0.6 =	**6** 3.14 × 5 =

7 $\frac{100}{0.2}$ = **8** $\frac{2.5}{5}$ = **9** $\frac{7.6}{0.001}$ =

Hint

When multiplying decimals, it is sometimes easier to start by multiplying them as whole numbers, temporarily ignoring where the decimal point comes.

For example, let's multiply 1.75 by 0.6.

```
 ₄1 7₃5
 X    6
 1 0 5 0
```

This simple multiplication gives an answer of 1050.

Next, all you have to do is count how many numbers came behind each decimal point in the problem and make sure that the same is true for your answer.

There were two digits behind the decimal point in the first number (1.75) and one digit behind the decimal point in the second ***multiplicand*** *(0.6). This means that the final answer must have three digits behind the decimal point.*

So, the final answer is 1.050.

HOW CAN WE VISUALIZE PERCENTAGES AND DECIMALS?

Fractions, percentages and decimals can be expressed as a portion of a whole with the help of a 10 × 10 grid, where each box shows one hundredth (1% or 0.01) of the whole.

ACTIVITY: What is your name worth?

ATL

- Communication skills: Interpret and use effectively modes of non-verbal communication

Draw another 10 × 10 grid and shade in enough boxes to make the shapes of the letters that spell your name.

What percentage of the whole is **your** name? What is the least fraction or percentage it **could** be (if you drew it differently)? What is the largest or most amount of space your name could take up as a percentage of the whole (and still be legible)?

Hint

Maybe your name is too long for a 10 × 10 grid! Can you think of a solution to this problem? You are free to change how you present your name or to alter the grid but, however you solve the problem, make sure you can still answer the questions accurately.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

ACTIVITY: Decimal numbers

ATL

- Communication skills: Interpret and use effectively modes of non-verbal communication

Using a long piece of paper (for example, wall paper or flipboard paper), you are going to write this number: 1.4935. But there is a catch. You have to represent the number by drawing it to a special scale, so that the number 1 will be 1 metre tall. The number following the decimal point (the tenths) must be drawn at one-tenth of the size of the whole number. The 9, which represents the hundredths, must be ten times smaller than the 4, and it will also be 100 times smaller than the 1. And so on, until the numbers are so small that you cannot see them!

Repeat the task for these numbers:

- **2.83011**
- **0.0003333**
- **1.0191919191**

Why would it be very difficult to draw a number greater than 10 in this way?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

Draw a 10 × 10 grid (or use squared paper) and colour in $\frac{7}{10}$. Draw another grid and colour in 70%.

Use another 10 × 10 grid. Now, represent 0.23 as part of the whole. You can be as creative with the representation as you like.

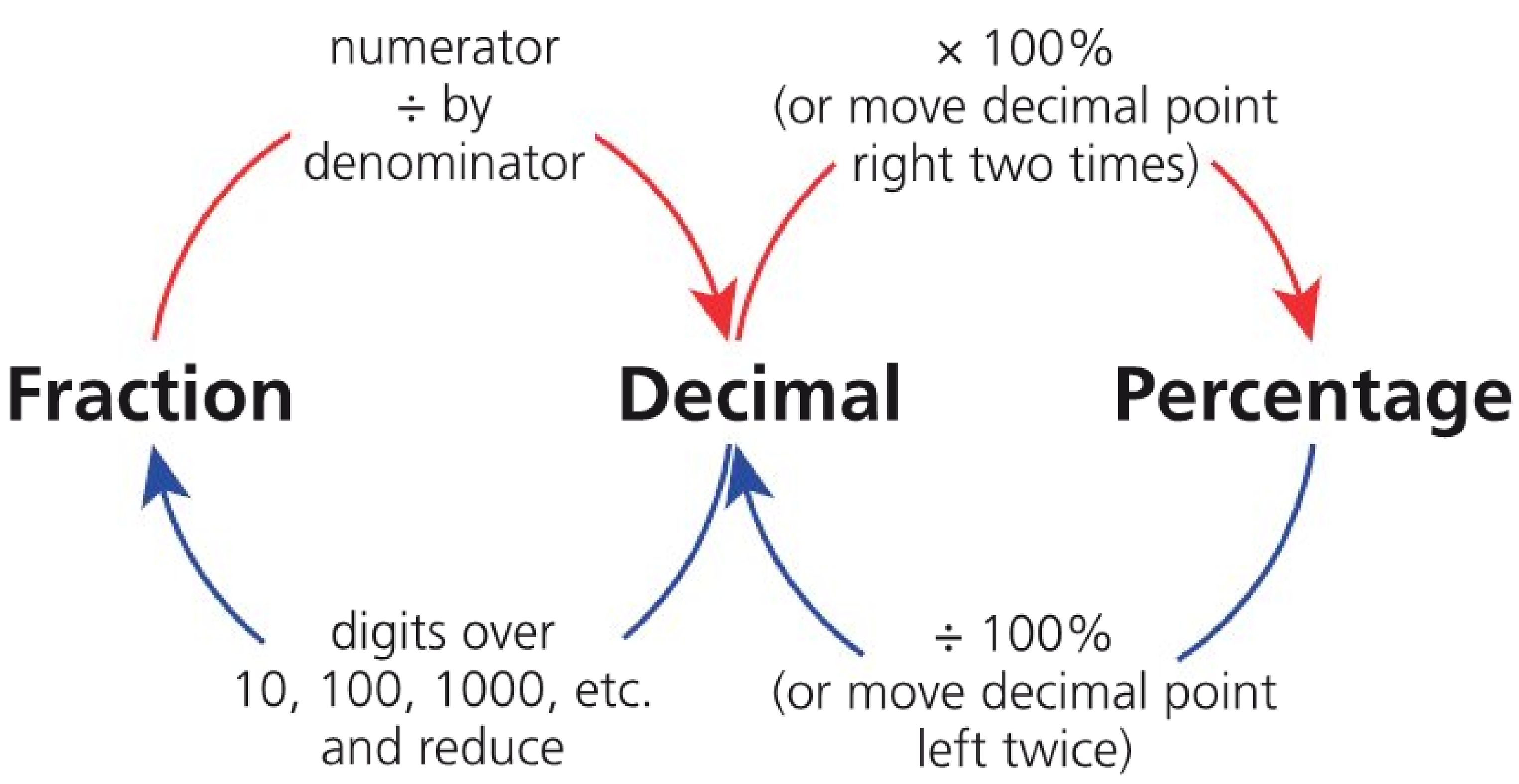
Given a 10 × 10 grid, it is only possible to represent from 0% to 100% in increments or steps of 1. What if we wanted to represent a value such as 0.2342? We must consider the parts of a whole with regard to the base 10. With a 10 × 10 grid, we can only colour in multiples of 0.01. But our number is more precise than that: 0.2342.

If we use a 100 × 100 grid, how many boxes will there be in the whole? What are the multiples that are possible now? Can you represent 0.2342 visually by colouring in the correct number of boxes?

How do we convert fractions and decimals?

REMEMBERING OUR CONVERSION TECHNIQUES

Previously, we practised the following rules:



When moving from a fraction to a decimal, we use **division of the quotient**.

Example 1 $\frac{1}{4}$

```
  0.2 5
4|1.0 0
  8
   2 0
```

Example 2 $\frac{1}{9}$

```
  0.1 1 1...
9|1.0 0
  9
  1 0
    9
    1 0
```

Example 3 $\frac{6}{29}$

```
   0.2 0 6 8
29|6.0 0 0
   5 8
     2 0 0
     1 7 4
       2 6 0
       2 3 2
         2 8...
```

Remember that a calculator can find these values, if you type in the fraction as a division problem. Many calculators can also switch between fractions and decimals (where it is possible) using a single button, or combination of buttons. Look at your calculator to see if you have such a button, often marked S↔D.

When we are converting a decimal into a fraction, our first job is to introduce a **vinculum**. As we saw in *Mathematics for the IB MYP 1*, the vinculum is the line that separates the numerator and the denominator. To start, we must write the decimal as a new fraction over 1. Then we can multiply above and below the line (or vinculum) until we no longer need the decimal point.

PRACTICE EXERCISE

Change these **fractions** into their equivalent **decimal** value.

1	$\frac{4}{5}$	**4**	$\frac{2}{15}$	**7**	$\frac{18}{14}$
2	$\frac{7}{11}$	**5**	$\frac{14}{35}$	**8**	$\frac{54}{7}$
3	$\frac{6}{9}$	**6**	$\frac{1}{200}$	**9**	$\frac{103}{102}$

Practice converting these decimals into fractions.

1	0.4	**4**	0.83	**7**	0.404
2	0.8	**5**	0.42	**8**	0.5432
3	1.3	**6**	1.50	**9**	3.005

Convert these percentages into decimals **and** fractions.

1	20%	**4**	71%	**7**	102%
2	75%	**5**	29%	**8**	62.5%
3	15%	**6**	3%	**9**	0.3%

Now check your answers to the above questions using your calculator.

Example 1

0.3

$\frac{0.3}{1} \times \frac{10}{10}$

$\frac{3}{10}$

Done ✓

Example 2

0.224

$\frac{0.224}{1} \times \frac{10}{10}$

$\frac{2.24}{10}$

Multiply again …

$\frac{2.24}{10} \times \frac{10}{10}$

and again …

$\frac{22.4}{100} \times \frac{10}{10}$

$\frac{224}{1000}$

Done ✓

Example 3

1.005

$\frac{1.005}{1} \times \frac{10}{10}$

$\frac{10.05}{10} \times \frac{10}{10}$

$\frac{100.5}{100} \times \frac{10}{10}$

$\frac{1005}{1000}$

Done ✓

Of course, the last two fractions could be simplified further by finding common factors. If you need to revise simplifying fractions, look at page 16 in *Mathematics for the IB MYP 1*.

What is the best way to find the percentage of something?

There are lots of ways to find the required percentage **of** a given amount. We will look at three different methods. These methods are all valid, and you can choose whichever makes the most sense to you, but you must always communicate clearly.

The problem: find 89% **of** 350.

Method 1: Treat the percentage like a set of fraction operations

If 89% is the same as $\frac{89}{100}$ and 350 is the same as $\frac{350}{1}$ and we know that 'of' between figures implies that we must multiply, then the problem becomes:

$$\frac{89}{100} \times \frac{350}{1}$$

How many ways can you think of to do this multiplication? Can you think of any shortcuts?

$$\frac{89}{10\cancel{0}} \times \frac{35\cancel{0}}{1}$$

$$\frac{89}{\cancel{10}_{2}} \times \frac{\cancel{35}^{7}}{1}$$

$$\frac{89}{2} \times \frac{7}{1} = \frac{623}{2} = 311.50$$

$$\begin{array}{r} 8^{6}9 \\ \times\ 7 \\ \hline 623 \end{array}$$

Examples

1 Find 21% of 18.

2 Find 65% of 1500.

Solutions

1 $\frac{21}{100} \times 18$

$\frac{21}{\cancel{100}_{50}} \times \frac{\cancel{18}^{9}}{1}$

$\frac{189}{50} = 3.78$

2 $\frac{65}{100} \times 1500$

$\frac{65}{\cancel{100}} \times 15\cancel{00}$

$= 65 \times 15 = 975$

Method 2: Treat the percentage as a decimal multiplier

For Method 2, all we need to do is change the percentage into a decimal multiplier instead of a fraction.

Who invented percentages and why?

To find out who invented percentages, read the following articles: **www.reference.com/math/invented-percentages-657eca4cb7048e89** and **https://solomon.io/mathematicians-roman-empire/** or search for **inventor percentages**.

0.89 × 350

$$\begin{array}{r} 3^{4}50 \\ \times\ 89 \\ \hline 3150 \\ 28000 \\ \hline 311.50 \end{array}$$

There were two numbers behind the decimal point in the question. The answer must match: 311.5

Examples

1 Find 21% of 18.

2 Find 65% of 1500.

Solutions

1 0.21 × 18

$$\begin{array}{r} 21 \\ \times\ 18 \\ \hline 168 \\ 210 \\ \hline 3.78 \end{array}$$

2 0.65 × 1500

$$\begin{array}{r} 1500 \\ \times\ 65 \\ \hline 7500 \\ 90000 \\ \hline 975.00 \end{array}$$

Method 3: Break it down, then build it up

Look back to *Mathematics for the IB MYP 1*, page 134, to see the basics of this idea. If we work out the 'easy' percentages, then the answer is just a combination of those we have already found.

Most percentages we come across are a combination of some of these:

- 100% =
- 50% =
- 10% =
- 5% =
- 1% =
- 2% =

So, for the example of 89% of 350, we can find:

$$\begin{array}{ccc} 100\% = 350 & \xrightarrow{\div 2} & 50\% = 175 \\ \downarrow{\div 10} & & \downarrow{\div 10} \\ 10\% = 35 & \xrightarrow{\div 2} & 5\% = 17.5 \\ \downarrow{\div 10} & & \\ 1\% = 3.50 & \xrightarrow{\times 2} & 2\% = 7 \end{array}$$

We want 89%, which we can make up in two ways:

1 50% + 10% + 10% + 10% + 5% + 2% + 2%

2 100% – 10% – 1%

Examples

1 Find 21% of 18. 2 Find 65% of 1500.

Solutions

1 100% = 18 50% = 9

10% = 1.8 5% = 0.9

1% = 0.18 2% = 0.36

21% = 1.8 + 1.8 + 0.18

= 3.78

2 100% = 1500 50% = 750

10% = 150 5% = 75

1% = 15 2% = 30

65% = 750 + 150 + 75

= 975

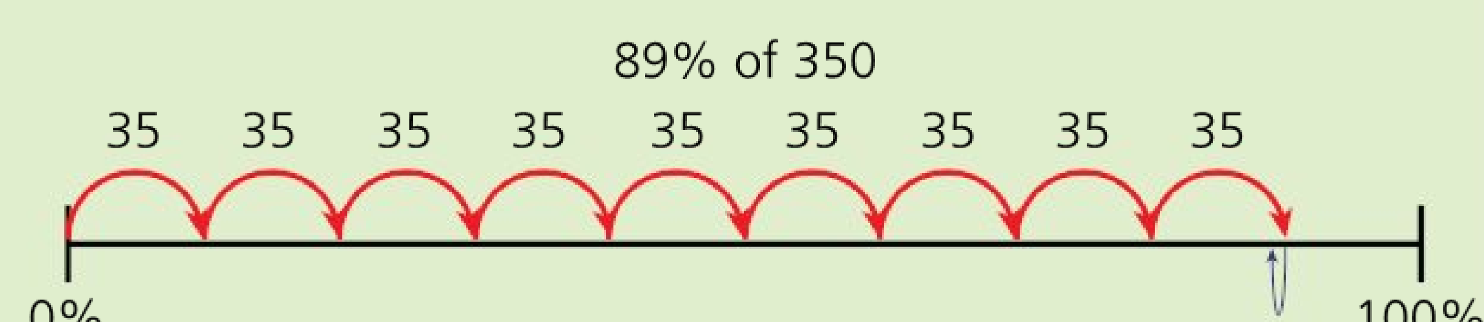
If visual representations work better for you, then you can imagine your calculation as a timeline of 100%, and each increase or decrease is a 'hop' of a certain quantity along the number line.

Examples

1 Find 89% of 350. 2 Find 24% of $40.

Solutions

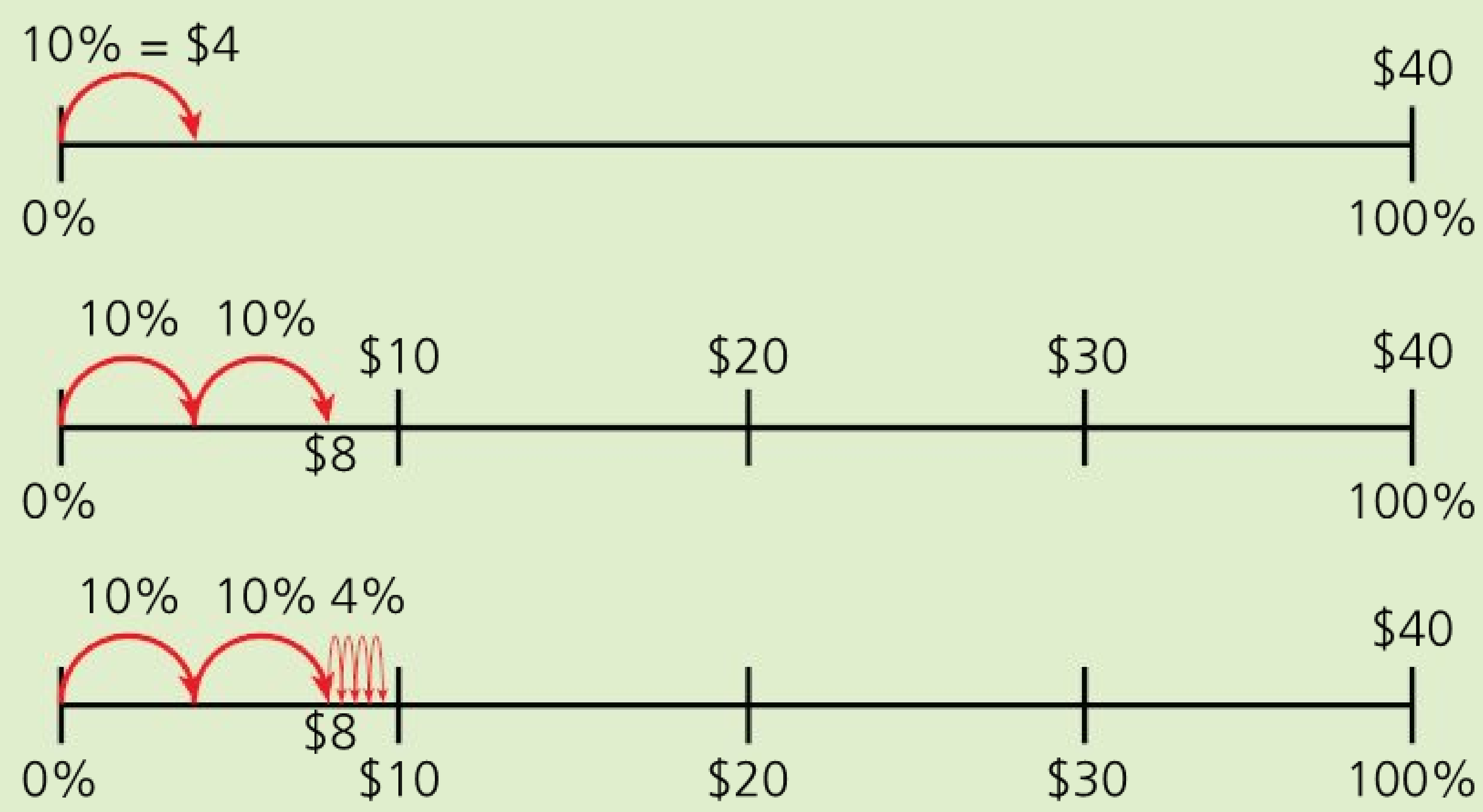
1



10% = 35
so 90% = 35 × 9
= 315

90%	=	315
– 1%		– 3.50
89%	=	311.50

2



10% = $4
1% = $0.40
4% = $1.60
24% = $4 + $4 + $1.60 = $9.60

PRACTICE EXERCISE

Now it is time to compare and contrast these three methods. Remember you are looking for the method that you find the fastest, most obvious, and easiest to remember – and which gets you the right answer most often!

Using all three methods, solve the following questions.

1 **Calculate 17% of 300.**
2 **Find 23% of $52.**
3 **Calculate 47% of 93.**
4 **Find 86.5% of 4340.**

THINK-PAIR-SHARE

Complete the Circle of viewpoints wheel for the three methods you have explored:

1 **Treat the percentage like a set of fraction operations.**
2 **Treat the percentage as a decimal multiplier.**
3 **Break it down, then build it up.**

In each section of the wheel, explain why someone who likes this method might prefer it. What would they say about the good aspects? Now shade in the section that shows your favourite method and think about what that says about you as a learner.

Think about how you use percentages in everyday life: service charges (or tips) are often given as a percentage; sales advertise how much discount you will receive; and banks often advertise percentages for credit cards, savings accounts or fees.

BRAINTEASER

Is 18% of 20 the same as 20% of 18? How could you **verify** this?

What is the difference between percentage *of* and percentage *off*?

HOW CAN I FIND THE PERCENTAGE OFF A QUANTITY?

Now that you have chosen your favourite method to calculate percentages, it is time to really master it.

Copy and complete the table by finding the correct percentage of the number that heads each column.

PRACTICE EXERCISE

Express each of the following as a percentage.

1	27 out of 27.	2	36 out of 60.
3	74 out of 620.	4	45 out of 150.
5	3.5 out of 1.2 million.	6	34 out of 4432.

	70	100	500	2000	120	380	42	15	6.5	11.3	9060	307 500
10%	7	10	50	200	12	38	4.2	1.5	0.65	1.13	906	30 750
50%												
20%												
100%												
25%												
15%												
90%												
75%												
30%												
18%												
66%												
51.5%												
134%												

Now that we can find the percentage of a quantity, can we do the reverse? What if we needed to find out how much a quantity is **as a percentage** of the whole?

How much is a $100 discount as a percentage of an $800 laptop?

To find the percentage, we must find out what proportion 100 dollars is *out of* 800. We recognize the 'out of' phrase as a division, so ...

100/800 is the fraction we want and we must multiply by 100 to change it to a percentage.

The deduction of $100 represents a 12.5% discount.

To calculate anything as a percentage of a whole, divide the number by the whole amount and multiply it by 100.

Many organizations publish recommended allowances for how much people should eat every day to remain fit and healthy. An example of a recommended daily allowance (RDA) can be seen here.

Typical values	Women	Men	Children (5–10 years)
Calories	2000 kcal	2500 kcal	1800 kcal
Protein	45 g	55 g	24 g
Carbohydrate	230 g	300 g	220 g
Sugars	25 g	31 g	23 g
Fat	70 g	95 g	70 g
Saturates	20 g	30 g	20 g
Fibre	24 g	24 g	15 g
Salt	6 g	6 g	4 g

Notice that the values are different for men, women, and children. What is a child's RDA of sugar as a percentage of a man's daily allowance? What is the RDA of fat for women as a percentage of a man's daily allowance?

! Take action

- Now that you are familiar with one set of dietary recommended allowances, put the information to good use in your school. If your school has a cafeteria, tuck shop or canteen, see what foods are on offer. Investigate any foods which have labels.
- Calculate how much each food item contributes as a percentage of your daily allowance of each food group. Are there some that are high and some that are low in certain types of nutrients? Why not raise awareness about this?
- Create some stickers with important percentages marked on them, so that people can make informed choices. Consider colour coding your stickers for even more information. Coordinate a campaign for healthier choices.

ACTIVITY: Gummi, gummi, gummi bears!

ATL

- Information literacy skills: Collect, record and verify data

Nutrition facts	
Serving size	17 pieces
Calories	140 kcal
Fat	0.1 g, 0% RDA
Sodium	20 mg, 0% RDA
Carbohydrates	43 g, 11% RDA
Sugar	18 g
Protein	3.2 g

If a child eats one serving (or 17 sweets), what is that as a percentage of their daily allowance of calories?

If an adult female eats 51 sweets, what is that as a percentage of her daily sugar allowance?

If an adult male decided to eat only these sweets for a day, how many should he eat to get his daily allowance of protein?

If he did eat that many sweets, what would that represent as a percentage of his daily allowance of calories? Is this a good idea?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

ACTIVITY: Money for nothing?

ATL

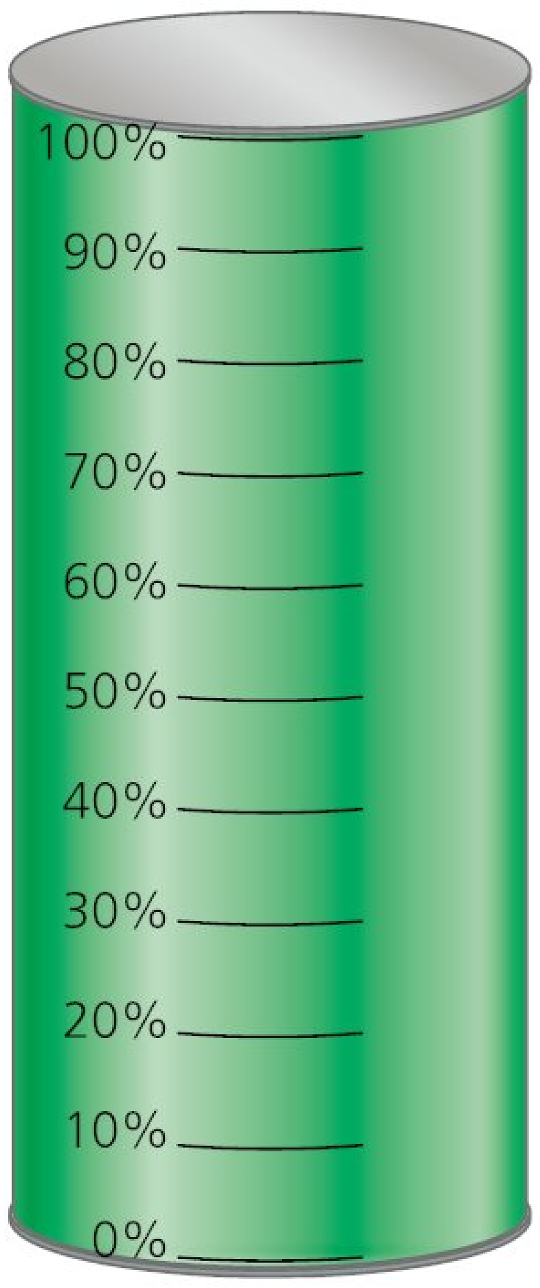
- Creative-thinking skills: Make unexpected or unusual connections between objects and/or ideas

Take a tube of potato crisps (chips) like those shown in the picture above. Notice that there are some empty spaces in the tube that are not filled with crisps. Discuss why this might be. How much empty space do you think there is? Do you and your partner have different predictions? Discuss and decide on a final answer together.

Remove the crisps, place them in a plastic food bag, seal and set aside.

Mark the outside of the tube (or the inside, but this is more difficult) with percentages. Comment on the accuracy of your markings. Did you mark increments (increases) of 10% or 25%, or something else?

Now crush the crisps inside the food bag, making sure not to lose any crumbs. Make the pieces as small as you can. Pour them back into the tube. Using your scale, estimate the percentage of the tube taken up by the crisp crumbs and the percentage of empty space in the container. What do you think now?



Hint

Don't waste the crumbs. If you don't want to eat them as crumbs, use them as a crispy coating or a topping for a meal.

Think now about the accuracy of your results. Did you lose any crumbs? How accurate are your observations?

Links to: Sciences

We often think about how to minimize experimental error in sciences. Instruments must be calibrated to ensure that they are accurate.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

Links to: PHE

Using your knowledge of healthy living and understanding of energy, together with your skills in Design, try to create informative and eye-catching food information labels. You might want to use the colour-coding system you came up with in the Take Action activity on the previous page.

PRACTICE EXERCISE

1 Which is bigger: 52.9% or 36.909%?

2 Which is bigger: 52% of 98 or 36% of 308?

3 A driving test has three stages. Learner drivers must score over 70% on each stage to pass. If Max's test scores are 11/20, 42/50 and 17/23, did he pass?

4 A teacher said she would postpone a test if 40% (or more) of the class is missing. If the class has 24 students, how many must be absent before she will cancel the test?

Can percentages go over 100%?

PERCENTAGE CHANGE

Often, we want to know a percentage of a quantity, but sometimes we want to know the final figure after the percentage has been added or removed. This is a **percentage increase** or **percentage decrease** and it is a very important concept to master.

Rita and Cliona worked at an airport tax refund office, where some travellers were entitled to claim back tax paid on their purchases. This is a real-life conversation that did actually happen!

Rita: There you are, sir. Your refund on 800 euro is 138.84.

Man: Wait … what? But the tax rate is 21%! I should be getting just over a fifth back! A fifth of 800 is 160 euro. I should be getting more than this!

Rita: Yes sir, the tax rate is 21% but that is on the original sum. Think of a pre-tax price of 100 euro; how much is the tax on that?

Man: 21 euro.

Rita: So the total would be 121 euro. Now if I gave you back 21% of 121 euro how much would that be?

Man (now has his phone in his hand): 25.41 euro.

Rita: So, I would be giving you back more tax than you actually paid, which wouldn't be right. The 21% is on the original sum, not the final. Actually, you get back roughly 17% of the final amount.

Man (looking a bit sad): Disappointing, but fair – I guess.

Cliona: Rita, I think you might end up being a Mathematics teacher!

To find the final amount (or price) after the percentage has been applied, you must add or subtract the percentage amount. If you want to give a 20% tip to an excellent waiter, you must first find 20% of the bill and then add it on to find the final amount. Likewise, if you had a '20%-off' voucher for a meal, you would find 20% of the cost and subtract it from the final bill.

There is an even quicker way to find the final amount, which eliminates the middle step. If we think of the final amount as a multiplier, we can find the final percentage in one step. If the tip of 20% brings the bill to 120%, then we can use this (or a decimal of 1.2) in our calculation. If we want the final amount after 20% has been taken off, we use 0.8 (or 80%) as the multiplier.

First, let's practise finding the multiplier. Copy and complete this table with the correct information.

Percentage	Increase or decrease?	Final %	Multiplier
5%	increase	105%	1.05
16%	decrease	84%	
26%	decrease		
91%	increase		
60%	decrease		
75%	increase		
2.5%	decrease		

Now let's use a multiplier from the table to find the total amount. A sale is taking place at a store where all damaged goods are being sold for 60% off. You see something for $36 that you want. How much will it cost after the discount has been applied?

60% off means 40% remaining, so the multiplier is 0.4.

$36 \times 0.4 = 14.4$

Notice how one letter makes a big difference?

60% **of** 36 = 21.6

But

60% **off** 36 = 14.4

PRACTICE EXERCISE

1. 45% off 150.
2. 15% increase on 500.
3. 15% decrease off 500.
4. 100% off 22.50.
5. 7% off 190.
6. 82% increase on 34.
7. 3.5% off 1.2 million.
8. Increase 40 by 50%.
9. 34% off 443.20.
10. If 15% of a number is 21, what is the number?
11. Find the number that, when decreased by 25%, gives you 16.

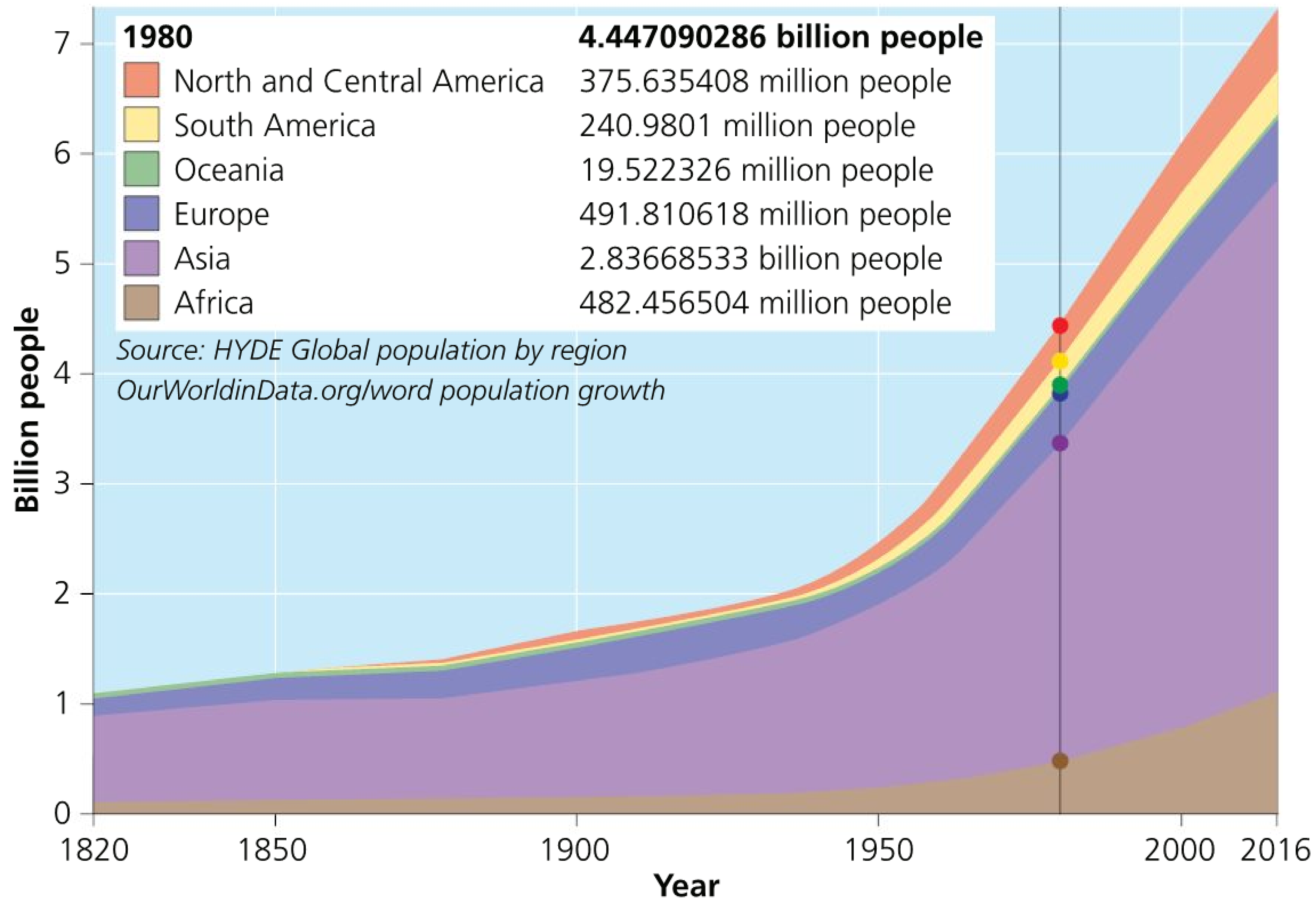
ACTIVITY: Ask the experts?

ATL

- Information literacy skills: Collect and analyse data to identify solutions and make informed decisions

1 The world population in 1980

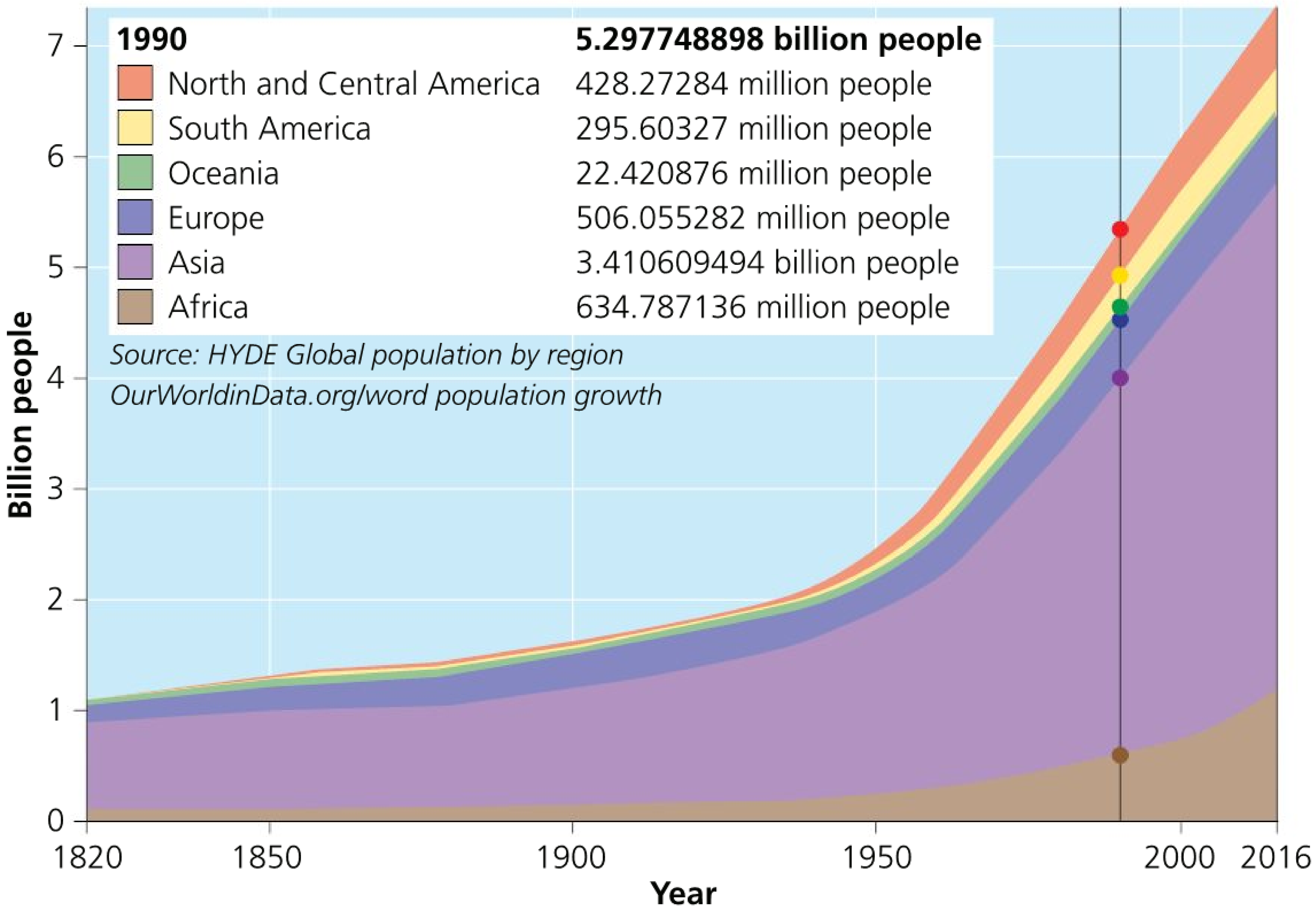
Some experts in 1980 predicted that population growth would be 3% over the following 10 years. This graph shows the world's population at that time, divided up by region.



1. Using the graph, write down the population of each world region for the year 1980.
2. If these experts were right, that the population would increase by 3% over the following 10 years, find out what the population increase would be for each region.
3. If these experts were right about the 3% increase, find out what the population **totals** would be for each region by 1990.
4. Comment on the accuracy of the 1980 data, using information from the graph or otherwise.

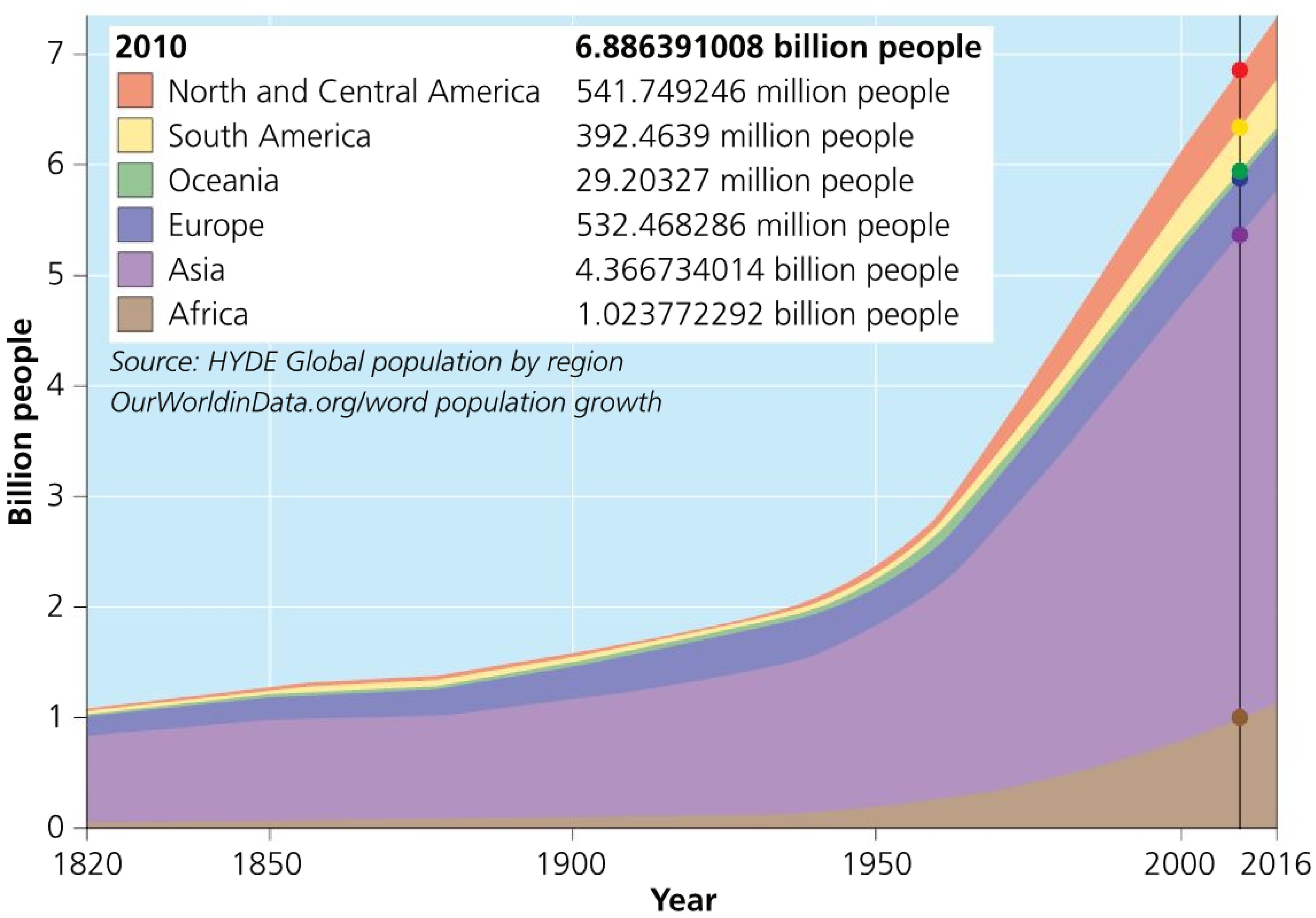
2 The world population in 1990

The actual data from 1990 gave this world population graph.



1. Using this graph, write down the population of each region for the year 1990.
2. Which regional populations increased or decreased between 1980 and 1990?
3. Find out how many more or fewer people there were in each region.
4. Calculate the percentage change in population by region for the years 1980 to 1990.
5. How accurate was the experts' prediction? Justify your answer with mathematical reasoning.
6. Based on your findings, predict the population growth from 1990 to 2010. Explain your reasoning.

3 The world population at 2010



1. Using this graph, write down the new population of each region for the year 2010.
2. Which regional populations increased or decreased?
3. Find out how many more or fewer people there were in each region.
4. Calculate the percentage change in population by region for 1990 to 2010.
5. How accurate was your prediction from Part 2? Justify your answer.
6. Compare the population growth over each of the three decades (1980 to 2010).
7. How accurate are your answers? Justify your opinion.

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Does money make the world go round?

ACTIVITY: Investing in the future

ATL

- Creative-thinking skills: Practise flexible thinking – develop multiple opposing, contradictory and complementary arguments

Imagine you are offered two different investments. One pays a constant amount every year; the other pays 10% of the value at the end of each year.

- **Investment A: ¥12 000 with ¥1000 added every year.**
- **Investment B: ¥10 000 with a 10% increase every year.**

You can invest the money for either three or five years – what is the best option? Explain your response, using mathematical reasoning.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

In business we depend on percentages to tell us if values are going up or down, if costs are rising, whether unemployment is falling and the exact amount of tax paid.

Some governments require their citizens to pay a high rate of tax on their earnings or purchases, others do not. Learn about the tax system in your country so that you know what to expect when you begin to work. Search for **income tax [name of your country]**.

ACTIVITY: Neither a borrower nor a lender be

ATL

- Communication skills: Give and receive meaningful feedback

Read the following scenario and answer the questions below.

Patrick lends Jacob 10 euro for a week on the condition that he pays him back 12 euro next week. If Jacob fails to pay, or forgets, then Patrick wants 14 euro the following week. When their parents hear this, they are not pleased with this deal.

1. **How much extra must Jacob pay in the first week?**
2. **What percentage is that on the original sum?**
3. **How much extra must he pay in the second week?**
4. **What percentage is that on the original sum?**
5. **What is this type of charge called?**
6. **Why might their parents be unhappy with this arrangement?**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

ACTIVITY: Complete the crossword, backwards!

ATL

- Creative-thinking skills: Create original works and ideas; Use existing works and ideas in new ways

A crossword is a game in which you solve clues and fit the answers into a fixed grid structure. This can be done with numbers as well as letters. The crossword here has already been completed but the clues have been lost!!!!

The only thing we know is that all of the clues were in the form of financial calculations, for example 'Find 10% of 32 410'.

Your task is to make up questions which give the answers that appear in the crossword, using any capital (amount) and any percentage increase or decrease that you want.

[1] 8	[2] 1	9	2.	9	[3] 9
	0				2
	[4] 5	1.	[5] 1	0	
	0.		2		[6] 5
	9		1		4
[7] 2.	2	1		[8] 7	3

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

Links to: Individuals and societies

'Neither a borrower nor a lender be' says the character Polonius in the play *Hamlet* by Shakespeare. What is the relevance of this quotation? What do you think it means? Do you agree or disagree with this statement?

In some cultures, and in many religions, money lending or profiting from finance is considered immoral or sinful. Research this idea to find examples of cultures or religions for which this is the case.

Search terms: **interest**, **usury**, **money lending**.

Listen to the following podcasts to find out more about the culture of money lending: **www.npr.org/2016/05/12/477758545/michigan-bank-discovers-the-need-for-islamic-finance-products** as well as **www.bbc.co.uk/programmes/p04kj3n5**

How do we simplify a ratio?

WHAT IS A RATIO?

A ratio shows the relationship between two quantities. It can show how an amount is shared between two or more parties, or explain how much bigger one quantity is compared to another. An example of a ratio might be a mathematics class in which the girls outnumber the boys 2 : 1. This tells us that there are two girls to every boy.

A ratio can also be considered as a way to express a fraction. The mathematics class has two-thirds girls and one-third boys. We can find the ratio by expressing the fractions compared to one another:

$\frac{2}{3}:\frac{1}{3} \qquad = 2:1 \qquad$ girls : boys

HOW DO I RECOGNIZE A RATIO?

The first thing to notice is that a ratio uses a **colon** (:) to separate the quantities or shares. Ratios are closely linked to the important idea of sharing, or **partition**. For example, 'I get two parts of the whole for every one that you get'.

Another way to recognize a ratio, when expressed in words, is the use of 'to' or 'for every'. In fact, whenever you hear these words in everyday spoken English, the person is using a ratio! Examples include:

- 'What is the teacher-**to**-student ratio at this school?'
- 'Ancient Romans got a day off **for every** two days they worked.'
- 'Our team scores three goals **to** every two of yours.'
- 'He completes four levels on that game **for every** one that I manage!'

In this computer game image, we can see there are 6 red counters, 6 yellow, 6 green, 4 blue, 5 purple and 5 pink. We can see the ratio of different colours relative to each other: 6 : 6 : 6 : 4 : 5 : 5

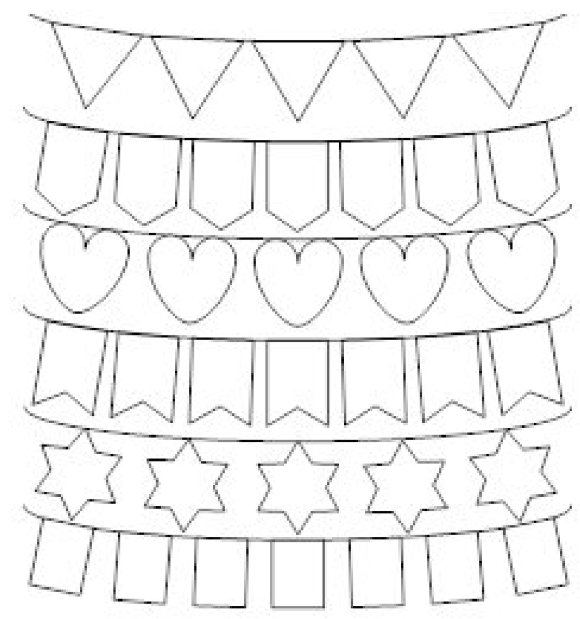
How could this be expressed as fractions? What are these fractions in their simplest form?

We can also look at the ratio of pairs of colours. The ratio of red **to** yellow is 6 : 6. It might also be obvious to you that this can be simplified to 1 : 1. There is one red counter **for every** yellow one.

ACTIVITY: Raise the ratio flags

ATL

- Creative-thinking skills: Apply existing knowledge to generate new ideas, products or processes



Colour in one set of flags (bunting) for each of the following ratios. You can choose any bunting for any ratio, but some will be trickier to use for some ratios than for others!

- **The ratio of red to pink is 2 : 3.**
- **The ratio of green, white and orange is 2 : 3 : 2.**
- **The colours of the bunting (flags) are in the ratios 1 : 4 for white and brown.**
- **Yellow, black and blue flags in this ratio 3 : 1 : 1.**
- **The flags of one bunting are in the colours of the rainbow in an equal ratio.**
- **Light blue to dark blue is in the ratio 3 : 4.**

How did you lay them out? In a pattern or randomly?

Assessment opportunities

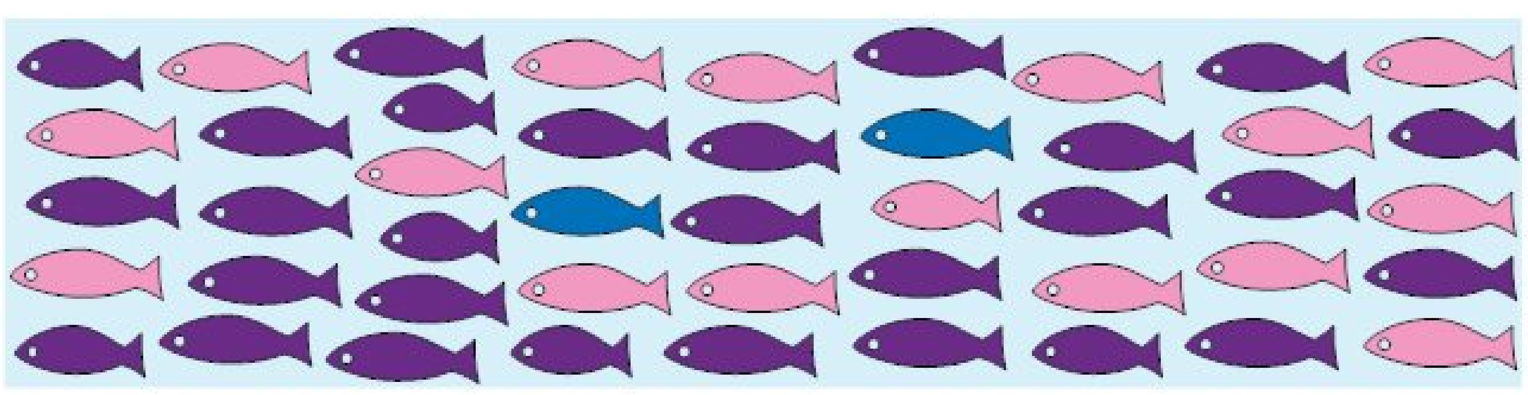
- In this activity you have practised skills that are assessed using Criterion C: Communicating.

ACTIVITY: Just keep swimming …

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

Look at this sample of wallpaper and answer the questions below.



1. **How many pink fish are there?**
2. **How many purple fish are there?**
3. **How many blue fish are there?**
4. **What is the ratio of blue to purple fish?**
5. **What is the ratio of pink to purple fish?**
6. **What is the ratio of blue to pink fish?**
7. **What is the ratio of purple to blue to pink fish?**
8. **If this had been a scientific investigation into a population of differently coloured fish, what observations could you make, based on the ratios?**

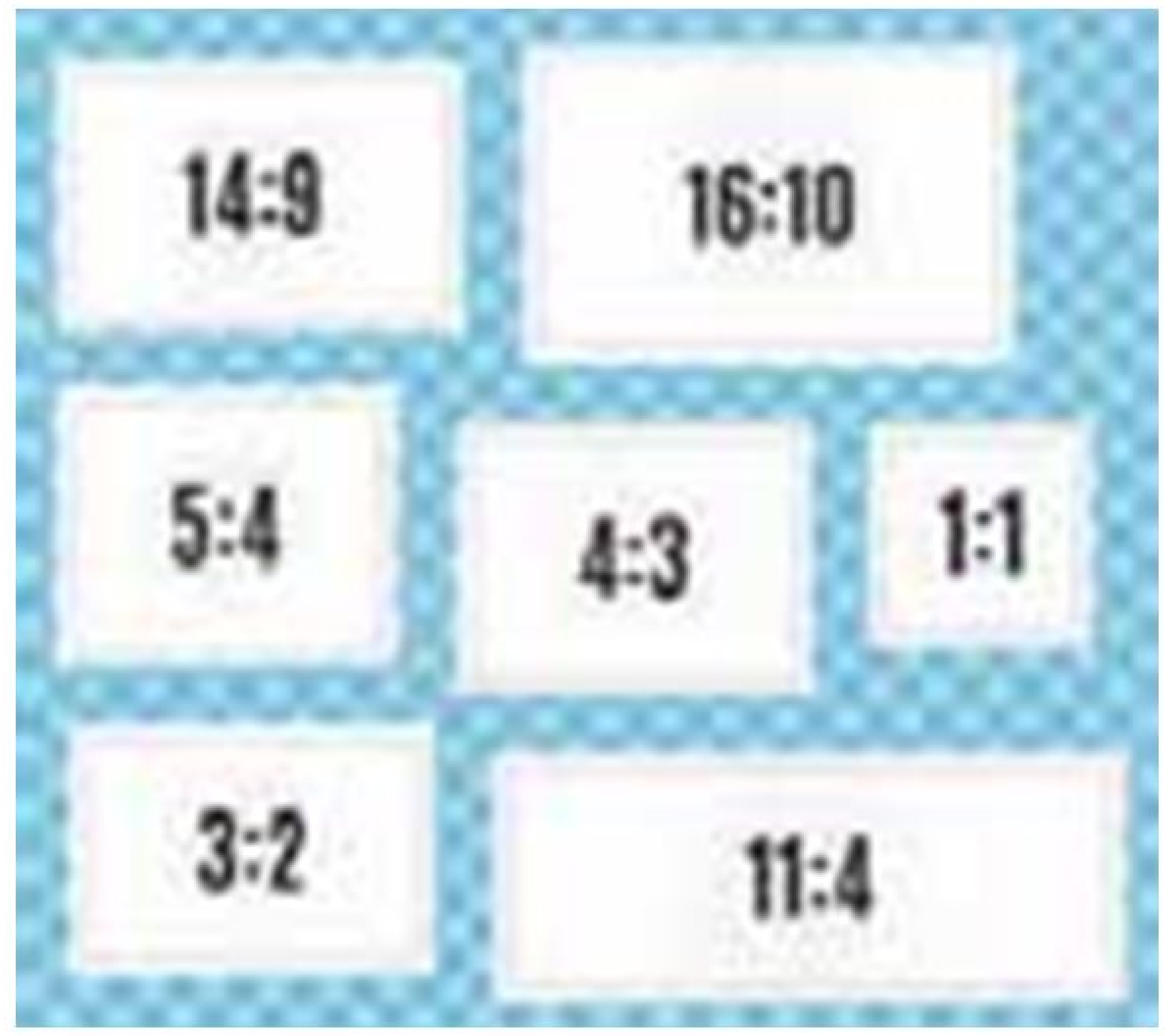
Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

Where else do we see ratios in real life? There are lots of examples of ratios, especially when talking about money and food: sharing out prize money or paying a fair share of a bill; scaling recipes up or down, or comparing the quantities of the ingredients. The probability of a certain event happening can also be quoted as a ratio: there is a 50 : 50 (50/50) chance it will rain tomorrow. The colon and the slash are interchangeable in this type of ratio.

THINK-PAIR-SHARE

Here are some more ratios in a real-world situation. What do you think they represent? What do ratios have to do with this context? Have you seen these ratios anywhere before? (Hint: think entertainment.)



HOW DO WE SIMPLIFY RATIOS?

Ratios are often expressed in their simplest form as the best way to indicate the relationship between the proportions. This is similar to the way in which we simplify fractions.

A ratio of 4 : 12

can be simplified to 1 : 3

by dividing both portions by four, as four is the **highest common factor**. This is the same process we use to simplify:

$\frac{4}{16}$ to $\frac{1}{4}$ and $\frac{12}{16}$ to $\frac{3}{4}$

A ratio of 5 : 7 cannot be simplified further, which is the same as being unable to simplify $\frac{5}{12}$ or $\frac{7}{12}$ any further.

PRACTICE EXERCISE

Simplify these ratios.

1	2 : 8	4	4 : 18	7	2000 : 60
2	8 : 2	5	4 : 20	8	17 : 51
3	3 : 30	6	44 : 11	9	11 : 121

ACTIVITY: Proportional reasoning, for a good reason

ATL

- Transfer skills: Change the context of an inquiry to gain different perspectives

Bermy Cafe

BREAKFAST

SIDES

Home Fries (Potatoes)	$3.00
Stir Fried Home Fries (Red Peppers, Onions, Parsley)	$4.75
English Beef Sausages or Turkey Bacon	$3.25
Pork Bacon or Pork Sausages	$3.00
Baked Ham, Turkey or Beef	$5.25
Italian Sausages	$5.25

OMELETTES

Plain	$6.75
Cheese	$7.75
Ham and Cheese	$9.75
Chourico	$9.75
Mushrooms	$9.00
Mushrooms and Cheese	$9.75
Mushrooms and Green Peppers	$9.00
Western	$9.00

We serve egg whites only for $2.00 extra. Just ask!

PANCAKES

Single Pancake $3.00 Single Flavoured Pancake	$4.00
Plain Pancakes	$6.00
Banana Pancakes	$8.25
Blueberry Pancakes	$8.25
Chocolate Chip Pancakes	$8.25

BREAKFAST

Hungry Breakfast $14.75
Two eggs, two pancakes, sausages, bacon, hash browns, OJ, Tea or Coffee.

Salmon Breakfast $18.75
Three eggs (any style), smoked salmon, cream cheese and a bagel, Tea or Coffee.

Full English $17.75
Two eggs (any style), English bacon, one English sausage, black pudding, baked beans, grilled tomatoes, fried bread & mushrooms.

Pancake Breakfast $15.75
Three eggs (any style), bacon or sausages, two mini pancakes & sliced banana.

Codfish and Potato Breakfast $15.75
Salt cod, potato and sherry pepper stew. Served with boiled egg, banana and avocado.

Special Beans $13.75
Two eggs (any style), turkey bacon & pork beans. Served with sliced homemade bread.

Bren and Alex ordered a meal of four dishes from this menu, one of which was Stir Fried Home Fries and another was a cheese omelette.

The bill came to $38.50. What else did they (probably) order? How can you tell? How should they split the bill?

Bren says that Alex ate way more than he did and should pay twice as much. What ratio does that mean? How much would each person pay?

Alex argues that, while he did eat *some* more, he is a vegetarian and Bren's meat dish cost a lot more than any of the others. What does this mean for the ratio? How much would each person pay then?

What would you recommend they do? Justify your answer. Does this make sense in the context of the real-world problem?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

DISCUSS

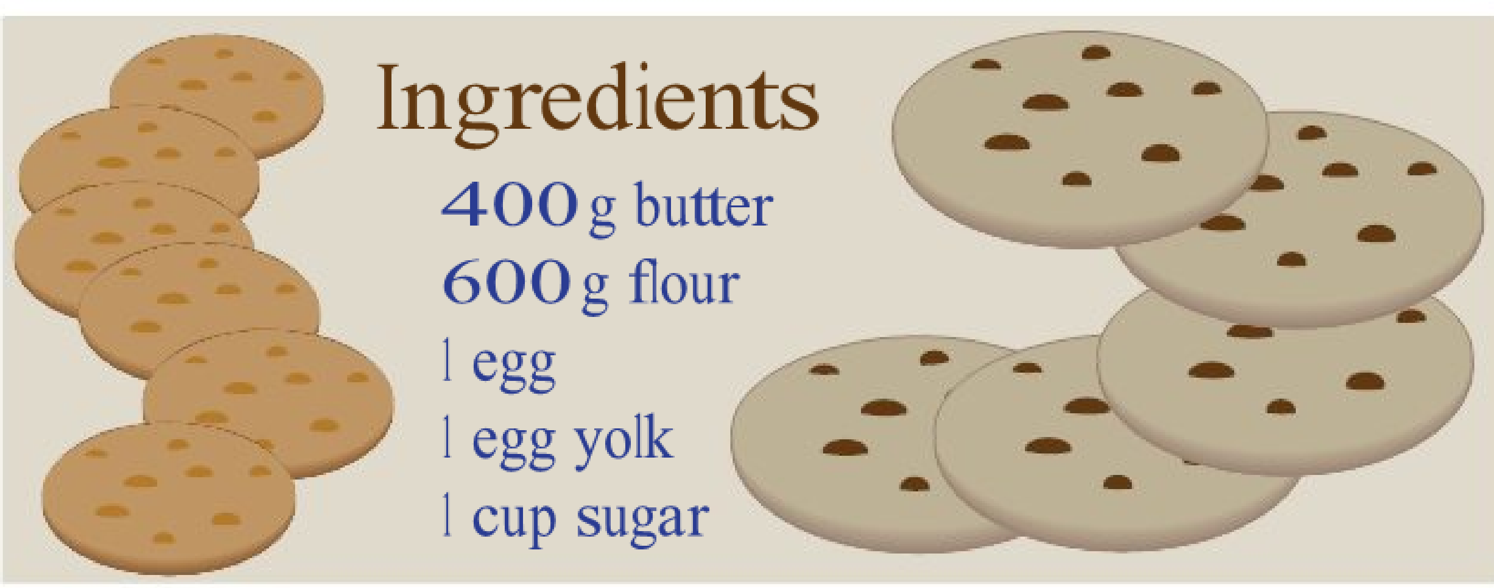
Maggie has three children – Mary, John, and Jimmy – who each have their own families. Maggie leaves an inheritance of \$200 000 for the education of her grandchildren. One family has an only child, another family has two children and the last has five children. In what ratio should the money be divided? Justify your answer.

ACTIVITY: Ratios in recipes

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

Sometimes students think of ratios like little recipes – instructions on how to get the quantities right.



Here you can see another simple baking recipe, this time for cookies. This recipe makes 10 cookies. You may need different numbers of cookies for different occasions and, as a global citizen, you don't want to waste ingredients. You want to cook in a more sustainable way.

1 **What is the ratio of butter to flour in the recipe?**
2 **How many eggs are required for 20 cookies?**
3 **How much butter is needed to make 5 cookies?**
4 **How much butter, flour and sugar would you need to make exactly a baker's dozen (13)?**
5 **Why would it be very difficult to make 7 cookies?**
6 **If you have 1.2 kg of butter, what is the maximum number of cookies you can make?**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

PRACTICE EXERCISE

Divide ₫120 (Dong) between two people, so that the following statements are true. In addition, give the ratio in each case.

1 Both people have the same amount.
2 One has twice as much as the other.
3 One has five times more than the other.
4 One has seven times less than the other.

Planning a dinner party? You should really watch this first: **www.stem.org.uk/elibrary/resource/30812**

A popular drink in Germany is Spezi, which is a mixture of cola and orange soda in the ratio of 1 : 1. What fraction of each bottle of Spezi is made of orange?

Andy visits a store and realizes that a 1 L bottle of Spezi costs 2.19 euro, a 1 L bottle of cola costs 1.19 euro and a 1 L bottle of orange costs 0.79 euro. What should he do? Why?

He purchases one bottle of cola and one of orange. On returning home he finds that he has three 1 L bottles of cola already and one litre of orange. How many litres of Spezi can he make? How many litres of drink will he have in total? If each of his glasses holds 250 mL, how many glasses can he fill? If he is having a party with eight guests, how many glasses can each of them have?

Let's take some time out to play with proportions. Visit **https://phet.colorado.edu/en/simulation/proportion-playground** to see how you can play with ratios in the real world.

ACTIVITY: Can ratios shed light on the problem?

ATL

- Information literacy skills: Understand and use technology systems

Colours on a computer are composed by 'mixing' primary colours. You can create any shade by changing the **ratio** of the three primary colours – green, red and blue.

When you change the colour of the text in many programs, including Microsoft Word or Paint, you can choose from a palette of colours like the one shown in the image.

How many colours do you have to choose from in the colour **hexagon**? How many colours can you choose from in total?

What if you want a different colour or shade that isn't shown here? You can create your own colour by clicking the 'Custom' tab. This tab gives you a much broader range to choose from and allows you to change the quantity of each primary colour in the virtual mix.

If we look at pure red, we can see the ratio of the numbers in the Red, Green and Blue boxes.

We can tell that the ratio is 255 : 0 : 0 between R : G : B

The highest value that can be entered for any colour is 255. 0 means **no trace** of that colour and 255 means that it is set to a maximum.

The ratio of 104 : 99 : 49 will result in an olive green.

Open a Microsoft Word document and type your name in. Experiment with the ratios to create colours by numbers. Find the ratio that gives you your favourite colour or colours.

Predict and record what colour you think each of these ratios will give. Then write down the true colour specified by the ratio.

- **91 : 109 : 165**
- **250 : 6 : 157**
- **20 : 97 : 166**
- **255 : 175 : 130**
- **0 : 255 : 255**
- **255 : 255 : 255**
- **0 : 0 : 0**

What does the last ratio tell you about the colour black? What does the second last ratio tell you about the colour white?

Copy and complete the following colour equations, using the ratio calculator on the custom palette, to test your answers:

(Full) Red + (Full) Blue = ________

Blue + Green = ________

Green + Red = ________

Links to: Sciences

Thinking about primary colours and light, were any of these results surprising? Why or why not? Does the information match with what you learned in Art? You can find out more on page 100 of *Sciences for the IB MYP 2: by Concept*.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

How do we reason with ratios?

MEET A MATHEMATICIAN: EUGENIA CHENG

Learner Profile: Communicator, Knowledgeable

Dr Cheng is Scientist in Residence at the School of the Art Institute of Chicago. Despite her title she is a mathematician, author and musician, as well as an excellent communicator on the joy and excitement of mathematics. Her book *How to Bake Pi* was published to widespread acclaim in 2015 and her videos have millions of views to date. In *How to Bake Pi*, each chapter begins with a recipe for a dessert, to illustrate the common threads in the methods and principles of mathematics and cooking. Eugenia's infectious love of explaining mathematics through cooking shows that she is knowledgeable about complex areas of pure mathematics, but can also communicate them clearly through the ideas of food and fun.

Watch these videos to see Dr Cheng in action:
www.youtube.com/watch?v=mA402F5K47o

www.tedxvienna.at/watch/what-if-mathematics-is-the-answer-for-progress-eugenia-cheng-tedxvienna/

https://twitter.com/hotelemc2/status/870367248229314564

https://t.co/Mp2mOgcqjZ

■ Dr Cheng uses mathematics to prove the perfect ratio of scone, jam and clotted cream, as well as to predict the proportions of the perfect doughnut.

What does Dr Cheng's work have to do with ratios?

> 'The secret to the "perfect" doughnut has been mathematically proven – and apparently it's all to do with the size of the hole.
>
> Dr Eugenia Cheng, Senior Lecturer of Pure Mathematics at the University of Sheffield, used calculus to find what gives the confectionery that can't-resist appeal. Dubbed the "Squidgy to Crispy ratio", her method proves that the smaller the hole in the middle of the doughnut, the squidgier it is, while a bigger hole will make the doughnut crispier.'

Source: **www.telegraph.co.uk**

Look at the doughnut poster at this link:
http://ow.ly/3mv530fZRMW

What does Dr Cheng mean by the 'squidgy to crispy ratio'? Do you agree with her findings? What might the letter r in the formula mean? Carry out a sample calculation, using a value for r that was either chosen by you or inspired by something mentioned on the poster. The poster shows the ratio in the form of two percentages; rewrite them as a ratio. Simplify if possible.

PRACTICE EXERCISE

Kristal is saving for her first car by working a part-time job. Her mom has agreed that for each \$90 CAD Kristal earns, she will give her \$10 to make an even \$100.

1 What is the ratio of Kristal's money to her mom's share?

2 If the car she wants costs \$2000, how much will Kristal have earned and how much will her mom have paid?

3 If her mom ends up paying \$360, how much must Kristal have saved?

4 If Kristal earns (and saves!) \$4050 over the year, what is the most expensive car she could afford?

ACTIVITY: Do you need to know about ratios to share and compare?

ATL

- Information literacy skills: Collect, record and verify data

Look at the extract from www.eurotopfoot.com shown below and answer the following questions.

1 **Considering the ratio of points to goals, how can you tell which are the top five leagues?**
2 **Identify one player who plays in a league which is not in the top five.**
3 **Identify a country whose league is not in the top five.**
4 **Would the top five change if there was no coefficient or ratio applied?**
5 **Would the top ten change if there was no coefficient or ratio applied?**
6 **Is this a fair method of comparison?**
7 **How accurate are these data?**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

www.eurotopfoot.com

Since the 1996–97 season, European Sports Media have awarded the Golden Shoe based on a points system that allows players in tougher leagues to win even if they score fewer goals than a player in a weaker league. The weightings are determined by the league's ranking on the UEFA coefficients, which in turn depend on the results of each league's clubs in European competition over the previous five seasons. Goals scored in the top five leagues, according to the UEFA coefficients list, are multiplied by a factor of two, goals scored in the leagues ranked six to 21 are multiplied by a factor of 1.5, and goals scored in leagues ranked 22 and below are multiplied by a factor of 1. Thus, goals scored in higher ranked leagues will count for more than those scored in weaker leagues.

Rank	Scorer	Team	League	Goals	Ratio	Points
1	L. Messi	FC Barcelona	ESP	37	2.0 : 1	74.0
2	B. Dost	Sporting CP	POR	34	2.0 : 1	68.0
3	P.E. Aubameyang	Borussia Dortmund	GER	31	2.0 : 1	62.0
4	R. Lewandowski	FC Bayern Muenchen	GER	30	2.0 : 1	60.0
5	E. Dzeko	AS Roma	ITA	29	2.0 : 1	58.0
6	H. Kane	Tottenham Hotspur	ENG	29	2.0 : 1	58.0
7	L. Suarez	FC Barcelona	ESP	28	2.0 : 1	56.0
8	D. Mertens	SSC Napoli	ITA	28	2.0 : 1	56.0
9	E. Cavani	Paris Saint Germain	FRA	35	1.5 : 1	52.5
10	A. Belotti	Torino	ITA	26	2.0 : 1	52.0
11	C. Ronaldo	Real Madrid	ESP	25	2.0 : 1	50.0
12	R. Lukaku	Everton	ENG	25	2.0 : 1	50.0
13	A. Modeste	1.FC Koeln	GER	25	2.0 : 1	50.0
14	G. Higuain	FC Juventus	ITA	24	2.0 : 1	48.0
15	M. Icardi	FC Internazionale	ITA	24	2.0 : 1	48.0
16	A. Sanchez	Arsenal FC	ENG	24	2.0 : 1	48.0
17	C. Immobile	SS Lazio	ITA	23	2.0 : 1	46.0
18	T. Werner	RB Leipzig	GER	21	2.0 : 1	42.0
19	A. Lacazette	Olympique Lyonnais	FRA	28	1.5 : 1	42.0
20	Diego Costa	Chelsea FC	ENG	20	2.0 : 1	40.0

Source: www.eurotopfoot.com

PRACTICE EXERCISE

This table shows the total number of streams for the top 16 songs on the week of 19 May 2017. For this real-world context, we will consider only these top 16 songs in our totals and problem solving.

	Track	Streams
1	Despacito – Remix by Luis Fonsi	52 724 885
2	I'm the One by DJ Khaled	40 760 183
3	Shape of You by Ed Sheeran	33 627 391
4	HUMBLE. by Kendrick Lamar	29 024 887
5	Mask Off by Future	22 902 602
6	Something Just Like This by the Chainsmokers	22 563 139
7	Stay (with Alessia Cara) by Zedd	21 881 484
8	It Ain't me (with Selena Gomez) by Kygo	21 843 638
9	Malibu by Miley Cyrus	21 397 603
10	Swalla (feat. Nicki Minaj & Ty Dolla $ign) by Jason Derulo	20 735 538
11	Despacito (Featuring Daddy Yankee) by Luis Fonsi	20 603 598
12	That's What I Like by Bruno Mars	20 591 664
13	There's Nothing Holdin' Me Back by Shawn Mendes	20 397 064
14	Symphony (feat. Zara Larsson) by Clean Bandit	19 219 377
15	Passionfruit by Drake	18 817 615
16	Sign of the Times by Harry Styles	18 023 563

1 Identify the most popular song.

2 Identify the most popular artist (singer or band).

3 If 'Stay' by Zedd is played another 3 million times, what place would it move to?

4 How many times was the most popular song streamed?

5 How many times were the top 16 songs streamed in total?

6 What fraction was the most popular song of the total streamed songs?

7 What was the streaming of 'Symphony' by Clean Bandit as a percentage of the total number of streams?

8 Express the exact ratio of Ed Sheeran streams to Drake streams. Comment on this ratio.

9 Express Shawn Mendes' share of the total plays as a decimal. Comment on what this tells you.

10 If there is a 24% increase in the streams of 'Mask Off' by Future, how many streams would it then have?

11 How accurate do you think these figures are?

12 Estimate how many more times 'I'm the One' by DJ Khaled was played for every time 'That's What I Like' by Bruno Mars was streamed.

13 How accurate is this answer?

14 Comment on the popularity of 'Despacito' compared to the other songs in the list. Use proportional reasoning in your answer.

SUMMATIVE ASSESSMENT: Coffee craze

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

Congratulations! You have been hired for your first part-time job at a new café in Edmonton! When your boss gives you the menu, you realize that you will have your hands full getting to know all of the different espresso-based drink recipes.

EXTENSION

The global context for this chapter is Globalization and sustainability. Research **Fairtrade coffee** to find out how coffee can be responsibly and ethically grown and sold.

Simplify the fractions where applicable.

1 In the café crème
- **a** Calculate the fraction of the drink that is heavy cream.
- **b** Calculate the fraction of the drink that is espresso.
- **c** Identify the inequality of the fractions using symbols. (Which is larger?)
- **d** What is the ratio of heavy cream to espresso?
- **e** Can you find another drink that has the same ratio, but different ingredients?

2 In the **mocha**
- **a** Calculate the fraction of the drink for each of the three ingredients.
- **b** What is the ratio of the ingredients?
- **c** Mr Jones visits your café and orders three mochas. In order to save time, you can make all three simultaneously. Scale the ratio of ingredients to ensure you have enough of each to make all three drinks properly.

3 Your boss decides to sell another size for all espresso drinks that is 40 mL, rather than 60 mL. He calls this size a 'mini'.
You are asked to make two **latte minis**. If you prepare 80 mL of espresso, 320 mL of steamed milk, and 3 mL of foamed milk, is this still a latte? Explain your reasoning.

The menu

Global context: Growing and consumption of coffee

In the coffee market, farmers receive a sum of money from a handling company. This company sells the coffee to the market (for example in the USA, UK and Europe).

The Fair Trade market for goods ensures that farmers are paid a fair salary that sustains their everyday living, as well as money to go towards supporting the community.

The following costs are per 100 kg of raw coffee beans.

	Farmers receive	Company sells	$ given back to coffee farms and community
Regular coffee	\$203	\$210	\$0
Fair Trade coffee	\$200	\$215	\$10

Your task

The school is thinking of changing their coffee supplier. What would you recommend they do, and why?

Hint

Consider the perspectives of the farmers and the school.

Things to remember in your answer:

1. **Use mathematical reasoning.**
2. **Comment on fairness.**
3. **Comment on the accuracy of your answer.**
4. **How true to real life do you think this activity is?**

Activity by Shannon Walsh and Huw Jones.

Reflection

<table>
<tr><th colspan="6">Use this table to reflect on your own learning in this chapter.</th></tr>
<tr><td>Questions we asked</td><td>Answers we found</td><td colspan="4">Any further questions now?</td></tr>
<tr><td>Factual: How do we convert fractions to decimals, and vice versa? What is the difference between percentage ‘of’ and percentage ‘off’? What is a ratio? How do we reason with ratios?</td><td></td><td colspan="4"></td></tr>
<tr><td>Conceptual: What is the relationship between fractions and decimals? How do we simplify a ratio? How do we share quantities in a given ratio? Can ratios shed light on problems?</td><td></td><td colspan="4"></td></tr>
<tr><td>Debatable: What is the best way to find the percentage of something? Can percentages go over 100%? Does money make the world go round? Is money good or bad, neither or both?</td><td></td><td colspan="4"></td></tr>
<tr><td rowspan="2">Approaches to Learning you used in this chapter:</td><td rowspan="2">Description – what new skills did you learn?</td><td colspan="4">How well did you master the skills?</td></tr>
<tr><td>Novice</td><td>Learner</td><td>Practitioner</td><td>Expert</td></tr>
<tr><td>Creative-thinking skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Communication skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Transfer skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Information literacy skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Learner Profile attribute(s)</td><td colspan="5">Reflect on the importance of being knowledgeable and a good communicator for your learning in this chapter.</td></tr>
<tr><td>Knowledgeable</td><td colspan="5"></td></tr>
<tr><td>Communicator</td><td colspan="5"></td></tr>
</table>

Form | Systems, Patterns | *Fairness and development*

2 Fact or fiction, truth or lies?

Fair forms of communication help us to reveal patterns and improve our truth-telling systems.

CONSIDER THESE QUESTIONS:

Factual: How do we get our hands on data? How do we organize data? What is a reality check?

Conceptual: What systems exist for measuring? In what forms can we represent data? How do we find patterns in data?

Debatable: How do we handle results fairly? How do we know what to trust? Do we need fact checkers?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

KEY WORDS

bias	in-built
city council	source
context	trend
credible	

IN THIS CHAPTER, WE WILL …

- **Find out** about traditional and innovative ways to collect, represent and analyse data.
- **Explore** the challenging world of fake news, fact checking and data detective work.
- **Take action** by informing yourself about voting and voting rights, especially in school and local affairs now, and in national and international affairs for the future.

These Approaches to Learning skills will be useful …

- Communication skills
- Transfer skills
- Information literacy skills
- Critical-thinking skills
- Collaboration skills

Assessment opportunities in this chapter:

- Criterion A: Knowing and understanding
- Criterion B: Investigating patterns
- Criterion C: Communicating
- Criterion D: Applying mathematics in real-life contexts

We will reflect on these Learner Profile attributes …

- **Principled** – We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.
- **Risk-taker** – We approach uncertainty with forethought and determination; we work independently and co-operatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how primary and secondary data are defined
- the difference between categoric (qualitative) and numeric (quantitative) data
- the differences between bar charts, pie charts, histograms, line graphs and scatter plots
- the most appropriate chart or graph to use for different types of data.

'**Lies, damned lies, and statistics**' is a phrase thought to have been first used by the British Prime Minister, Benjamin Disraeli. It is used to highlight the persuasive power of numbers, particularly when trying to strengthen weak arguments. It is also often used to cast doubt on the statistics that prove an opponent's point.

Although the phrase is attributed to Disraeli, it is not found in any of his works and its earliest known appearance was years after his death. It has also been attributed to many other people, often wrongly to Mark Twain, who popularized the term in the United States.

In this chapter, we will be dealing with collecting, organizing and representing data to see patterns or trends. Later, in Chapter 5, we will look at 'number crunching', that is finding measures of spread and tendency, such as mean, mode, median and range. In this chapter we will talk about the accuracy of data, as well as causes for error or bias.

ACTIVITY: Match up!

ATL

- Communication skills: Use and interpret a range of discipline-specific terms and symbols

First, let's start with some prior knowledge. How much statistical vocabulary can you remember?

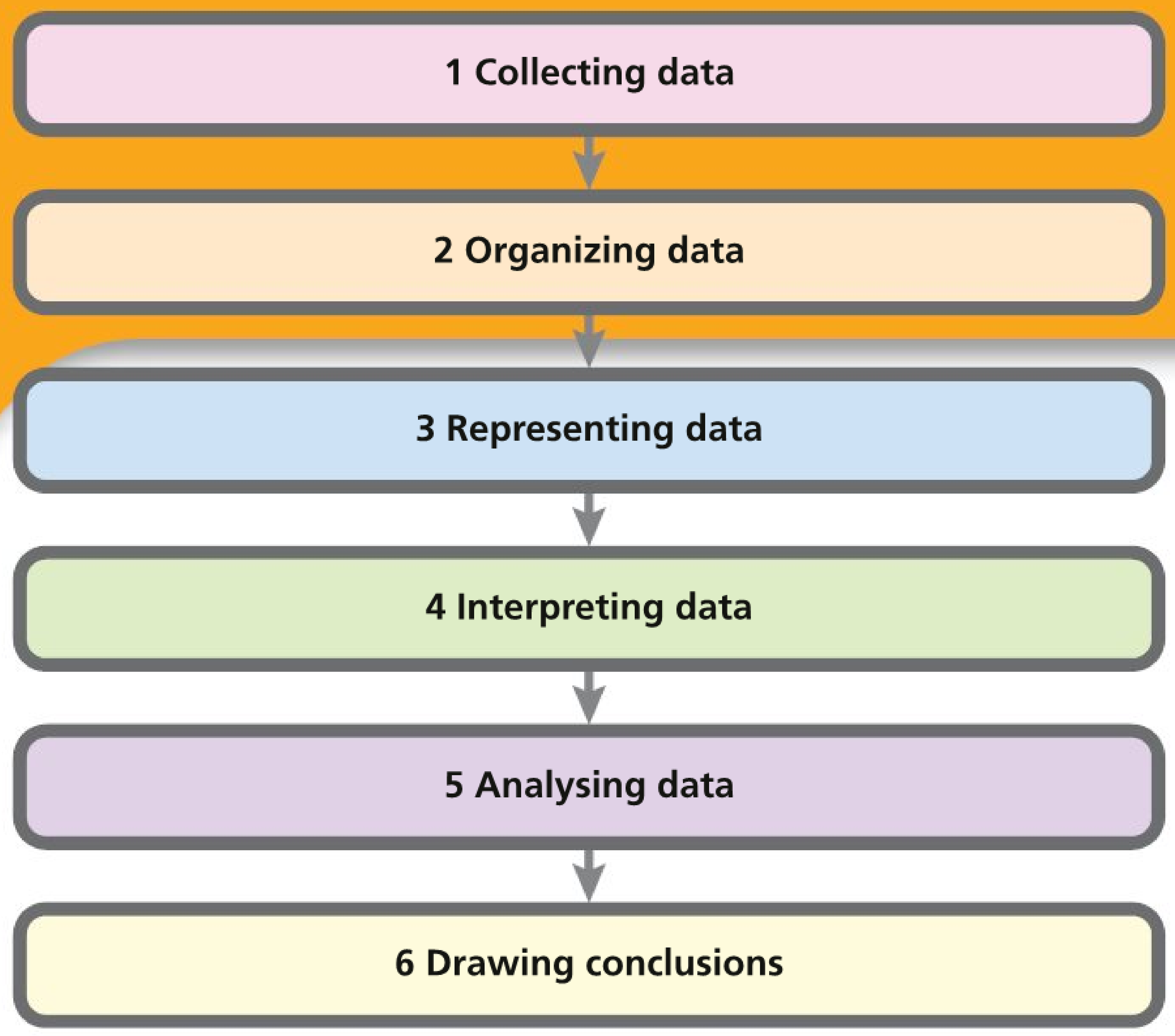
Match each data term to the correct definition.

Datum	a collection of pieces of information or quantities
Data	data that we have collected and checked ourselves
Primary data	a single piece of information or a quantity
Secondary data	this refers to the data we are measuring or studying
Outliers	data that do not follow, or that lie outside, the general trend
Quantitative (numeric) data	data that have been collected by someone else
Qualitative (categoric) data	a single variable that we wish to study
Variable	data that have numeric values
Uni-variate data	two variables that we wish to study at the same time to discover whether there is a relationship
Bi-variate data	data without numeric values, but which have descriptive values
	the data we are measuring or studying

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion C: Communicating.

How do we get our hands on data?



WHERE DO WE BEGIN?

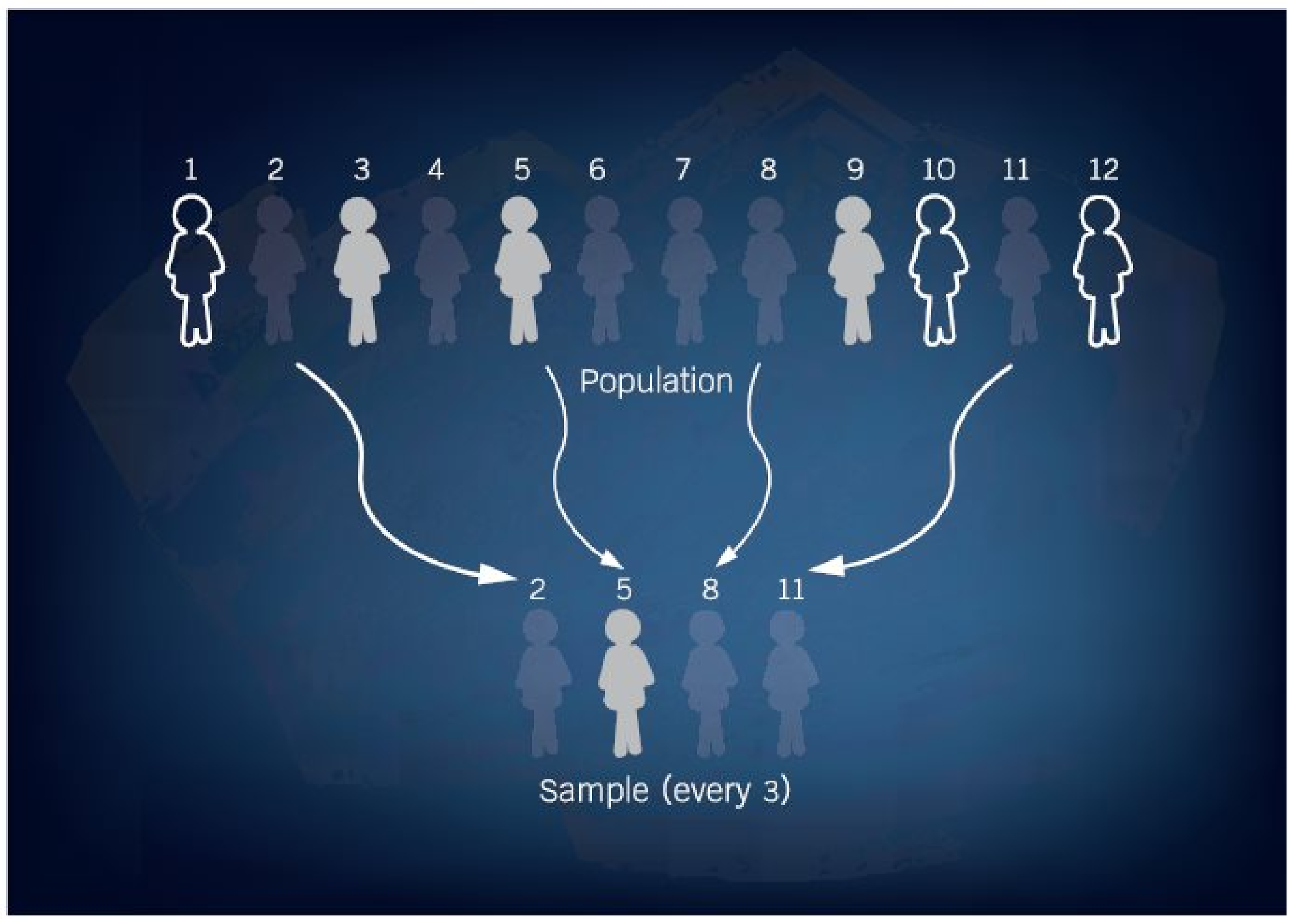
You might remember, from *Mathematics for the IB MYP 1*, that the process of carrying out an investigation with data can be shown in a flowchart like the one opposite.

If our first step involves collecting data, what does that mean exactly?

An individual point of data is called a **datum**. The plural for datum is data, so make sure that when you say data, you follow it with a plural form of a verb; for example, 'the data are highly accurate', 'irrelevant data were collected', 'the data tell us that the trend is increasing'.

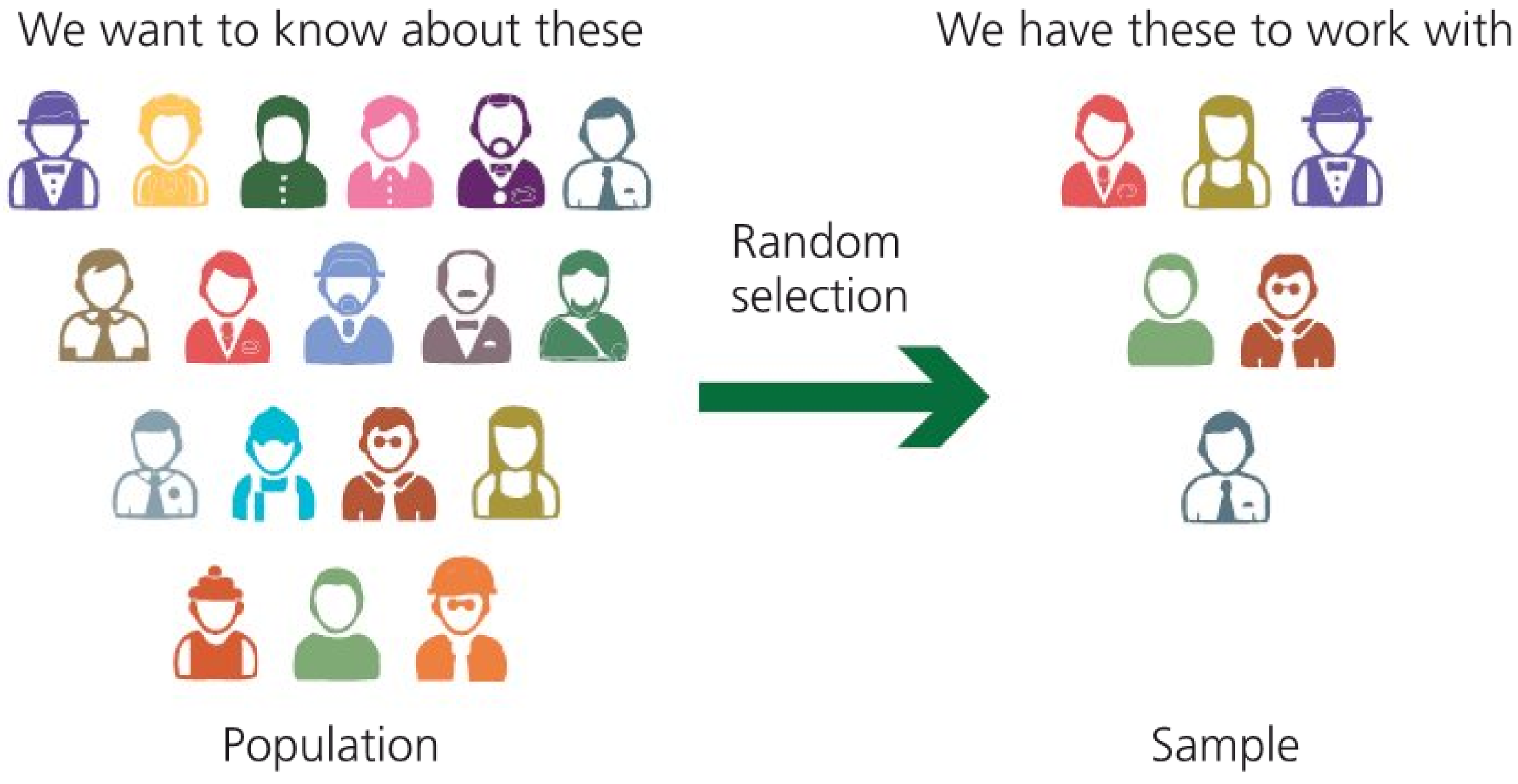
'**Population**' in statistics means the total **set** of all of the measurements that can be made. If we are studying the most popular music types among teenagers, then the population is all of the teenagers in the world. If we were studying the movements of sharks, then the population would be all of the sharks in the oceans and in aquaria.

Surveying an entire population can be too expensive, too difficult, too time-consuming or just downright impossible. Imagine trying to ask every teenager about music or to track every shark! To make the research possible, we often use a method called **sampling**. This means we ask a smaller group, or a **subset** of the population. We can ask this subset and try to make observations or identify patterns that apply to the whole population.



In this image, we can see that a sample of 4 people was taken from 12 people. This was done in a planned way as every third person was selected.

Sampling can be planned or strategic, in this ordered way, or it can be random. Sometimes people do not want to answer questions and you have to work with what you have got. The relationship between the sample and the population is a vital one.



So, once we have chosen a sample, whether it is random or strategic, how do we collect the data? Useful data can be collected through **observation** or **experimentation**. It may be that we are carrying out an experiment, and so making lots of measurements. Alternatively, we might be seeking feedback by asking for people's answers or opinions directly. We can count the **frequency** of objects, quantities and responses; this means we record how many times they occur.

WHAT OTHER THINGS SHOULD WE CONSIDER?

When we survey people for opinions or personal details, we must think carefully about these questions.

- What is **realistic**? How many people can we reach? How many people are likely to help us find the data we need or to answer our questionnaire?
- What is **affordable**? How much time or money can we spend on this survey? How long will the data collection take? How many people will we need in order to carry it out?
- What is **fair**? How can we make sure that our questions are not biased? How can we avoid our own opinion influencing the question or possible answers? How can we avoid questions that just confirm what we suspect?
- What is **ethical**? Are people likely to share this information with us? Are they worried about how it will be used?

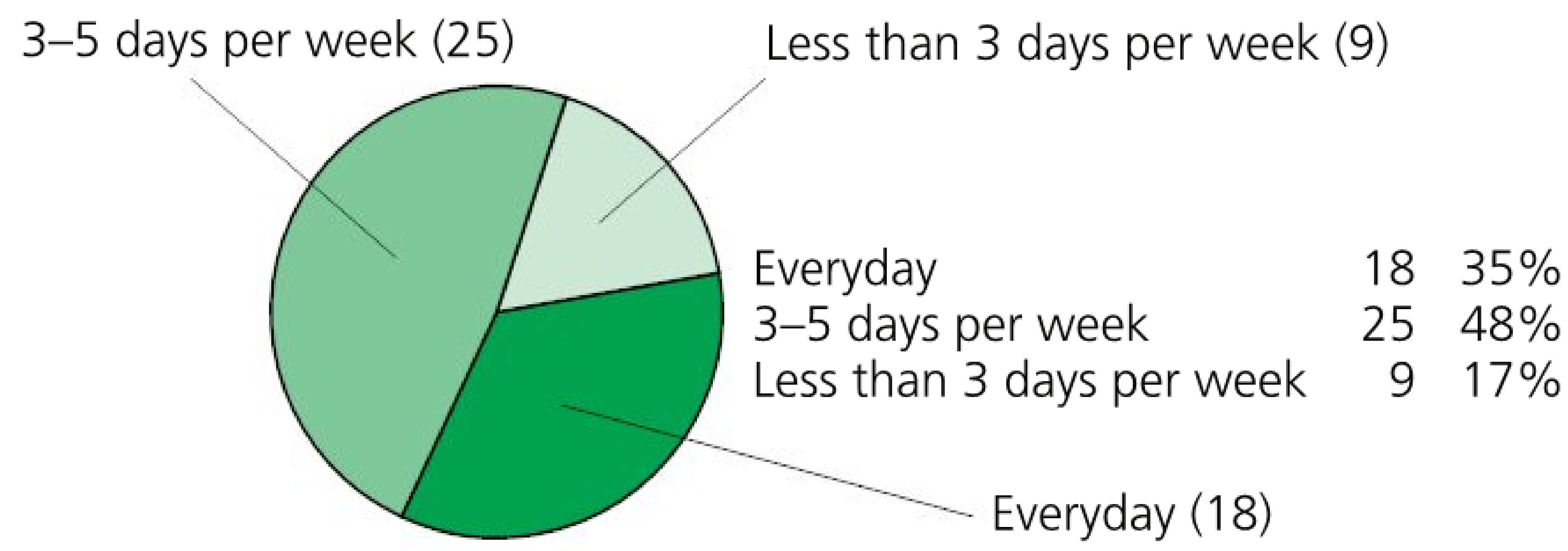
Once you have considered the questions above, you are ready to collect data by surveying. You can create a questionnaire to be completed in person or online, using websites like SurveyMonkey or Google Forms.

In the example shown above right, a teacher contacted her students' parents to ask them two questions using Google Forms. The teacher was able to view a spreadsheet with responses as raw data and could click on the visual representations you can see.

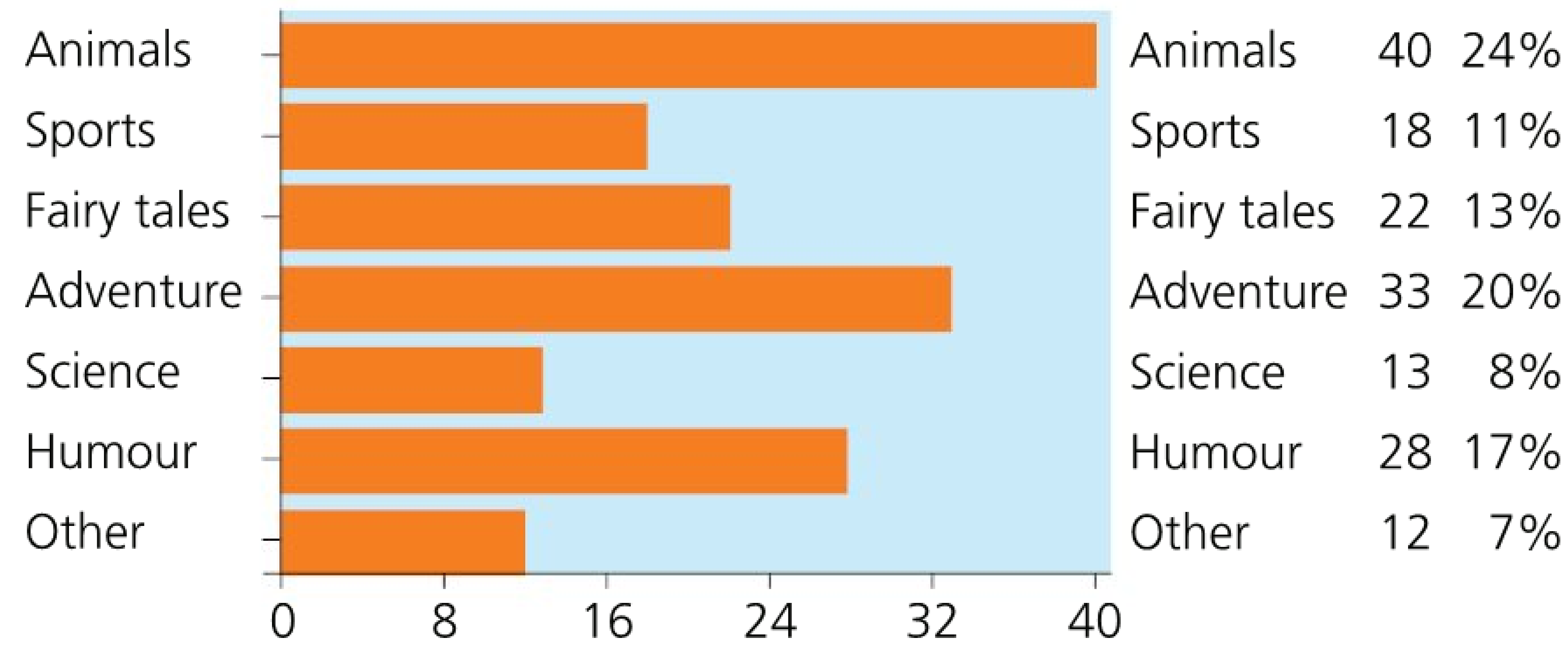
THINK-PAIR-SHARE

Consider the ethical side of data collection. What information might people **not** want to share with you? Why? Give examples. Does the desire for privacy change between genders? Nationalities? Ages?

How often do you read with your child at home?



What types of books does your child like to read?



Source: ashley-smith-s-google-site/google-forms

PRACTICE EXERCISE

1. **Imagine that there is a traffic jam every day on the way to your school. The city council has received lots of complaints about it. They want to investigate. Suggest what data they should collect before they try to fix the problem.**
2. **Your class cannot agree on where to go on a class trip this year. Write a survey to collect relevant data. How can you use these data to resolve the disagreement fairly?**
3. **The Design teacher has heard about your next unit on 3D printers (see Chapter 3) and wants to buy one; create a questionnaire or a data-collection worksheet that would be appropriate to help them choose the best one to buy.**
4. **A politician recently made the claim that 'There are more German cars on US roads than American ones'. Is that true? How could we find out if it is true or not? Design a plan to collect suitable data on this statement.**

How do we organize data?

WHAT ARE THE SYSTEMS FOR MEASURING?

Tallying is a system of recording (and counting) results using lines grouped in sets of five. Four vertical lines are drawn and a diagonal strike through them gives a group of five. It's easy to keep track of a number over a period of time using tallying and the system reduces the chance of error, because it's so straightforward to count up in fives.

Tally charts record the data and allow us to total the frequency of each category or number. To organize this carefully, we use a frequency table.

PRACTICE EXERCISE

1 A class was asked about their favourite subject and the results were tallied. Copy the table and complete the frequency column to show many people are represented by the tally marks.

Subject	Tally	Frequency
Arts	IIII	
Design	𝍸 I	
English	III	
Finnish	II	
Mathematics	IIII	
Music	𝍸 𝍸	
Physics	𝍸 II	

2 How many students were in the class?

3 The following is a list of favourite subjects from a different class of 34 students. Complete a tally chart for these data.

PHE, Mathematics, PHE, Music, Spanish, Music, Urdu, Sciences, PHE, Design, English, PHE, Design, Arts, History, Arts, PHE, Design, Urdu, Arts, English, PHE, Mathematics, Sciences, PHE, History, Arts, PHE, Spanish, Arts, Design, Mathematics, Arts, Urdu

What was the most popular subject?

4 One of the students in the second class, Marius, was interested in finding out if students' favourite subjects were connected to their teachers' ages. The age of each of the teachers was as follows:

22, 67, 62, 32, 31, 55, 52, 66, 24, 49, 41, 38, 29, 35, 42, 27

How could you group (or classify) these data in a sensible way?

Complete a frequency table, including a tally column, to represent this information.

Is it possible for Marius to prove a connection between the age of the teacher and whether the subject is a student's favourite? Why or why not?

MEET A (MATHEMATICAL) DESIGNER: DAVID McCANDLESS (1971–PRESENT)

Learner Profile: Risk-taker

Meet David, in his own words:

'I'm David McCandless, a London-based author, writer and designer. I've written two beautiful infographic books and for *The Guardian*, *Wired* and others. I'm into anything strange and interesting.

These days I'm an independent data journalist and information designer. A passion of mine is visualizing information – facts, data, ideas, issues, statistics, questions – in graphical ways anyone can understand.

I'm interested in how designed information can help us understand the world, cut through BS & fake news, and reveal the hidden connections, patterns and stories underneath. Or, failing that, it can just look cool!

My pet-hate is pie charts. Love pie. Hate pie-charts.'

Source: **www.informationisbeautiful.net**

ORGANIZING DATA IN ASCENDING ORDER

Statistics is the study of data that comes from observation or experimentation. Until now we have been using observed or recorded (researched) data. Now it is time to generate some data of your own!

ACTIVITY: From youngest to oldest

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

The last ten Japanese Emperors, including the current one, are listed below with the age at which they each ascended to the throne (became Emperor).

- **Momozono – 7**
- **Go-Sakuramachi – 22**
- **Go-Momozono – 13**
- **Kōkaku – 9**
- **Ninkō – 17**
- **Kōmei – 15**
- **Meiji – 15**
- **Taishō – 33**
- **Shōwa – 25**
- **Akihito (Reigning Emperor) – 56**

Rewrite the list of Japanese Emperors, in ascending order of age (this means from lowest to highest).

Rewrite the list of Japanese Emperors in descending order of age (from highest to lowest).

Given that the list includes Emperors from the past to modern day, what trends or patterns can you see in the age at which an Emperor ascends to the throne? Why might this be?

Research the length of each Emperor's reign (how long they ruled for). Using this information, rewrite the Emperors' reigns in ascending order. Who reigned for the longest time and who reigned for the shortest?

Are there any outliers in the group? Explain why this person is an outlier.

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

ACTIVITY: Loads of graphs!

■ ATL

- Information literacy skills: Collect, record and verify data

In these four exercises you are expected to collect the data, organize and represent them as accurately as you can. The population depends on your school and class context, and the sample in each case is the entire population.

1 Qualitative data

Choose one of the following variables: car colour, manufacturer or country of origin.

In pairs, collect data on the frequency of that variable in your school car park (or any suitable car park nearby). Be careful to watch out for variability of the sample if anyone leaves or arrives during your data collection!

2 Quantitative discrete data

In this data collection, the variable is the number of words that a person can remember from a specific list.

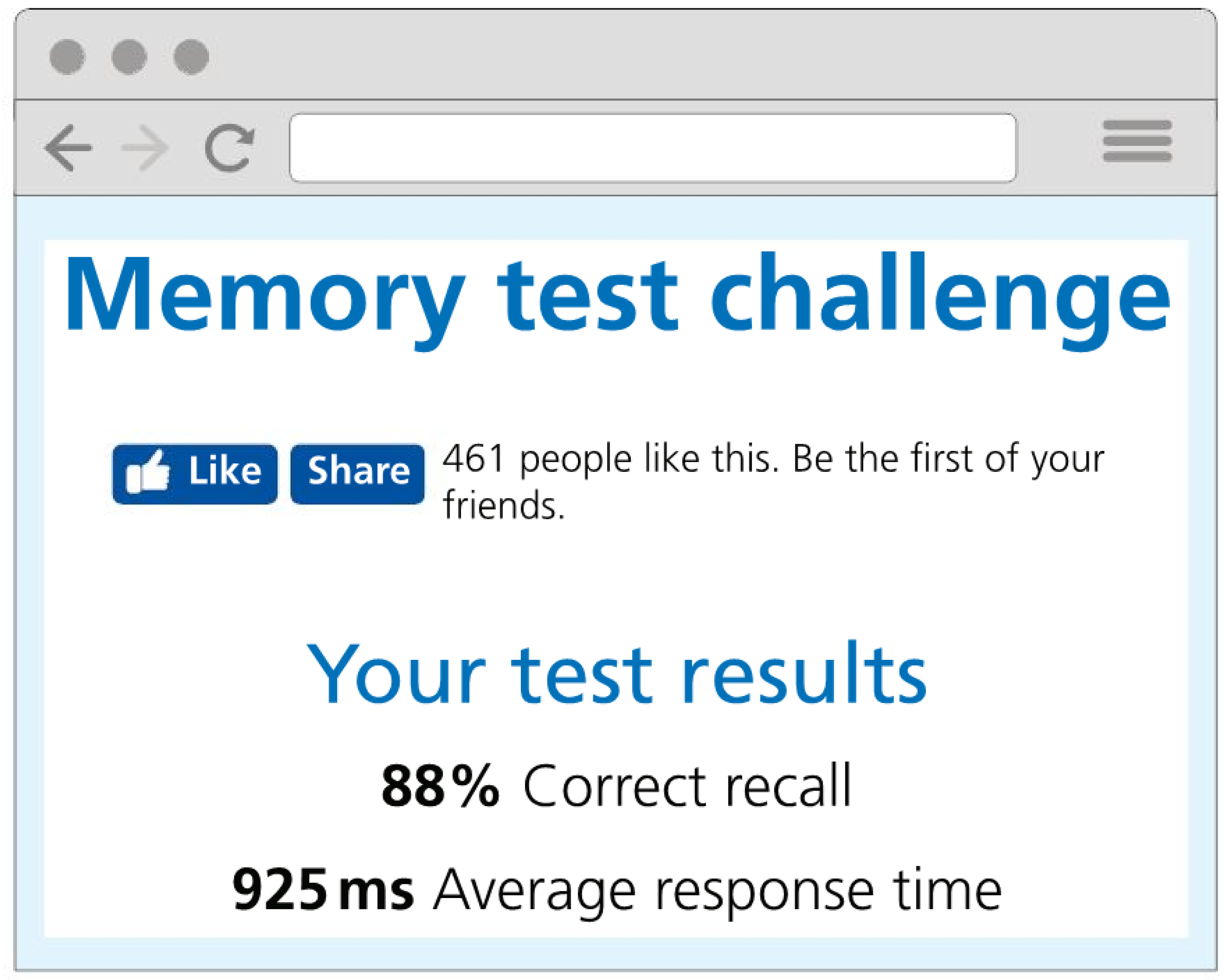
As a class, create a list of 13 words of the same type, all nouns or all adverbs for example. These words could be displayed on a board in the classroom. Allow each person 30 seconds to memorize as many as they can. After 30 seconds, each person must write down as many as they can. Gather and process (tabulate) these data.

3 Quantitative continuous data

To find out about testing short-term visual memory visit this website: **www.memorylosstest.com/free-short-term-memory-tests-online/** or search online for a reliable **short-term memory test**. Complete the test and collect all of the results for the recall (or memory) for everyone in the class.

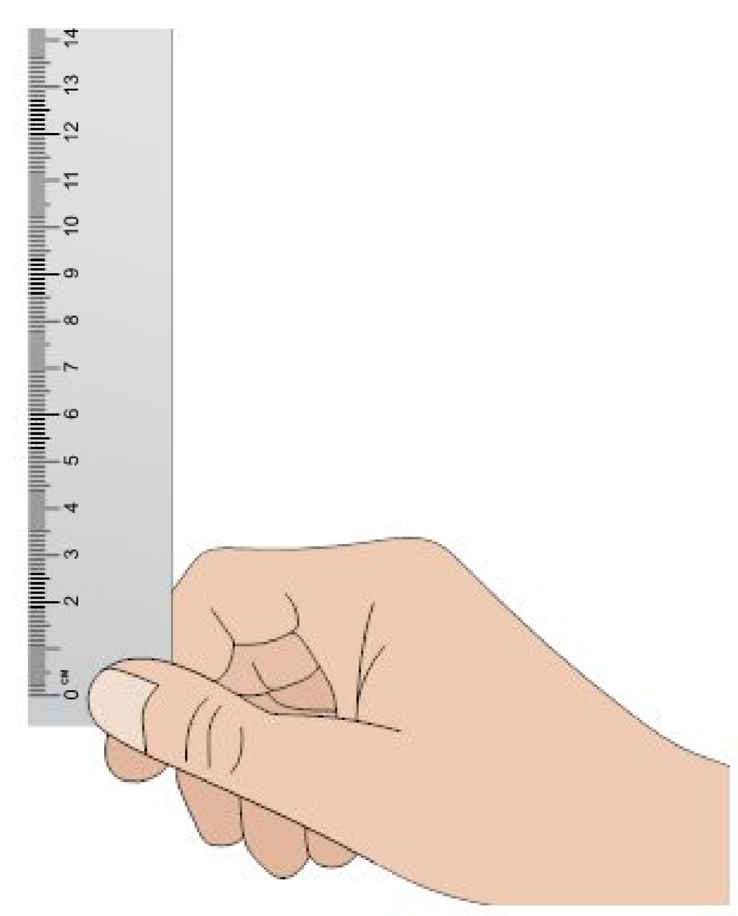
Gather and process these data.

What should you do if some people want to repeat their test? Should you include this in your data? Justify your answer, with at least one reason why or why not.



4 Quantitative continuous data

In this experiment to measure reactions, the variable is the length of ruler, determined by reaction time. Hold a metre stick or long ruler in your hand, pinching it at the bottom, on the zero mark, between your thumb and your index finger.



Release your grip and then catch the ruler again, as quickly as you can. Record the length at which you caught it.

Repeat this exercise with everyone in the room. Gather, organize and process the data.

▼ Links to: Biology

Your reaction time is controlled by your nervous system. To carry out a more rigorous and scientific version of this experiment, visit: **www.education.com/science-fair/article/biology_oops/**

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

In what forms can we represent data?

It is important to choose the graphical representation that best suits your data type. In what situation would you need:

- a histogram
- a line graph
- a pictogram
- a stem-and-leaf diagram
- a scatter plot?

ACTIVITY: Food banks

ATL

- Information literacy skills: Use critical-literacy skills to analyse and interpret media communications

The chart opposite shows seven years' worth of data from a food-bank charity called the Trussell Trust, based in the UK. A food bank is an organization that helps people who cannot afford food by giving them three-day emergency food supplies. The charity has provided the data to create this chart. They measure or record a whole year, from April to the following April, so 10/11 means April 2010 to April 2011.

1 What type of chart or graph is this?

2 What is the overall trend or pattern for the amount of supplies being given out?

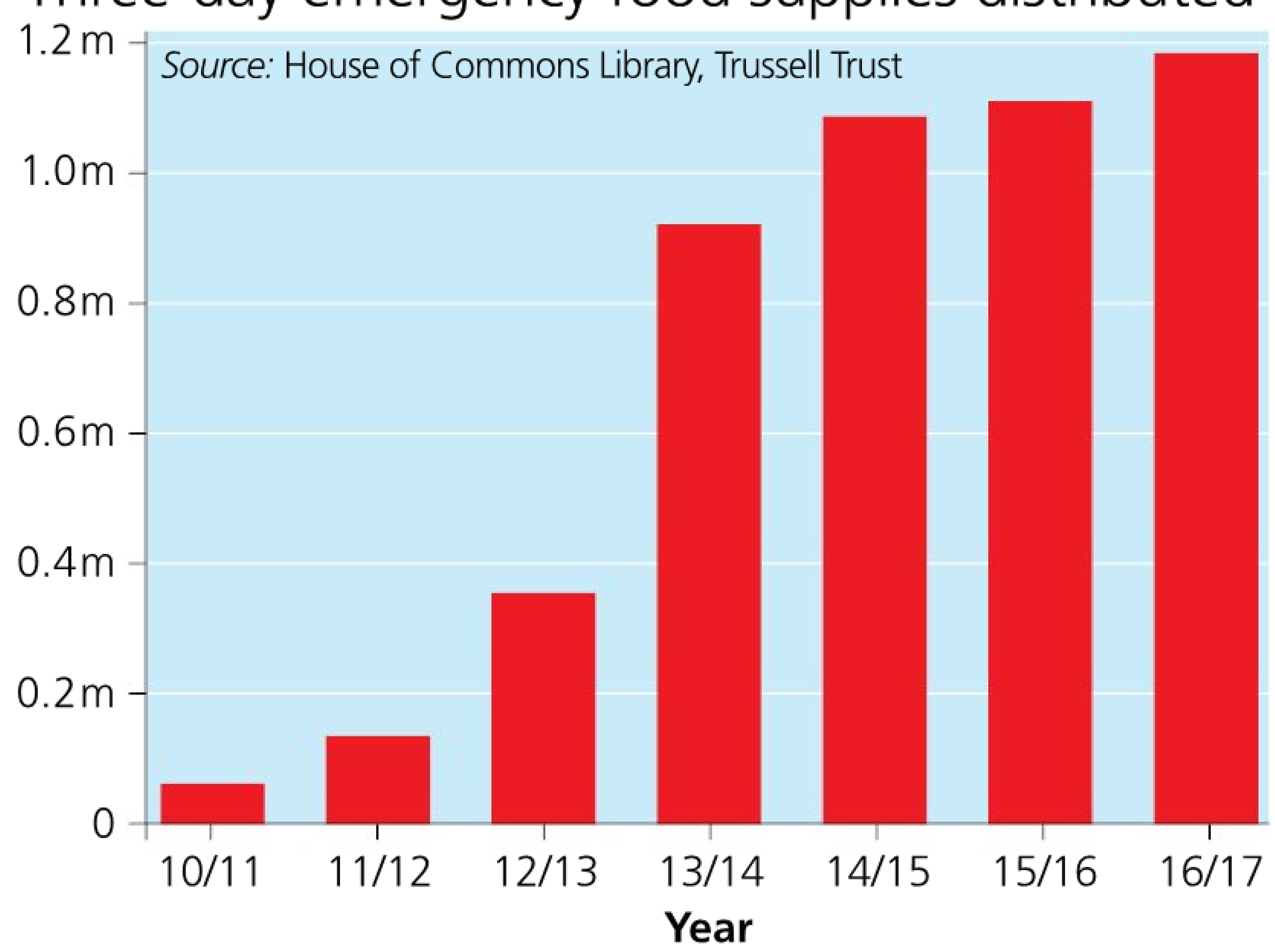
3 What has been the biggest increase?

4 In which year were the most emergency food supplies given out?

5 Estimate how many supplies were distributed that year.

Use of food banks

Three-day emergency food supplies distributed



6 Draw a table to represent the data shown on the graph.

7 What observations or conclusions can you make from this data set?

8 What, if anything, would you recommend that the designer do to **improve** this chart?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

MEET A MATHEMATICIAN: WILLIAM PLAYFAIR (1759–1823)

Learner Profile: Principled

William Playfair was an engineer and economist but we remember him as the founder of graphical methods of statistics. He was the inventor of several of the most popular, and regularly used, diagrams in statistics: the line graph, the pie chart and the bar chart to name a few. Playfair was born in Scotland, the fourth son of a Reverend and had many different careers in his lifetime. He ran a variety of 'failed' businesses, but this never deterred him from bouncing back and trying again. Playfair moved to Paris in 1787 and, two years later, took part in the Storming of the Bastille, a famous moment in French history.

Above is an example of one of the very first published line graphs by Playfair. What does it represent? What information is contained in the chart? Notice how similar it is to the ones we still use today! What do you think he would say about our use of his charts today, over 200 years later?

Playfair was a prolific innovator and argued that charts were better communication tools than tables of data. He is credited with drawing the first example of a pie chart, published in 1801. This technical innovation was a milestone in the development of statistics and data analysis.

This pie chart by Playfair shows how much of the Turkish Empire was located in Asia, Africa and Europe prior to 1789. It is one of the oldest pie charts in existence. What similarities or differences can you see compared to the pie charts we use today?

Hint

Quantifying *means giving a number or a numeric value for something, for example we might say 'let's quantify the size of the problem' or 'it is impossible to quantify the number of stars in the sky or grains of sand on a beach'.*

The four types or representations (histogram, line graph, pictograms and scatter plots) are not the only ones that are available to us. Other popular types of graphs or charts include dot plots, stem-and-leaf diagrams, pie charts and bubble graphs. We will look at how to draw and read some of these types of charts, so that you can be more data literate.

You might know that Florence Nightingale, a woman remembered primarily for her dedication to nursing, was also a successful statistician who invented the polar area diagram to clearly represent the proportion of avoidable deaths in hospitals at the time.

ACTIVITY: Grouping and classifying data

■ ATL

- Communication skills: Understand and use mathematical notation

1. **Gather** the data: Write down the first name of every person in the room in any random order.
2. **Describe** the data: Are these data primary or secondary? Qualitative or quantitative?
3. **Order** the data: Rewrite the list of names in ascending order, from shortest to longest.
4. **Quantify** the data: Instead of writing down names, record the quantity of letters in each name instead, for example Lida, Chris, Tanya, Xavier: 4, 5, 5, 6.
5. **Classify** the data: Put the results into different groups or classes (depending on the length of the name) and **tabulate** them (by putting them into a table like this).

Name length (in letters)	Frequency
2–3	
4–5	
6–7	
8–9	
10 or more	

6. **Display** your results as a pie chart **and** as a pictogram, and compare the two forms of representation.

Repeat this process for the family names or surnames of everyone in the room. Is there a difference in the results? Compare both pie charts and both pictograms to one another. Are most first names shorter or longer than surnames for this population? Can you make any other observations or conclusions?

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

WHAT IS A DOT PLOT?

Another form of representation is a dot plot. Dot plots display a dot (or other mark) for each observation along a number line. If there are multiple occurrences of an observation, or if observations are too close together, then dots will be stacked vertically.

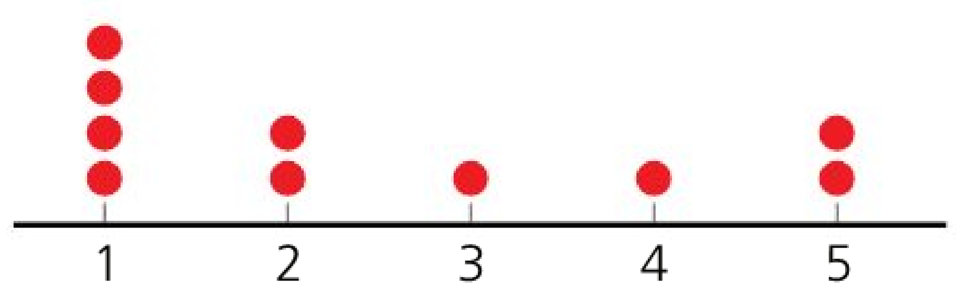
Let's consider a data set that shows the number of days between tests for a class.

2 4 1 1 5 2 1 5 3 1

First, we draw a horizontal or vertical line.

Then we take the set of raw data and mark each possible value within the range on the number line.

Finally, we represent each observation (or number) as a dot at the correct location.



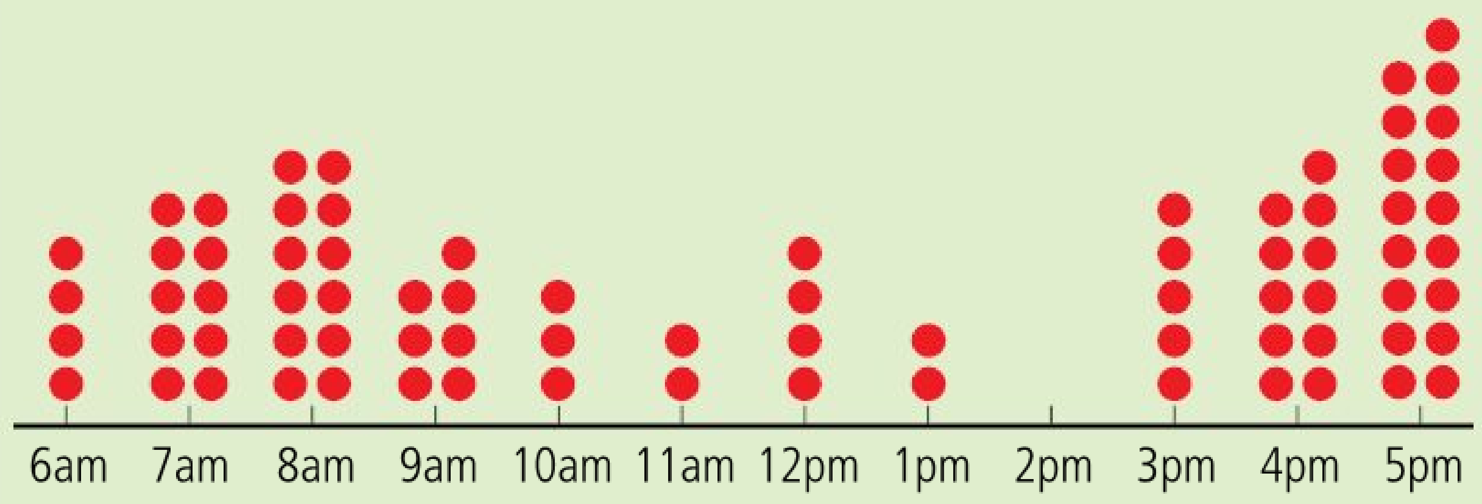
From this dot plot we can see the spread of data and how often a certain value appears. (See Chapter 5 for more on the value of mode.) A dot plot is considered a useful tool for a set of less than 50 data points.

PRACTICE EXERCISE

1 Create a dot plot to represent the number of siblings for the people in your class.

2 The buses arriving at a bus stop are represented on the dot plot below.

Comment on some patterns or trends you see in the data, giving reasons or explanations.



3 The times for buses arriving at a different bus stop were as follows:

6:13	6:45	7:03	7:09	7:58
8:10	8:29	9:37	10:17	10:55
11:00	11:21	12:36	13:55	13:57
14:04	15:31	15:35	15:44	16:05
16:30	15:05			

Identify a mistake made by the person recording the data and correct it.

4 Represent the data for the second bus stop on a dot plot.

5 Comment on some patterns and trends you see in the data, giving reasons or explanations.

6 Compare the data for the two bus stops.

If one bus stop is in a city and one is in the countryside, which do you think is which? State reasons for your answer.

7 Is a dot plot the best representation for this information? Justify your answer.

STEM-AND-LEAF DIAGRAMS

Another data visualization, which would have been useful for the previous bus-stop exercise, is called a **stem-and-leaf diagram**. These are good for data sets with no more than 120 to 150 points. This type of diagram represents quantitative data, grouped by the 'stem' with individual data as the 'leaves'.

Example 1

Here is a data set showing the heights of trees in a greenhouse.

134 cm 137 cm 162 cm 141 cm 152 cm 157 cm
139 cm 147 cm 162 cm 159 cm 131 cm 168 cm

To place these on a stem-and-leaf diagram, we first decide on the stem (how the data will be grouped). In this data set we have values in the range from 130 to 170 cm.

We can represent the numbers in the data set in the following way: we add each datum as an individual leaf. We must be careful to check we have organized our data in ascending order.

13	1	4	7	9
14	1	7		
15	2	7	9	
16	2	2	8	

These are 131, 134, 137, 139

Key
13|1 = 131 cm

Notice how the numbers are arranged in increasing order. Also, see that identical entries of 162 must be shown twice, given that height appears twice. There must also be a key to explain how the stem has been devised.

Example 2

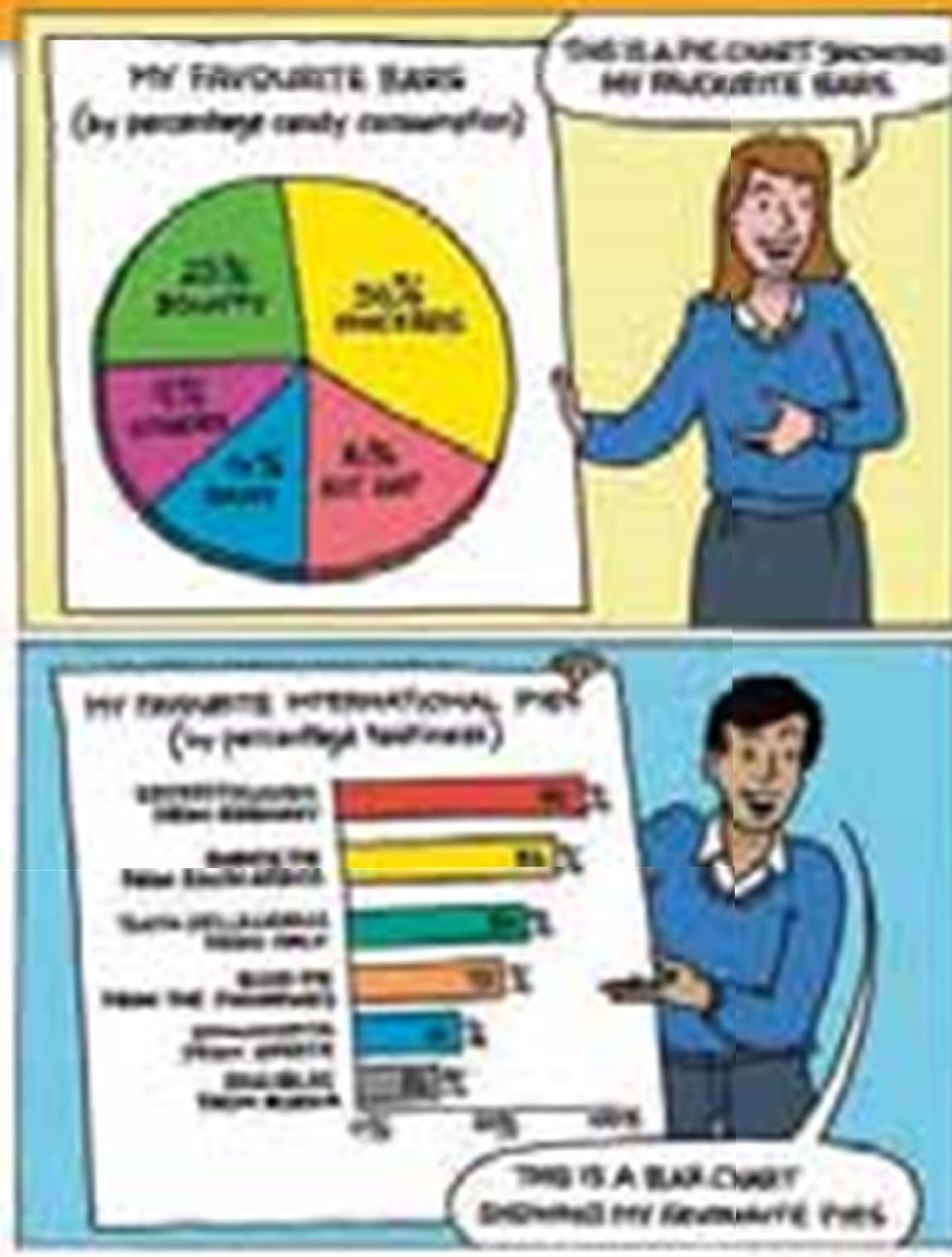
If we think back to the bus-stop exercise, we could use a stem-and-leaf diagram to represent that data better, as we would not lose information about the exact times of the buses.

6:13 6:45 7:03 7:09 7:58 8:10 8:29 9:37
10:17 10:55 11:00 11:21 12:36 13:55 13:57

These data can be represented on a stem-and-leaf diagram, as follows:

6	13	45	
7	03	69	58
8	10	29	
9	37		
10	17	55	
11	00	21	
12	36		
13	55	57	

Key
Where 6|13 = 6:13



A stem-and-leaf diagram is a good way to see **distribution** at the same time as seeing the original data. It is good for data sets below 150 entries. Any more than this, it becomes unwieldy (hard to use). In fact, this representation is used for bus and train timetables around the world, from Japan to Switzerland.

Example 3

Here we are using a stem-and-leaf diagram to compare two sets of data.

If we have two sets of data, measuring the same variable but for two different populations, then we can make a two-sided stem-and-leaf diagram! This enables us to compare the data quickly.

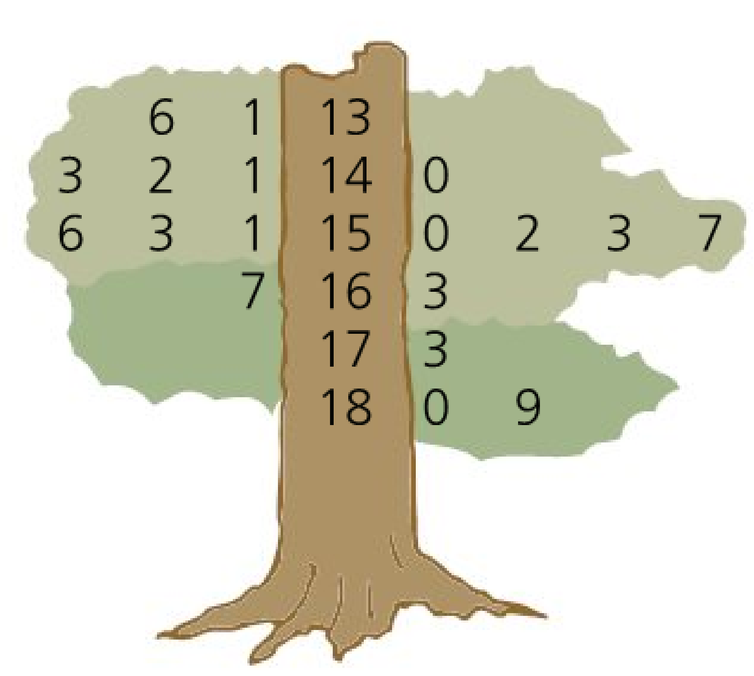
This data set shows girls' heights and boys' heights for a class, in centimetres.

Girls: 131 141 167 143 151 156 136 142 153

Boys: 140 150 189 152 163 157 153 173 180

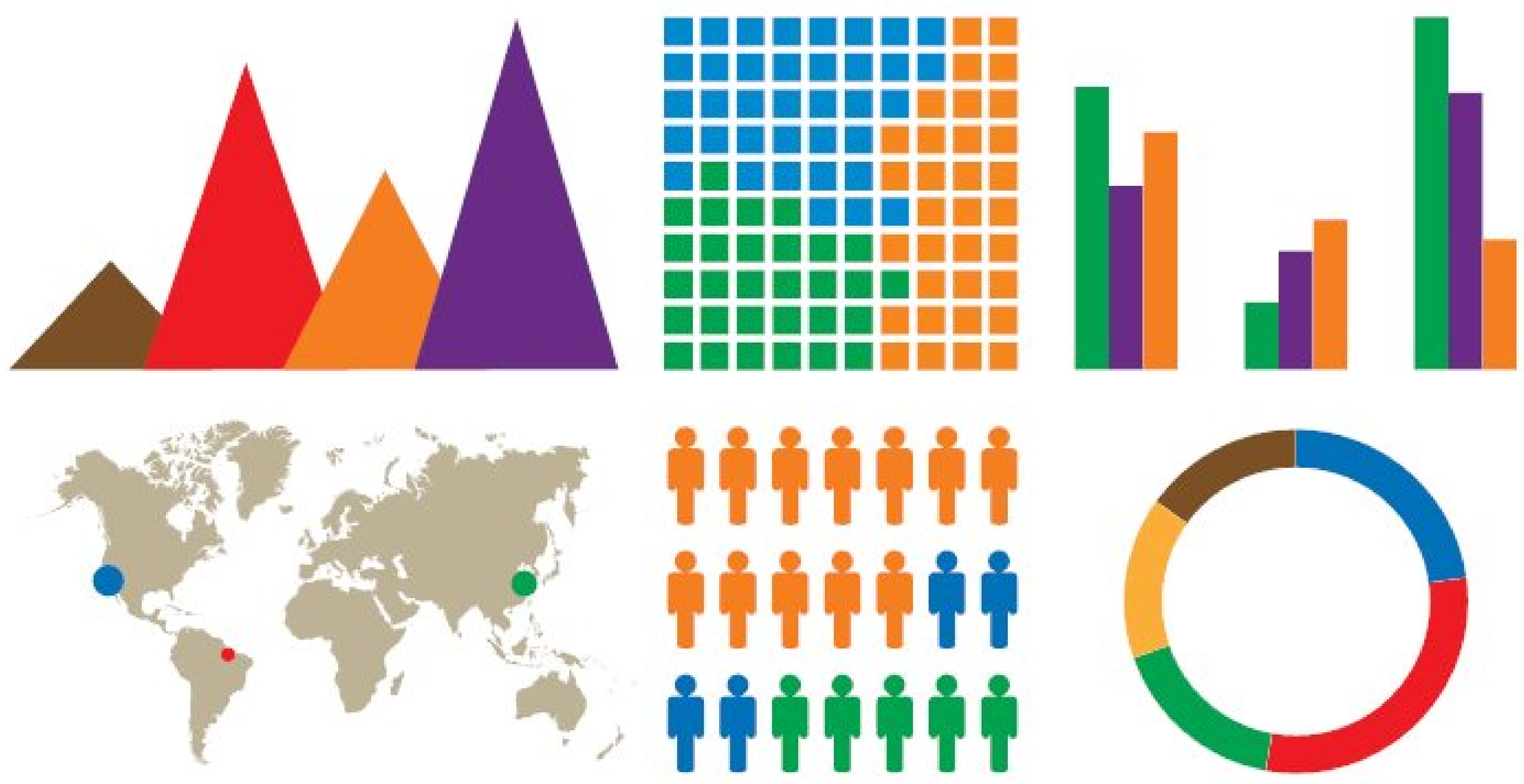
The data can be represented with a common stem in the middle, and the girls' data going to the left and the boys' to the right.

			Stem				
	6	1	13				
3	2	1	14	0			
6	3	1	15	0	2	3	7
		7	16	3			
			17	3			
			18	0	9		



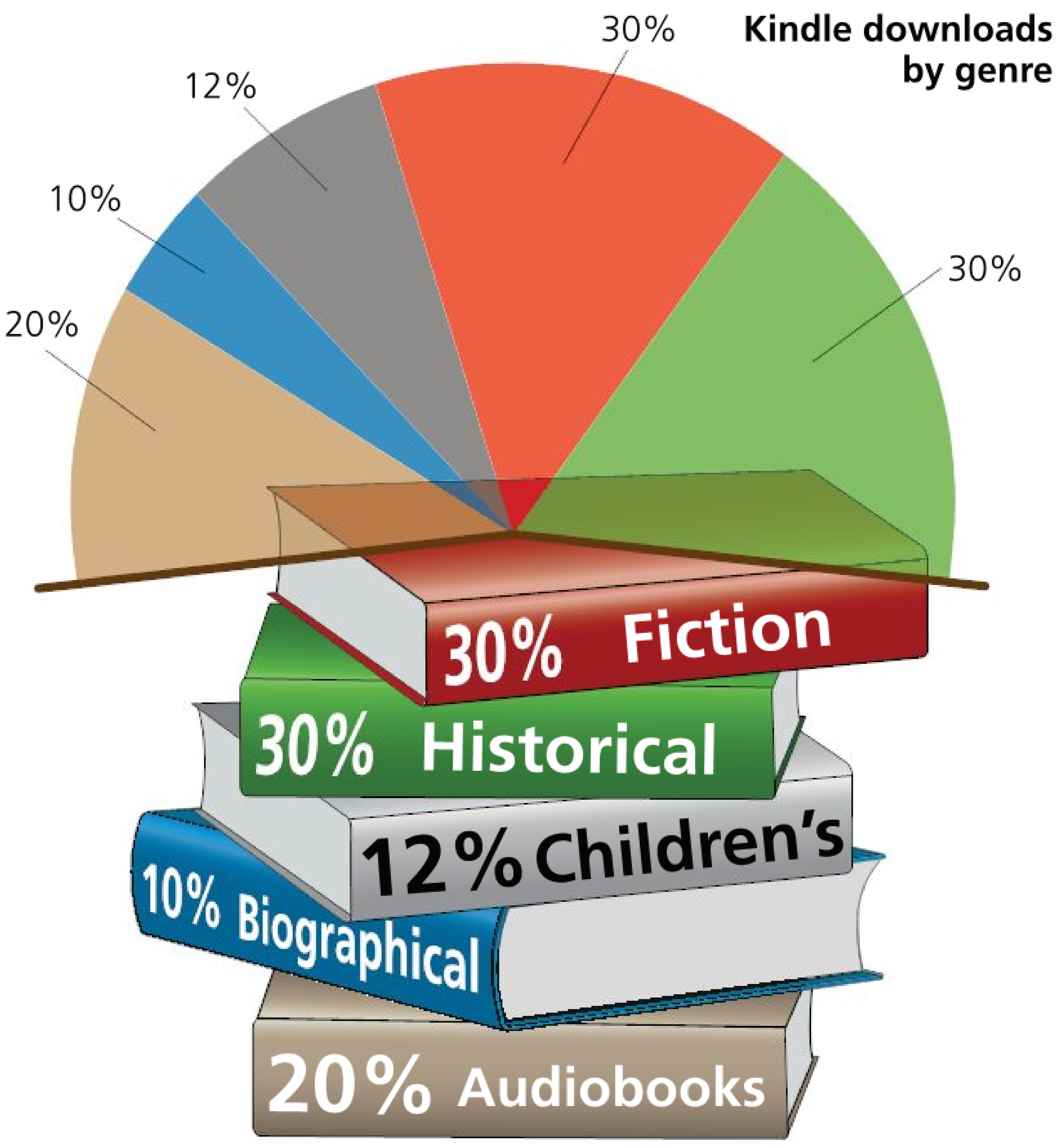
Now we see how the distribution of heights differs between boys and girls. The girls are considerably shorter than the boys. Can you see the 'tree' now?

What exactly are infographics?



A data visualization is any illustrative tool we use to represent data in a visually appealing or accessible way. Modern infographics are visually appealing, colourful and easy to make, thanks to innovations in design and computing. But it is important to remember that all graphs, including infographics, are not just pictures for communicating ideas; they are also tools for analysis.

One problem with infographics can be accuracy. They may look attractive, but it is important to be **extra careful** that the stories they tell are true and fair to the data. The book infographic above has been developed to show one set of data in two ways, as a pile of books and as an open book, just over half of a pie chart. This is a creative way to represent data but on close inspection we see several mathematical flaws. How many errors or misrepresentations can you find?



THINK-PUZZLE-EXPLORE

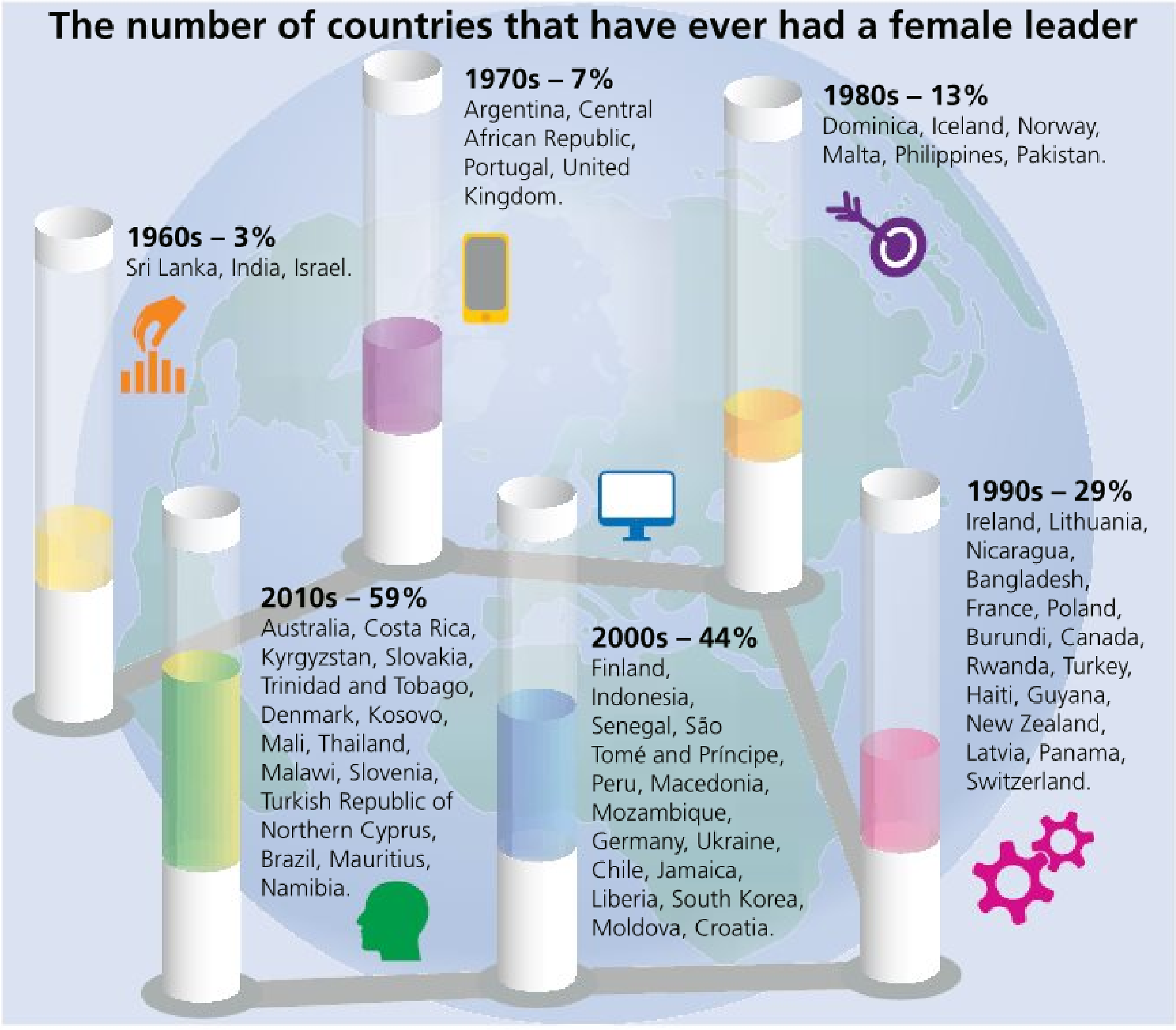
Discuss your results as a class and come to an agreement about what the flaws are and how you might fix them. Create the same infographic, but make it mathematically accurate this time. One error was that children's books accounted for 10% of all downloads and **not** 12%. Explore how this mistake changes the infographic.

THINK-PAIR-SHARE

This infographic shows us five different categories of information and we can see how large the share is within each one. Rate the infographic, on a scale from 1 (lowest) to 10 (highest) on how:

- **good it looks**
- **easy it is to read**
- **easy it is to understand**
- **internationally minded it is**
- **innovative it is.**

Discuss your answers with a partner and then share with the class.



ACTIVITY: Create your own visually appealing infographic

ATL

- Information literacy skills: Present information in a variety of formats and platforms

Scenario

You are a journalist working on an article about innovative buildings and how we make room for them in cities. You have carried out research into demolitions (when a building is knocked down) in your city and found the following data.

Your task is to represent the data provided in a visualization to go with your article.

Month	Demolitions
Jan	2
Feb	4
Mar	4
Apr	5
May	6
Jun	8
Jul	9
Aug	8
Sep	8
Oct	3
Nov	1
Dec	0

What representation might you choose? Discuss why this would be an appropriate choice.

Where do you think the data might have come from? How trustworthy do you think the data might be?

Is this likely to be a sample or is it the entire population?

While looking through the numbers for a pattern, you are reminded of this picture you came across during your research. You are inspired to include it somehow in your visualization, to bring out the theme.

Using the data and the image provided, create a data visualization to go with your article. For fun, call the city after yourself. Comment on any trends or patterns you observe.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

How do we find patterns in data?

ACTIVITY: Statistical analysis of texts

ATL

- Critical-thinking skills: Practise observing carefully in order to recognize problems

In this activity, you are going to use data collection and analysis to **describe** patterns as a general rule and then to **verify** the rule.

You are given three different types of text – part of a poem, an online article from an entertainment website and an academic text from a university publication.

Source A – a poem

Invisible kisses

By Lemn Sissay

If there was ever one
Whom when you were sleeping
Would wipe your tears
When in dreams you were weeping;
Who would offer you time
When others demand;
Whose love lay more infinite
Than grains of sand.

If there was ever one
To whom you could cry;
Who would gather each tear
And blow it dry;
Who would offer help
On the mountains of time;
Who would stop to let each sunset
Soothe the jaded mind…

Source: http://lemnsissay.com/writing-2/pttt/

Source B – an article from an entertainment website for news and gossip

Politics Tech Lifestyle Sports News

We still do not know who the next Doctor in *Doctor Who* will be, so let me just say right here, right now … It should be Olivia Colman. If you are an alien planning on attacking Earth, you would not get past Olivia Colman. She would destroy you. She would also negotiate with them like this. Everything that Olivia Colman is in is absolutely stellar.

Source C – academic text

Confirmation bias occurs from the direct influence of desire on beliefs. When people would like a certain idea/concept to be true, they end up believing it to be true. They are motivated by wishful thinking. This error leads the individual to stop gathering information when the evidence gathered so far confirms the views (prejudices) one would like to be true.

Once we have formed a view, we embrace information that confirms that view while ignoring, or rejecting, information that casts doubt on it. Confirmation bias suggests that we don't perceive circumstances objectively. We pick out those bits of data that make us feel good because they confirm our prejudices.

Source: www.psychologytoday.com

Your task

In each source you may have noticed a difference in the style and the types of words used. In this investigation you will look for patterns in the lengths of words used in different texts to see if you can find a general rule.

For each of the sources A, B and C, complete a copy of the following table.

Number of letters in each word	Tally	Frequency
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		

Comment on any patterns that you see in the data.

Create three bar charts to represent these results (or you could create one using different colours for each data set).

Comment on any trends or patterns you might observe in the data representation. State your findings as a general rule.

Now it's time to verify that rule by testing it on some new sources. Let's see if it holds up with new data. This time, the letters in each word have been replaced with a symbol (so that the context is not obvious). Can your rule help you to determine what type of text each source is?

Source D

Τηισ ωορκ αδοπτσ α χομπαρατιϖε στανχε βορροωινγ φρομ ετηνογραπηιεσ, νοϖελσ ανδ χονϖεντιοναλ πηιλοσοπηιχαλ ωορκ, βυτ τηε μαιν φοχυσ ρελιεσ ον τηειρ φυνδαμενταλ χομπαραβιλιτψ το αντηροπολογψ, ασ ωελλ ασ τηε οπποσιτε. Το χλοσε, σομε ιν–ροαδσ χονχερνινγ τηε ποσσιβιλιτψ οφ συντηεσισινγ βοτη περσπεχτιϖεσ αρε ουτλινεδ ωησσε σπεχιαλ ρελεϖανχε το υνδερστανδινγ αντηροπολογψ ισ μορε χοντινγεντ υπον α σηιφτ ιν περσπεχτιϖεσ ανδ δισχιπλινεσ ωηιχη ρενδερσ αντηροπολογψ βοτη α συβφεχτ ωηιχη σεεκσ το υνδερστανδ ασ ωελλ ασ αν υνδερστανδινγ συβφεχτ.

Source: **http://compaso.eu/wp-content/uploads/2013/01/Compaso2012-32-Dubhghaill.pdf**

Source E

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Which is the poem? The article? The academic text? Having examined sources D to F, state which type of text you think each one is, giving reasons for your answer. Your teacher will reveal the results and you can evaluate how successful your rule was.

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

How do we handle results fairly?

ANALYSING DATA

One way of collecting survey data is by telephone. Political polls are conducted by calling people and asking them a series of questions about who they might vote for and why. It is also possible to collect data in this way about how a politician or policy is being received by the public.

Based on the script extract on the right, what is the **response rate**, as a percentage? Recently, the response rate for phone polling has been around 3 to 5%. Is this an increase or a decrease? Suggest a reason why the response rate has changed. Why is participating in this type of data collection less popular now? Nowadays, we also get asked our opinions through online surveys and in face-to-face interviews, as well as by phone.

The last few years have been difficult ones for pollsters (people who carry out research to predict who will win elections). They have predicted several important votes incorrectly and people have much less confidence in them than in previous times. Some surprising results within the past decade, which did not match the predictions, include the USA, UK, Serbian and French Elections, as well as the UK's infamous 'Brexit' vote. Many pollsters have been asking themselves what went wrong.

Not all variation is explainable, but your job here is to identify trends and to try to understand them, sometimes despite the variation.

Perhaps there are 'shy voters' who won't say how they will vote or who are not completely honest when questioned, and this affects the data. If they are private people, 'shy voters' might think that voting intensions should be confidential, or they may not want to say how they will vote if they feel that could be judged by others as inappropriate.

SAM: Hey, Ginger.
BONNIE: How's it going in there?
SAM: I popped Mandy with my tranquilizer gun. She's doing fine.
GINGER: Bonnie wanted to know why it takes 48 hours.
SAM: We need 1500 responses.
BONNIE: It takes 30 people, 48 hours to make 1500 calls?
SAM: It takes them about 12 hours to make 1500 calls. We need 1500 responses, which means we need to make 6000 calls.
TOBY: [enters] Sam.
SAM: Yeah.
BONNIE: Only 1 in 4 people don't hang up?
SAM: That's if you're lucky.

An extract from 'The West Wing', a TV show based in the White House, which was popular in the early 2000s. Season 1, Episode 21 – 'Lies, Damn Lies and Statistics'.

Source: **http://westwingwiki.com**

Another possibility is that people don't tell survey takers the truth about their intended participation. When one polling company followed up on their respondents, they checked the voting register to see who actually turned up to vote. They found that 11% of the people who told them that they were definitely going to vote in 2015 didn't go on Election Day.

Another idea put forward by the pollsters was that they simply asked the wrong people (an unrepresentative sample) or relied too heavily on online data. They said that their models needed to be better next time around.

One consequence of the failure to predict the UK General Election in 2015 was that many polling companies subsequently played down the effect that young people have on the vote. They suggested the data told them that young people don't really turn up, even when they say they will, so they applied a 'lying factor' to their results for their 2017 predictions. The pollsters also tried to track social media posts, viral images and status updates to take these into account.

As it turns out, the pollsters were wrong again and 2017 saw a huge turnout of younger voters, which led to another unexpected result and many polling companies were embarrassed once again!

The youth for today

How the 2017 election changed the political landscape

Despite their calamitous campaign, the Conservatives increased their share of the vote to 42% – up five points since 2015 – which in any other election in the past three decades would have been enough to build a commanding majority.

But Labour outperformed even that achievement as a unique alliance of enthused younger voters and previous non-voters combined with older austerity-hit, anti-establishment Ukippers to deliver a 10-point rise in Labour's vote compared with two years ago, to 40%. This is just below the 41% secured by Tony Blair in his 2001 landslide victory.

Turnout

The 'youthquake' was a key component of Corbyn's 10-point advance in Labour's share of the vote – exceeding even Blair's nine-point gain in his first 1997 landslide. No official data exists for the scale of the youth vote but an NME-led exit poll suggests turnout among under-35s rose by 12 points compared with 2015, to 56%. The survey said nearly two-thirds of younger voters backed Labour, with Brexit being their main concern.

Source: **www.theguardian.com**

! Take action

! Remember, it's really important to cast your vote whenever you get a chance, even if your choice doesn't end up being the winning one. It means that your voice is heard and that you have had a chance to express your preference. Whether it is a class election, a local government vote or a country-wide election, you have a responsibility and a right to inform yourself and to express your opinion.

BUT IT'S NOT ALL ABOUT POLITICS

Data are not just used for elections and politics. Important decisions are often data-driven and people use statistics to convince us of things all the time. Market research, medical testing and advertising often use statistics to draw conclusions or make persuasive arguments.

Look at the advertisement opposite.

- What percentage of people agree that this shampoo makes their hair shinier?
- How large was the sample size?
- What is the population?
- How many people agreed with the statement that the shampoo made their hair sleeker and shinier?
- How many people disagreed with the statement?
- What observations can you make about the sample size versus the probable sales?

In this shampoo example, the sample size seems quite small, especially when we consider that the shampoo will probably sell thousands or tens of thousands of bottles, maybe even millions of bottles. Some people might say that such a small sample is **statistically insignificant**. The researchers might argue that the size is not a problem as long as it accurately reflects the population. What do you think?

How do we know what to trust?

CRITICAL-THINKING SKILLS

Statistics and statistical reasoning depend on context. This means that you need to know as much as you can about how, where and when the data were collected. Sometimes who collected them and why also matters.

A recent innovation in data collection is the phenomenon of 'crowdsourcing'. Crowdsourcing is the idea that instead of a small data collection done by an individual or team of researchers, a large number of people can collect data and submit them to a central location for analysis. Smartphone apps and websites have made this type of large-scale data collection popular. Scientific studies conducted in this way have included pollen count collection and information on star counts.

One similar innovation was an app developed in Boston, USA that allows road users to record and report serious potholes and other problems with public streets. Potholes can be damaging to cars and are extremely dangerous for cyclists. Most city authorities rely on people reporting such problems to know where to fix things. Crowdsourcing this process seemed like an excellent approach to finding this raw data.

However, before long, there were concerns about the bias which might be built into the process. People began to wonder whether there was more reporting from economically advantaged areas than from poorer, less advantaged areas. People from prosperous neighbourhoods were more likely to have smartphones, cars and bicycles for cycling to work. This might mean that the richest areas were getting the most repairs.

What do you think? Would this have an effect on the data collected? Is this innovation a good idea or a bad idea? What improvements might be made?

ACTIVITY: Does this look like a good sample?

ATL

- Collaboration skills: Manage and resolve conflict, and work collaboratively in teams

You are carrying out a survey on people's attitudes to the environment in your country. The people in the picture above were the first to respond to your survey.

Let's think back to our Statement of Inquiry for this chapter: Fair forms of communication help us to reveal patterns and improve our truth-telling systems. The activity below reminds us that it is important **who** we ask to ensure that data collection and representation is fair. Next we must consider **how** we ask the questions in our systems of data collection.

For all of these questions, think about the context of your country. You and your team must agree on all of the answers as a group. You must use persuasive speech to convince one another, if there are differences, to come to a joint decision.

Think about the following questions:

- **How big is your sample size?**
- **How big is the population of your country?**
- **Comment on the suitability of the sample size.**

Now let's think about the group's ages, genders, ethnicities and attitudes. Comment on the representation of people:

- **by age**
- **by gender**
- **by ethnicity.**

How good a mix is this, as a fair sample of humankind?

How good a mix is this, as a fair sample of **your** country? Is there a difference? Justify your answer

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion D: Applying mathematics in real-life contexts.

ACTIVITY: What should you do when your app is racist?

ATL

- Critical-thinking skills: Recognize unstated assumptions and bias; Consider ideas from multiple perspectives

Read the following article: **www.newyorker.com/business/currency/what-to-do-when-your-app-is-racist**. The SketchFactor app was designed to collect crowdsourced data from people about how unsafe or 'sketchy' they felt a neighbourhood was. These data were then processed and represented on a map, which people could use to decide whether to avoid a place or not.

There were serious concerns about the cultural insensitivity and potential for racism. What do you think? What is the potential for bias here? How would you feel if your home or area was identified as 'sketchy'? What if your business was in one of those areas?

Write a short paragraph considering the **accuracy** of the data that this app collects. You must make sure to explain the mathematical flaws and the possible consequences of this app to the developers.

Assessment opportunities

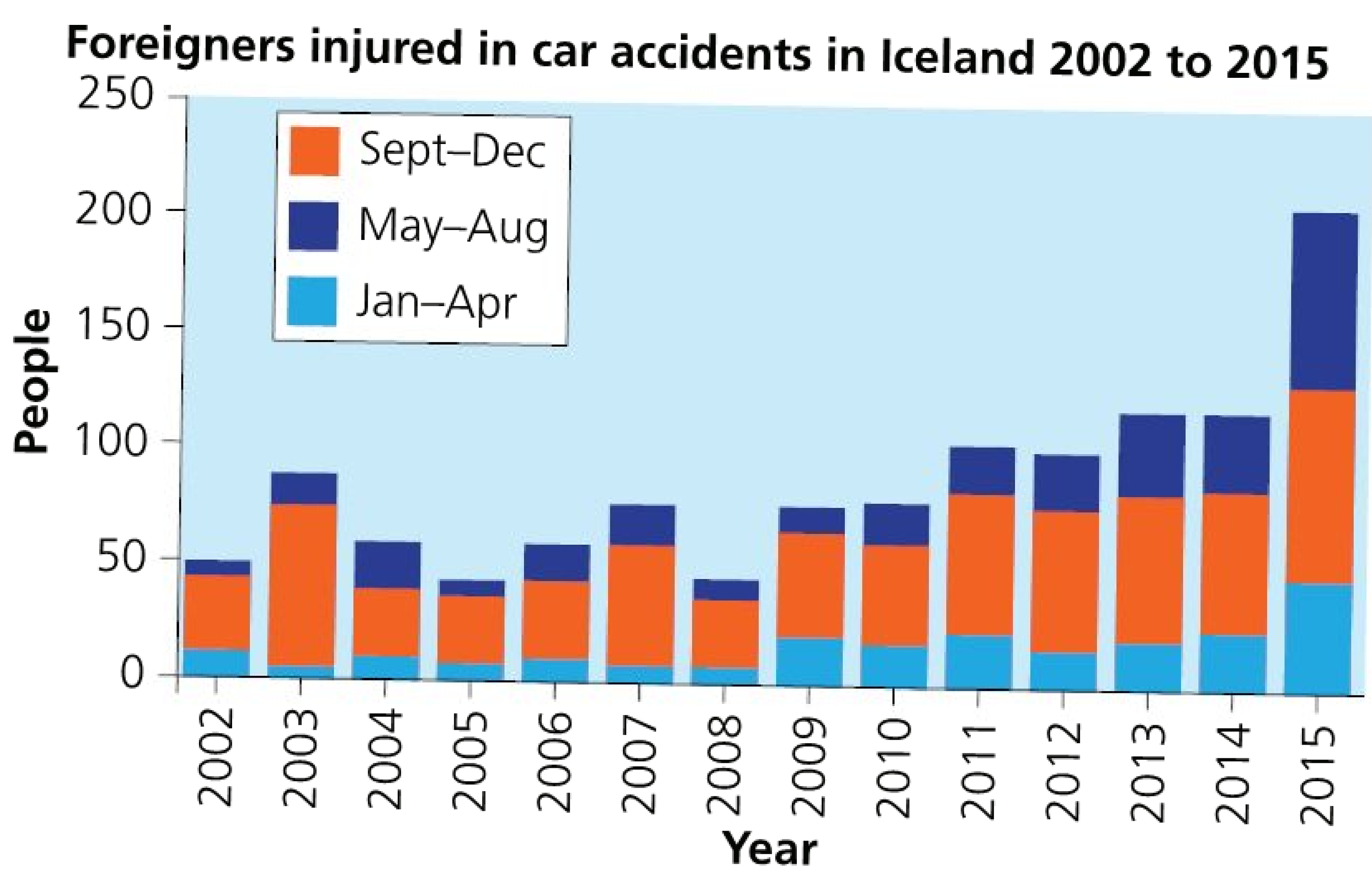
- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

THE IMPORTANCE OF CONTEXT

Knowing as much as possible about the source, the collection and the accuracy of the data is very important to making data-based decisions.

Case study: Car accidents in Iceland

The graph below shows the number of foreigners (or non-Icelanders) who were injured in car accidents between the years 2002 and 2015, inclusive.



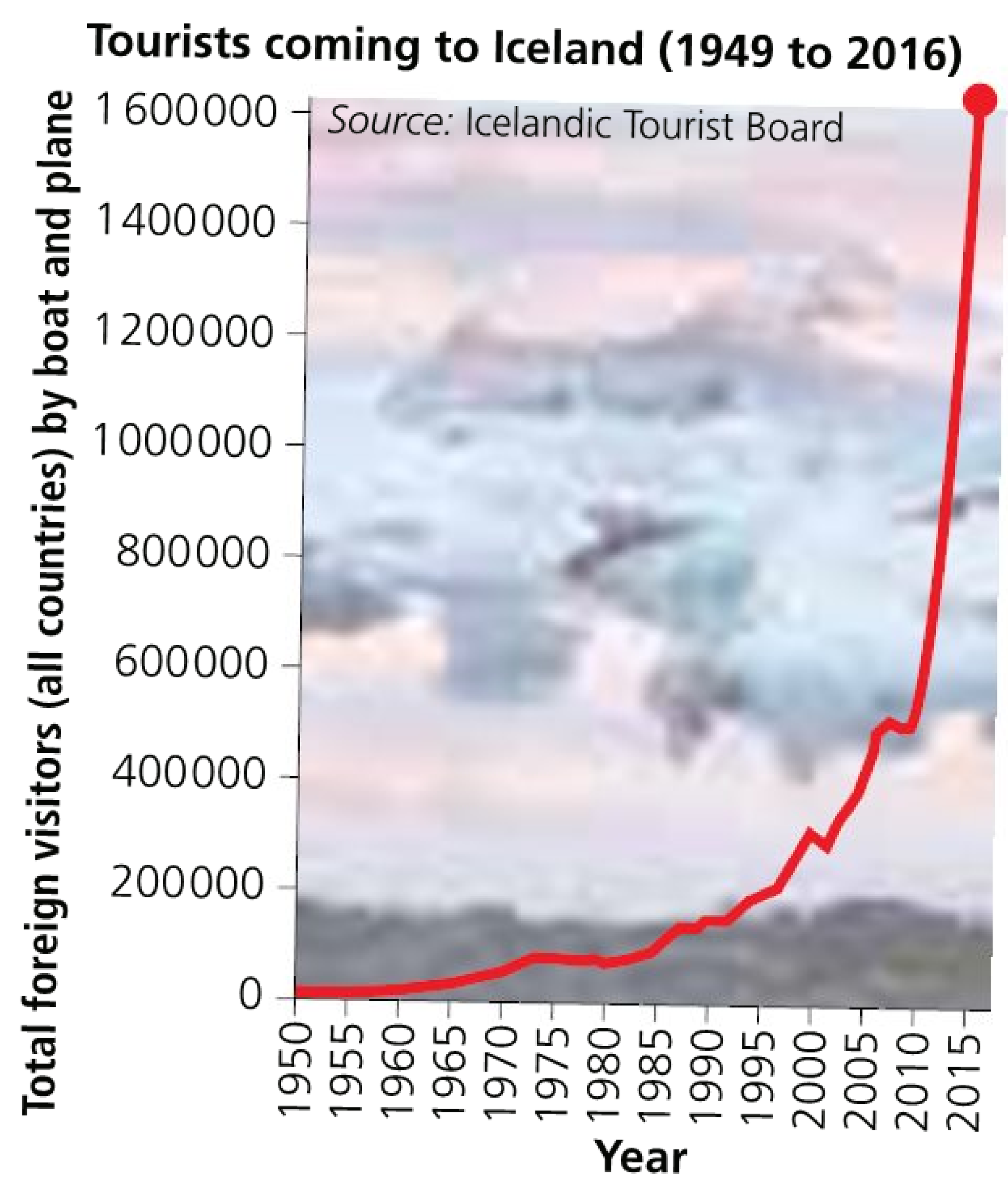
Source: www.goiceland.com

We notice that for the years 2002 to 2008 there seems to be a cyclic pattern of increasing and decreasing numbers of injuries. The total number of injuries seems to fall in the range of 45 to 90 annually; 2009 and 2010 are similar to this, staying in the same range. Then, from 2011, the numbers steadily increase, with a surge (or uptick) in the number of injuries in 2015. Why might this be? Did Iceland's roads get more dangerous? Did drivers suddenly get worse?

Another pattern we can see from the chart is the timing of these accidents. In the earlier years, from 2002 to 2011, the majority of the accidents took place in the summer months, between May and August. This appears to have been a more likely time to be injured, so we can conjecture that there were more foreigners, possibly tourists, in Iceland during those months. However, from 2012 onwards, we can see growth in the number of accidents happening in the other, colder, months. The data might lead us to wonder whether the roads get worse in winter. What is happening here?

The clue that will lead us to the real story behind the data is knowing a little bit about the context (background). If we assume that the numbers of injuries can be trusted (as they probably come from hospital or police records), then the reason for the change must come from another factor. Perhaps more data might unlock the truth …

If we look at the following line graph, we can identify an important factor. Notice how the number of tourists is increasing over time.



There has been a huge increase in tourism in Iceland, particularly since 2010, which is no coincidence. In that year, an Icelandic volcano called Eyjafjallajökull erupted and caused travel chaos due to its ash cloud. Worried that this might discourage visitors from coming to Iceland, the tourist authorities responded with an international publicity campaign that encouraged people to visit, especially during the winter when they might see the impressive display of the Aurora Borealis (or the Northern Lights). It appears that this campaign was extremely successful.

Mathematically speaking, this context helps us to explain the change in the injury numbers. If there are more tourists, particularly in the winter months, then sadly the possibility of foreigners being injured in accidents increases. Knowing about the second data set has allowed us to better understand the first set of data.

What is a reality check?

WHO ARE THE 'DATA DETECTIVES'?

A recent phenomenon has been the rise of 'fake news', stories shared online which are inaccurate or based on lies. This doesn't mean that all news is fake news, so we shouldn't question every news report without a good reason. There are organizations dedicated to fact checking, which means that they try to find the sources of stories, statistics and even rumours, to verify if the data are true and accurate.

These fact checkers aim to separate facts from fiction and mistakes from misinformation. This will allow people to make up their own minds about information, instead of being misled, either by accident or intentionally.

1 On live TV – BBC Reality Check

(British Broadcasting Corporation)

> *'The BBC is to assemble a team to fact check and debunk deliberately misleading and false stories masquerading as real news. Amid growing concern among politicians and news organisations about the impact of false information online, news chief James Harding told staff on Thursday that the BBC would be 'weighing in on the battle over lies, distortions and exaggerations'.*

Source: **www.theguardian.com**

The Reality Check team were extremely busy during the 2017 UK General Election and 'live tweeted' at the same time as TV debates were happening. This meant that while people were listening to a politician speak, the team were researching their claims and would quickly confirm or correct the information. This allowed people to decide on the trustworthiness of individuals or political parties. They are continuing this work on social media sites such as Twitter and Facebook and are now a permanent team at the BBC, working with these sites to help them to improve their content.

2 Online – Africa Check

> *'Africa Check is a non-profit organisation set up in 2012 to promote accuracy in public debate and the media in Africa. The goal of our work is to raise the quality of information available to society across the continent. Devised by the non-profit media development arm of the international news agency AFP, we are an independent organisation with offices in Johannesburg, Dakar and London, producing reports in English and French testing claims made by public figures, institutions and the media against the best available evidence.*
>
> *Since 2012 we have fact-checked hundreds of claims on topics from crime and race in South Africa to population numbers in Nigeria and fake health cures in countries around Africa.'*

Source: **https://africacheck.org/about-us**

3 On Twitter – Fact Check

> *'Our Mission – We are a nonpartisan, nonprofit "consumer advocate" for voters that aims to reduce the level of deception and confusion in U.S. politics. We monitor the factual accuracy of what is said by major U.S. political players in the form of TV ads, debates, speeches, interviews and news releases. Our goal is to apply the best practices of both journalism and scholarship, and to increase public knowledge and understanding.*
>
> *FactCheck.org is a project of the Annenberg Public Policy Center of the University of Pennsylvania. The APPC was established by publisher and philanthropist Walter Annenberg to create a community of scholars within the University of Pennsylvania that would address public policy issues at the local, state and federal levels.'*

Source: **www.factcheck.org**

4 On the radio – *More or Less: Behind the stats*

In this BBC Radio 4 show, and podcast recording, the team led by Tim Harford investigates the statistics all around us. People often ring in with claims or statistics they have heard being used and the team will track down the source, or expose the lack of a source, and explain the mathematical – as well as real-life – meanings of the numbers.

But who checks the checkers?

DISCUSS

How can mathematics be used in this philosophical way? Does this make claims and fact-checking free from bias?

Can marijuana cure measles?

No evidence for Kenyan claim

Kenyan parents are flocking to have their children treated for measles by a group of women who make use of marijuana.

The women, based in Kisumu, reportedly blow marijuana smoke into the face and ears of children as young as 2 years old, according to the Sunday Standard.

…

While there is global interest in the use of marijuana to treat a host of conditions, more research is needed, according to the World Health Organization.

Due to the difficulties of carrying out well-designed studies – such as strict legal requirements – it has not yet been rigorously tested in studies involving humans. Most of the early trials are concentrated in Europe and North America.

In June 2015, the Journal of the American Medical Association published a systematic review of 79 eligible trials out of 505 deemed relevant. The review looked at evidence for the use of cannabinoids to treat 11 pre-specified illnesses. (Note: Cannabinoids are among the most active of the more than 500 known compounds in marijuana.)

The review found 'moderate-quality' evidence to support the use of cannabinoids for the treatment of chronic pain and spasticity (unusual muscle stiffness).

'As part of our review we screened all randomized trials of medical cannabinoids and I'm fairly sure that there were none targeting measles,' Dr Penny Whiting, one of the review authors and a senior research fellow at the University of Bristol, told Africa Check.

Whiting, who specializes in reviews, added that there may be uncontrolled or non-randomized studies out there that she was unaware of, however.

A report commissioned by British lawmakers found 'good evidence' for the use of cannabis in managing chronic pain and spasticity. Covering studies published until April 2016, it also found 'moderate' evidence for its use in managing sleep disorders and low appetite.

This review found varying results for 23 illnesses, but not for measles.

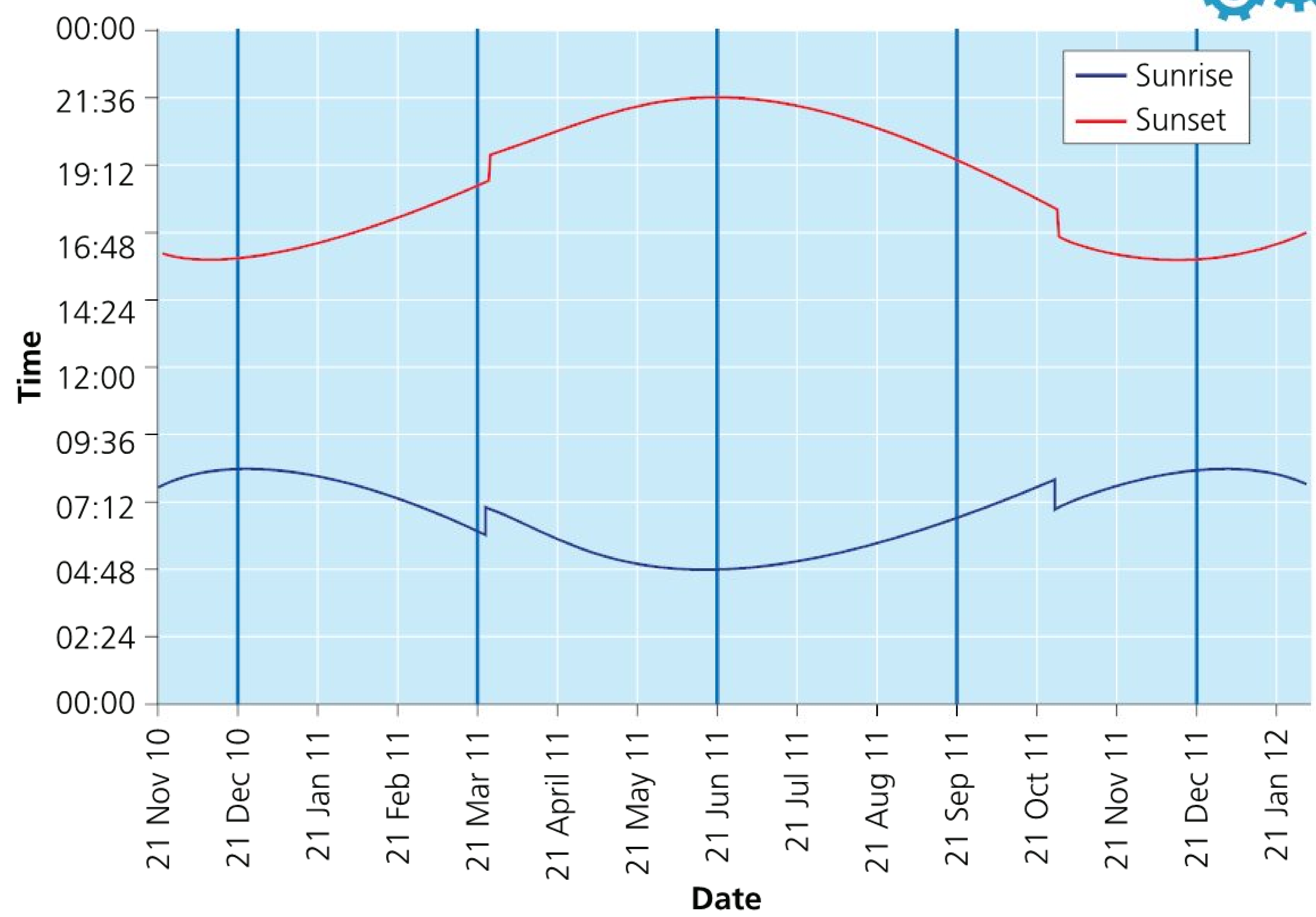
It is a 'total myth' that marijuana can cure measles, Kenya's director of public health, Dr Kepha Ombacho, told Africa Check.

'Where marijuana is being used, it is for the purpose of managing chronic pain; the cannabinoids in marijuana are a strong painkiller because of their sedative effects on the body,' he said.

Source: **https://africacheck.org**

Be a data detective!

You have been presented with this graph and no other information. Using your skills in data representation and analysis, describe the story behind the graph in as much detail as you can. What does it tell us? Where might the data have come from? Comment on the accuracy of the graph, where possible.



Source: **www.poptasticdave.co.uk**

SUMMATIVE ASSESSMENT

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

1 How would you find out which is the most popular night to go the movies? Suggest a sample that would be suitable for collecting data about going to the cinema.

2 State whether you think each of these questions is suitable or unsuitable for a questionnaire on school life. State reasons for your answers.

- What is your favourite subject?
- What is your least favourite subject?
- Do you agree that the new Mathematics teacher is excellent?
- What do you think of the amount of homework?
- Isn't the Design teacher being paid too much?

3 The design teacher has bought a 3D printer and the first thing he wants to print are some Lego-compatible bricks. He prints some with the following colours and weights.

Red (2 g)	Yellow (3 g)	Red (3 g)
Blue (5 g)	Blue (3 g)	Green (2 g)
Yellow (3 g)	Green (6 g)	Green (5 g)
Blue (2 g)	Green (2 g)	Green (4 g)
Yellow (3 g)	Blue (3 g)	

a How many bricks did he print?

b Are these primary or secondary data?

c Create a tally chart to show the number of each colour of brick.

4 The following questions on a survey have been identified as inappropriate (not suitable). Suggest a reason why this might be.

a State your age, in months.

b Circle the correct group for your age.

6–10 11–17 21–29

c How often have you been arrested?

d Why do you like that rubbish team, Manchester United?

e How many tmes have u been lat to school this week?

f What is your skin colour and how much do you earn?

5 Your cousin Kenneth is having a birthday next month and you don’t know what to get him. Suggest two ethical ways to collect data on his likes and interests.

Suggest one unethical way to collect the same data.

6 Create a dot plot to show the birth month for each person in the class.

7 The design teacher wants to print some more Lego-compatible bricks. He prints some with the following colours and weights.

Red (2 g)	Yellow (3 g)	Red (3 g)
Blue (5 g)	Blue (3 g)	Green (2 g)
Yellow (3 g)	Green (6 g)	Green (5 g)
Blue (2 g)	Green (2 g)	Green (4 g)
Yellow (3 g)	Blue (3 g)	

8 Your family is considering moving to a new area in a big city. Discuss the variables or factors on which they should collect data.

9 The design teacher wants to print even more Lego-compatible bricks. He prints some with the following colours and weights.

Red (2.8 g)	Yellow (3.4 g)	Red (3.6 g)
Blue (5.1 g)	Blue (3.3 g)	Green (2.7 g)
Yellow (3.7 g)	Green (6.1 g)	Green (5.0 g)
Blue (2.2 g)	Green (2.9 g)	Green (4.5 g)
Yellow (3.2 g)	Blue (3.9 g)	

10 The design teacher is printing another batch of Lego-compatible bricks. He prints some with the following colours and weights.

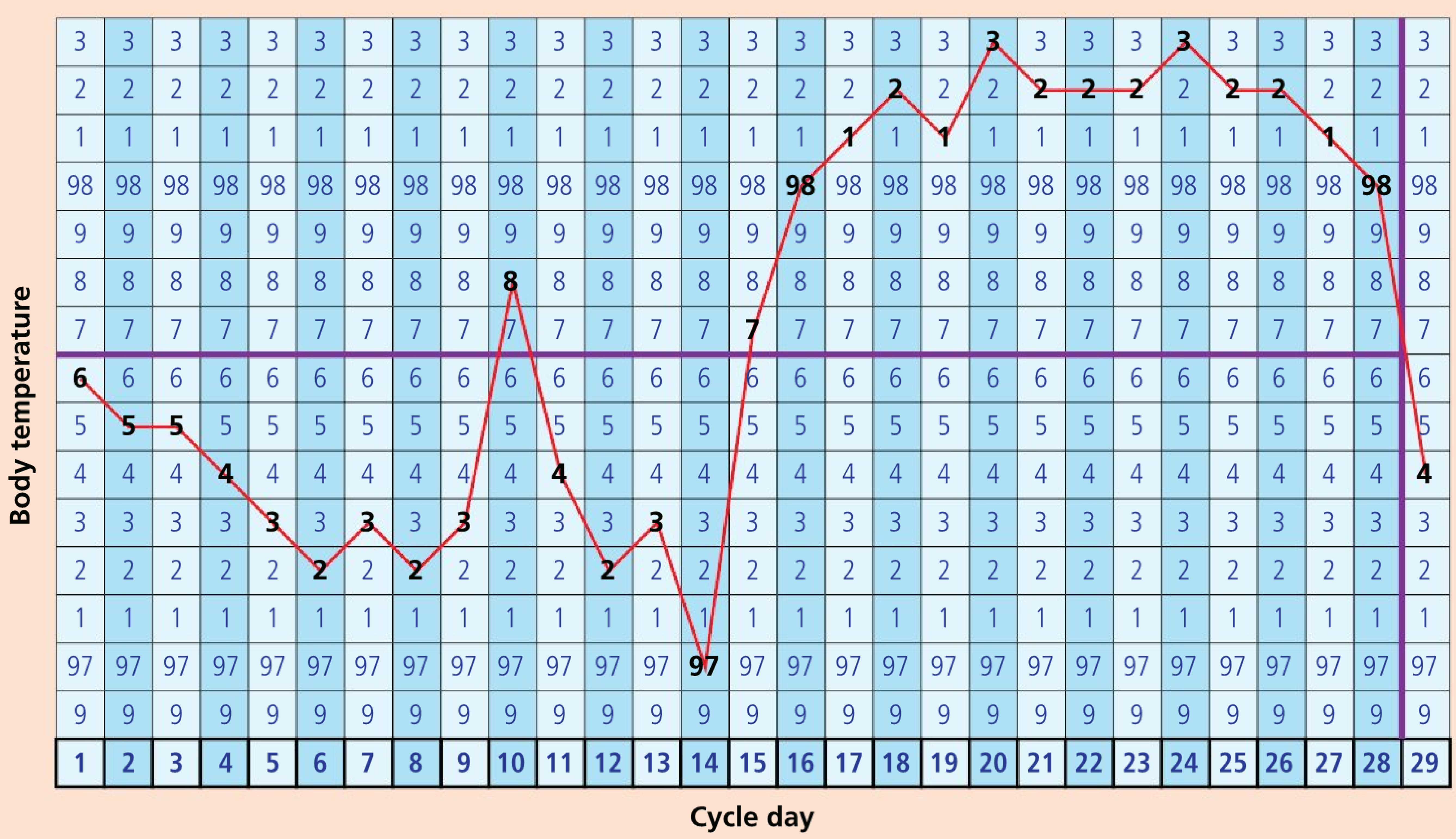
Red (2.8g)	Yellow (3.4g)	Red (3.6g)
Blue (5.1g)	Blue (3.3g)	Green (2.7g)
Yellow (3.7g)	Green (6.1g)	Green (5.0g)
Blue (2.2g)	Green (2.9g)	Green (4.5g)
Yellow (3.2g)	Blue (3.9g)	

a Calculate the mean weight of the bricks.

b The teacher inspects the last two bricks and realizes that they have split during printing. He throws them away. Now represent the remaining colours of bricks as a pie chart.

c Is this data set uni-variate or bi-variate?

d Why is it not possible to complete a scatter plot? Explain your answer.



a Are the colours of the bricks qualitative or quantitative data?

b Are the weights of the bricks qualitative or quantitative data?

c Create a frequency table for the colours of the bricks.

d Create a dot plot to represent the weights of the bricks.

a Are the colours of the bricks discrete or continuous data?

b Is weight a continuous or a discrete variable?

c Represent the weights of the bricks on a stem-and-leaf diagram.

d Represent the frequency of colours in the most appropriate form.

11 As you may have learned in biology, ovulation is the stage of the reproductive cycle in which a woman's body releases an egg. This ovulation causes an increase in the woman's basal body temperature, so some people who wish to get pregnant chart their temperatures to give them an estimate for the day of ovulation.

a What type of graph is shown opposite?

b Research to find out how the graph has been modified (changed from the usual presentation).

c On which day(s) has ovulation *probably* occurred?

d The y-axis has no units. State the unit of measurement for this axis.

Reflection

Use this table to reflect on your own learning in this chapter.

Questions we asked	Answers we found	Any further questions now?			
Factual: How do we get our hands on data? How do we organize data? What is a reality check?					
Conceptual: What systems exist for measuring? In what forms can we represent data? How do we find patterns in data?					
Debatable: How do we handle results fairly? How do we know what to trust? Do we need fact checkers?					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Communication skills					
Transfer skills					
Information skills					
Critical-thinking skills					
Collaboration skills					
Learner Profile attribute(s)	Reflect on the importance of being principled and a risk-taker for your learning in this chapter.				
Principled					
Risk-taker					

3 How can we travel between dimensions?

The **general** properties of shapes and our spatial environment can be **measured** by **logic** and manipulated and created by **technology**.

CONSIDER THESE QUESTIONS:

Factual: What do we know about shapes? How do we measure what is 'inside' a shape? What are the mathematics of Snapchat? What is a 3D printer?

Conceptual: What is inside these shapes? How does the measurement of shapes appear in our everyday lives? How can logic help us map 2D to 3D? What general rules can we find for objects? How can we have fun with shapes?

Debatable: Is nature made from shapes or visa versa? Do we need to understand shapes to innovate?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- the definition of a polygon as any closed shape with straight lines, and a regular polygon as a shape with equal sides and interior angles
- the names of regular polygons, from three sides to eight sides
- how to find the interior angle of any polygon
- how to name angles based on their size
- the meaning of perimeter, area and circumference, and how to find them
- the shapes around us in the real world and that these are mostly three-dimensional.

IN THIS CHAPTER, WE WILL …

- **Find out** how technology has made it easier to manipulate and manufacture shapes into desired forms and how mathematics can help us design these shapes.
- **Explore** the differences between dimensions and expand our minds to imagine beyond what we can see and touch.
- **Take action** by celebrating π Day.

We will reflect on these Learner Profile attributes …

- **Inquirer** – We nurture curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.
- **Reflective** – We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion B**: Investigating patterns
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

These Approaches to Learning skills will be useful …

- Creative-thinking skills
- Affective skills
- Collaboration skills

DISCUSS

'We inhabit a spatial world inhabited by spatial objects' (Nuffield 2013). These spatial objects crowd into our consciousness and fade from view with time. If we look, we will see them all around us, all the time. They are hiding, but hiding in plain sight.

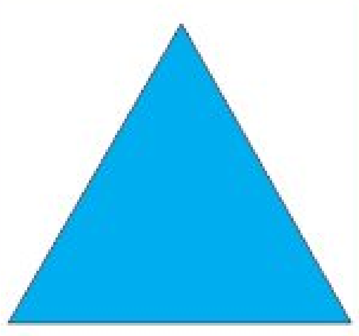
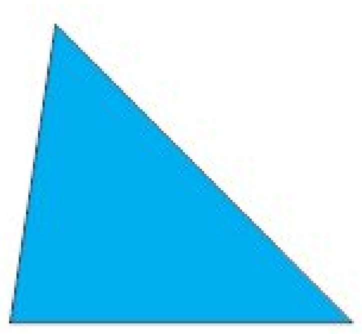
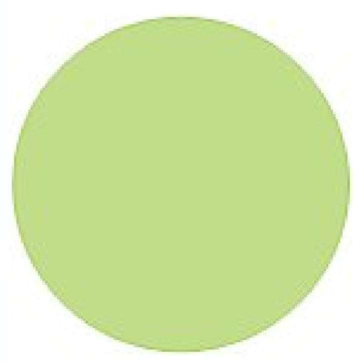
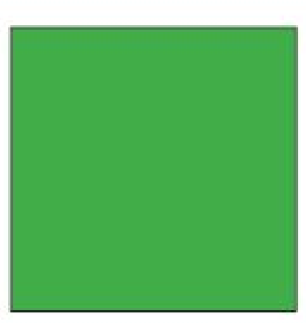
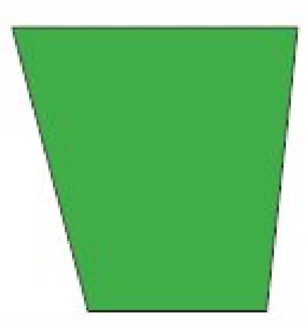
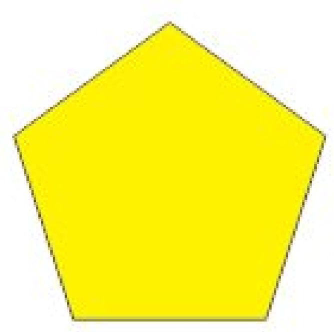
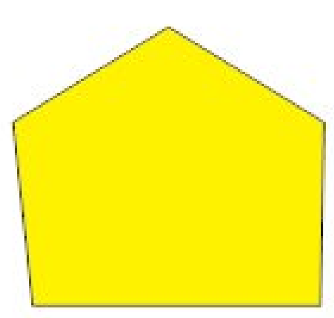
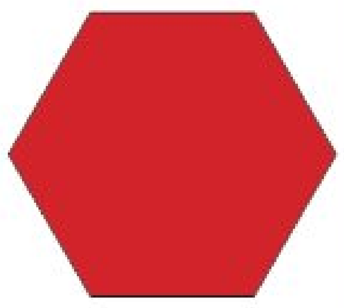
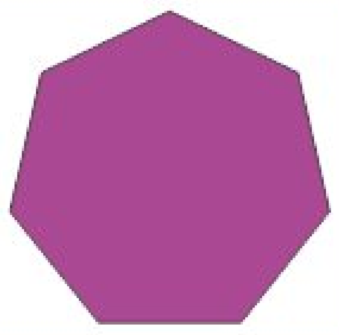
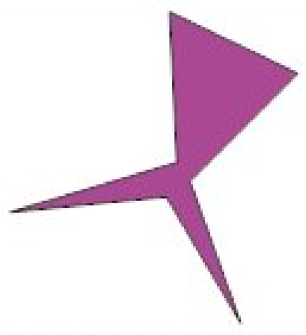
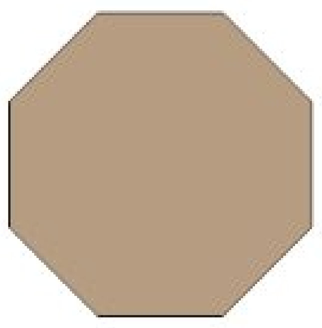
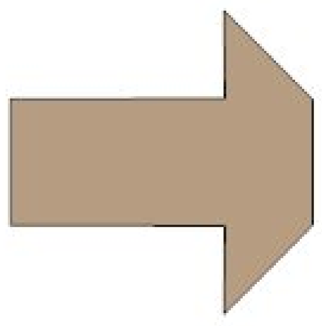
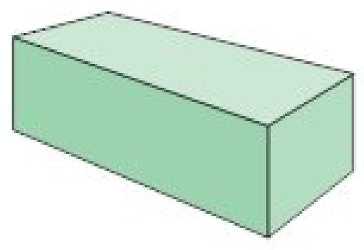
What do you think this means? How can things hide in plain sight? When do you see the shapes around you and when do you not notice them?

KEY WORDS

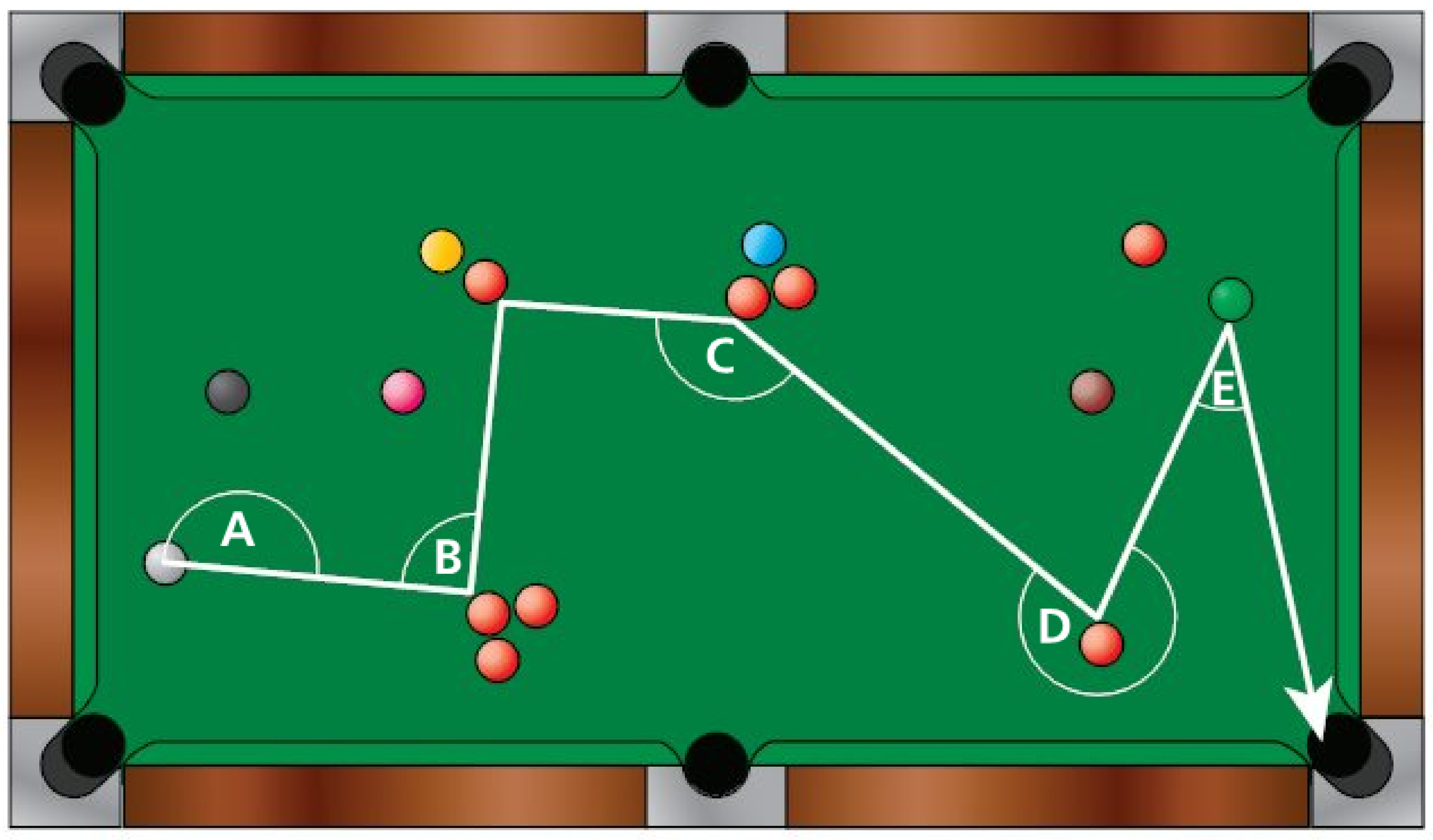
diagonal	spatial
floor	sum
polygon	surface
space	

What do we know about shapes?

All **closed** two-dimensional shapes without any curves are called polygons. A closed shape is one that does not contain an opening.

A polygon can have three or more sides	Regular polygons All sides are equal length and all internal angles are equal	Irregular polygons Any polygon that is not regular	Not polygons
3 sides Triangle			Circles 
4 sides Quadrilateral			Any shape that includes a curve
5 sides Pentagon			
6 sides Hexagon			Any shape that isn't closed
7 sides Heptagon			
8 sides Octagon			Three-dimensional objects 

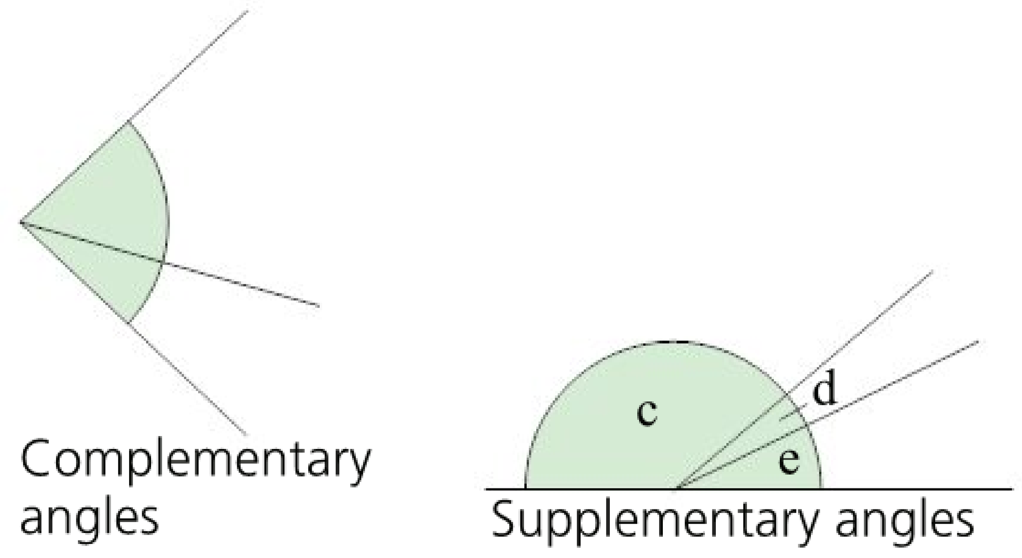
PRACTICE EXERCISE



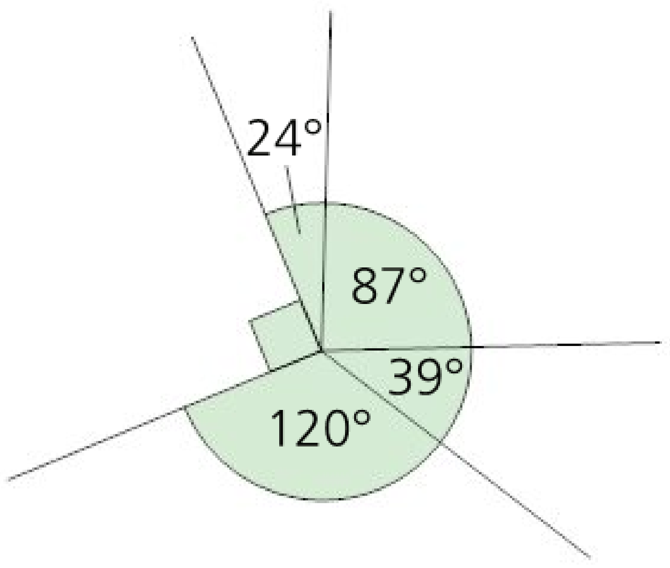
State the names of each of the angles from A to E, choosing from acute, obtuse, right angle, reflex angle and straight-line angle. How could you check without a protractor?

Don't forget that there are other quadrilaterals that are given special names, such as rhombus, kite, parallelogram, arrowhead and trapezium (trapezoid), among others. Make sure you are familiar with these shapes and can draw and identify them.

The angles shown in the diagrams below are examples of complementary and supplementary angles. **Complementary angles** are those that sum to make 90°, which means together they make a right angle. **Supplementary angles** are ones which sum to make 180°, or a straight-line angle.

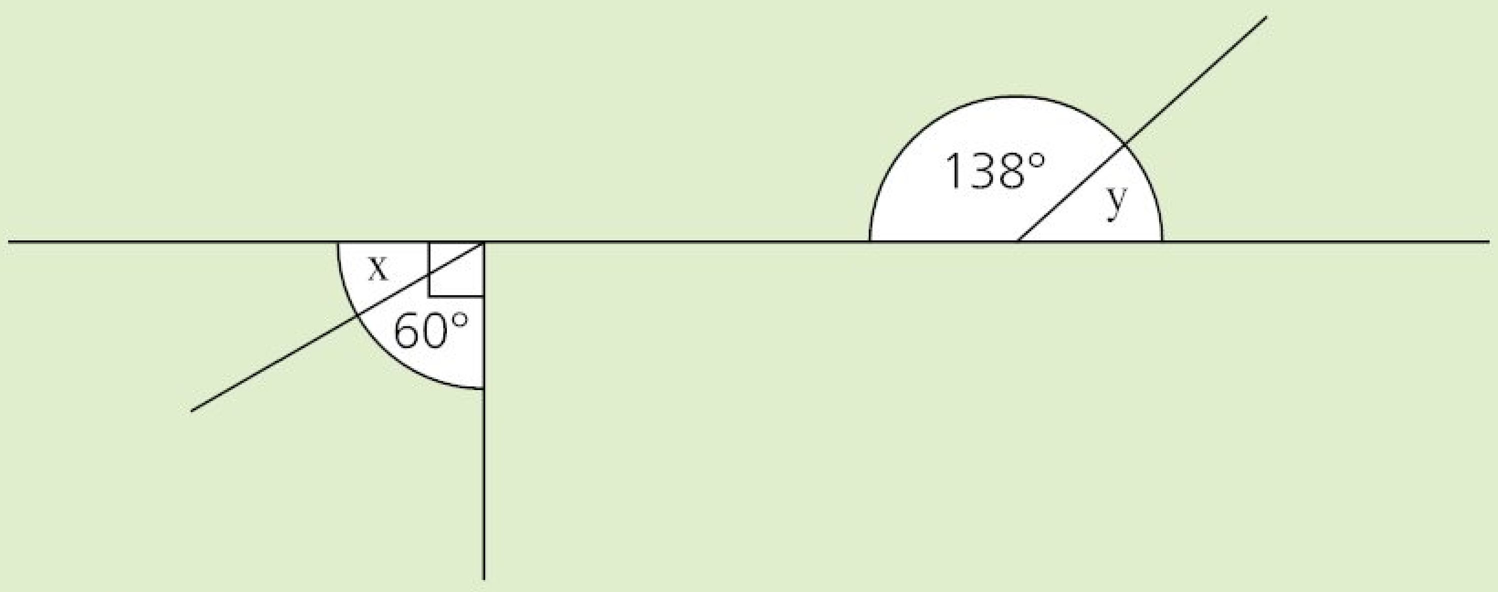


Another useful geometrical fact to remember is that angles at a point, or inside an invisible circle, sum to give 360°.



Example

Find the missing angles.



Solution

Complementary angle, x

$x + 60° = 90°$

$x = 90 - 60$

$x = 30°$

Supplementary angle, y

$y + 138° = 180°$

$y = 180 - 138$

$y = 42°$

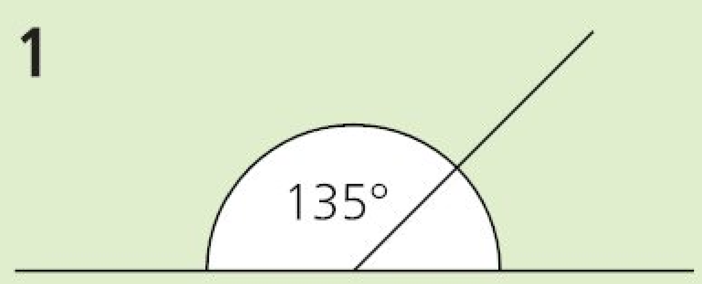
Hint

Remember how to solve algebraic expressions.

PRACTICE EXERCISE

Find the missing angles for each question, giving a reason for your answer.

1



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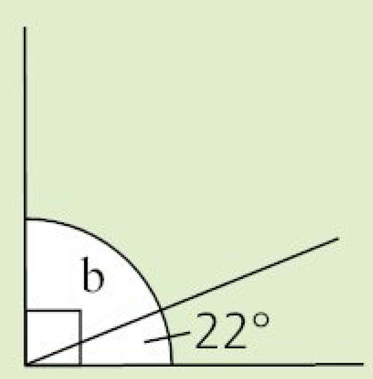

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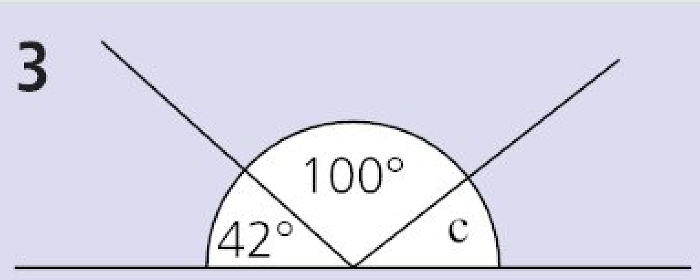

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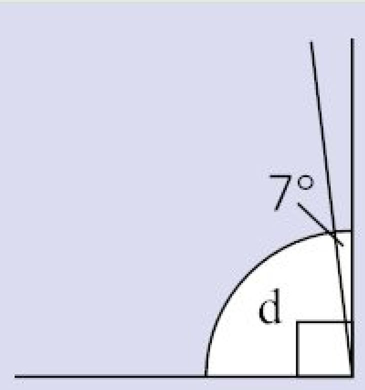

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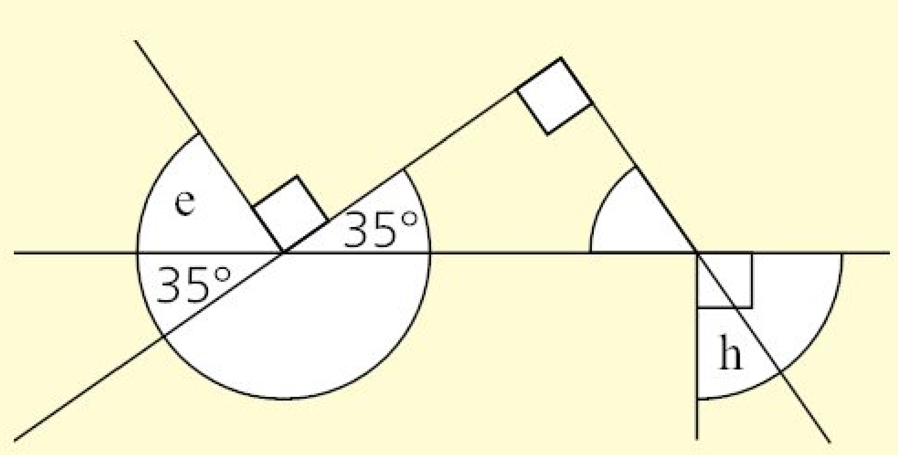


PONS ASINORUM

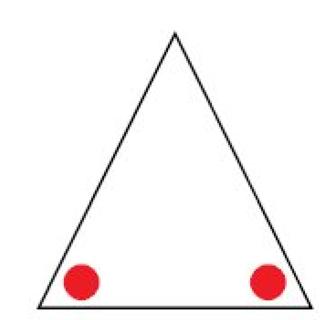
'This statement … will separate the sure of mind from the simple, the fleet thinker from the slow, the determined from the dallier; to represent a critical test of ability or understanding.'

Another theory states that 'It is said that students are as reluctant to tackle these problems as donkeys (asses) are to cross over a bridge.'

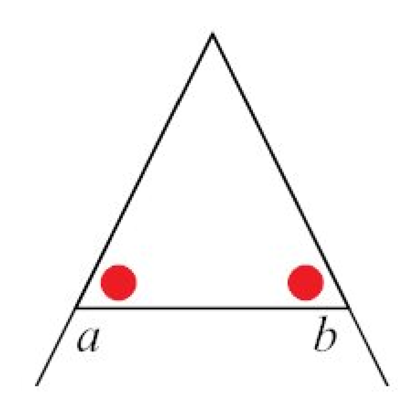
Source: **www.newworldencyclopedia.org**

The Bridge of Asses, or the 'Pons asinorum' to give it the original name, is an ancient geometric fact which states that the angles opposite to the equal angles in an isosceles triangle are themselves also equal.

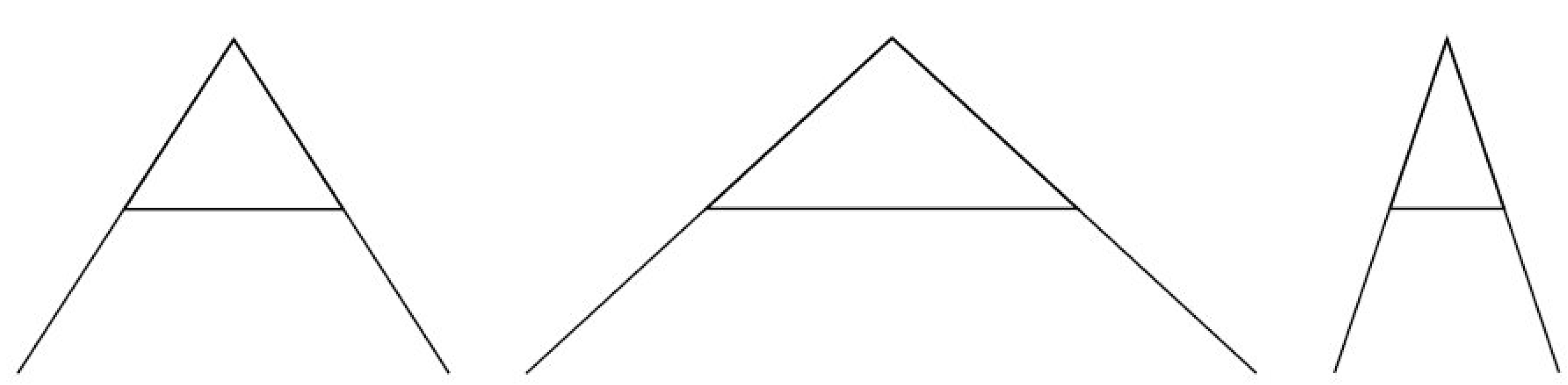
This means that for an isosceles triangle like this:



the angles opposite the red dots, a and b, are also equal.



Verify this for the following diagrams, constructing and measuring the angles for yourself.



What do you observe? Why do you think this is called the 'Bridge of Asses'?

According to legend, this was the last theorem learned by mathematics students in medieval times. Only the strong mathematicians would continue on to further study and all others escaped, never crossing the bridge to enlightenment!

ACTIVITY: Guess which?

■ ATL

■ Creative-thinking skills: Design improvements to existing machines, media and technologies

Guess who? is a game where two players each choose a card showing a character. They both have a board which shows all the characters. Using careful questioning, the players must use logic to find out their opponent's character.

Now that you are familiar with lots of types of polygons, shapes and angles, create two identical sets of cards to replace the people.

Inspiration source

Why not personalize your characters? Giving human characteristics to non-human or inanimate objects is called anthropomorphizing. You could give the shapes a theme – superheroes or Pokémon. Check out this website for some great mathematical characters: **https://solvemymaths.com/category/blog/mr-men-blog/page/3/**

◆ Assessment opportunities

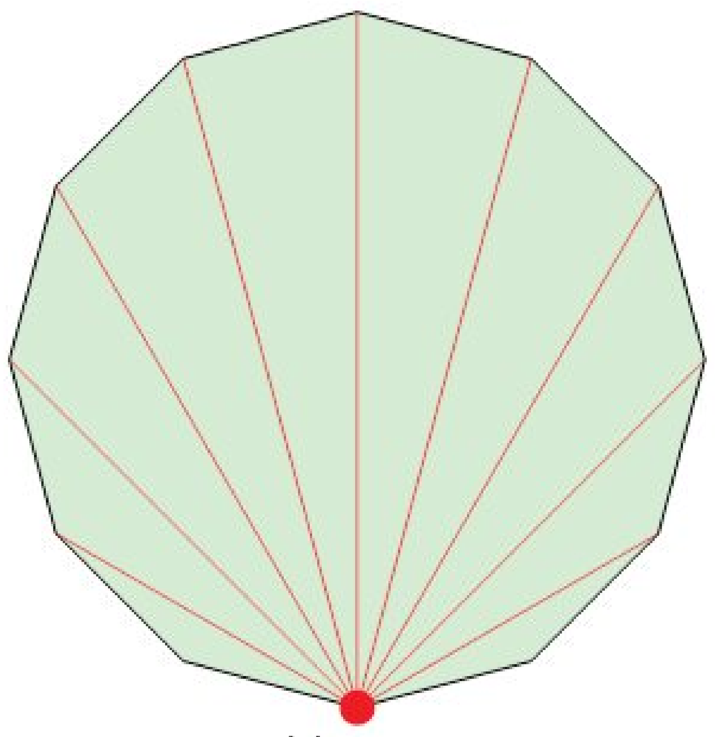
◆ In this activity you have practised skills that can be assessed using Criterion A: Knowing and understanding.

ACTIVITY: Diagonals in polygons

■ ATL

■ Affective skills: Demonstrate persistence and perseverance

In this activity, you will investigate the patterns in the diagonals of a shape by applying the same rule to each shape. A diagonal is a line segment which joins two vertices of a polygon. These vertices must not be on the same edge. For example, this diagram shows all the diagonals in a shape coming from a single vertex.



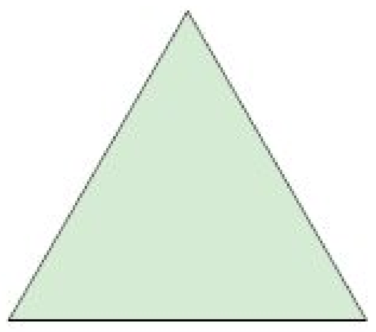
In the following polygons (shapes), you must draw all the diagonals coming from the same vertex. If you have any diagonals crossing over each other, then something has gone wrong!

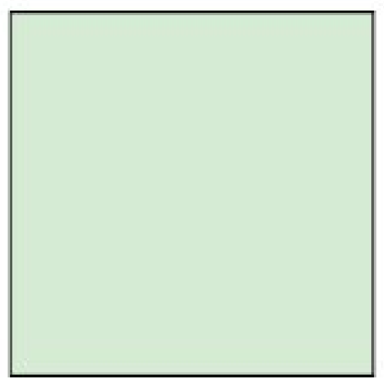
1 **Using the equilateral triangle provided, explain why triangles have no diagonals.**

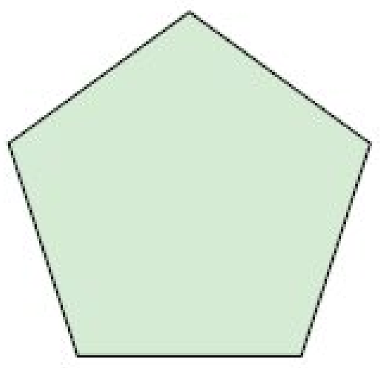
2 **Draw the diagonal(s) in a quadrilateral, using the square provided.**

3 **How many diagonals will a pentagon have? Support your answer with a diagram.**

4 **Predict how many diagonals a hexagon will have, before you draw it. Explain how you decided on your prediction by describing the pattern.**







5 **Draw a hexagon and all the diagonals. Was your prediction correct?**

6 **Now, using a copy of the table below (or another method), find a general rule which relates the number of sides (n) to the number of diagonals (d).**

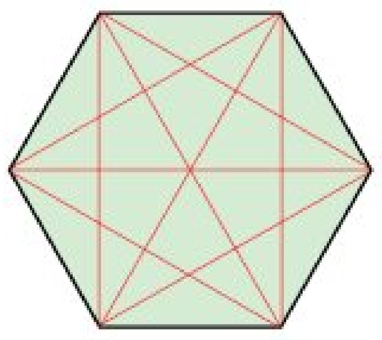
Polygon name	Number of sides (n)	Number of diagonals (d)
triangle		
quadrilateral		
pentagon		
hexagon		

7 **Using your rule, calculate how many diagonals a seven-sided shape (a heptagon) will have. Verify your answer by drawing a heptagon and its diagonals. Repeat this for an octagon (eight-sided shape).**

8 **Reflection: All the shapes you used were regular polygons. Would it make a difference to your rule if the shape was irregular?**

EXTENSION

Now, let's consider **all** the diagonals in a polygon.



The formula to find the total number of diagonals in any polygon of n sides is given by

$$d = \frac{n(n-3)}{2}$$

Using both the formula and drawings, show that this is **true**.

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

WHAT ABOUT SHAPES WITH NINE SIDES OR MORE?

So far, we know the names of polygons with sides numbering from three (triangle) to eight (octagon). Some shapes have interesting and impressive names like myriagon (a 10 000-sided shape), megagon (a million-sided shape) and octakischiliagon (an 8000-sided shape).

Find the names for shapes with:

- 9 sides
- 10 sides
- 11 sides
- 12 sides
- 14 sides
- 18 sides
- 20 sides
- 50 sides
- 90 sides
- 100 sides
- 1000 sides
- 10^{100}
- 27 sides
- 96 sides
- an infinite number of sides.

Links to: Languages

Where do the names for these shapes come from? Why do you think that is? Are there different names for the same shapes? Why do you think that might be?

ACTIVITY: When does a polygon stop?

ATL

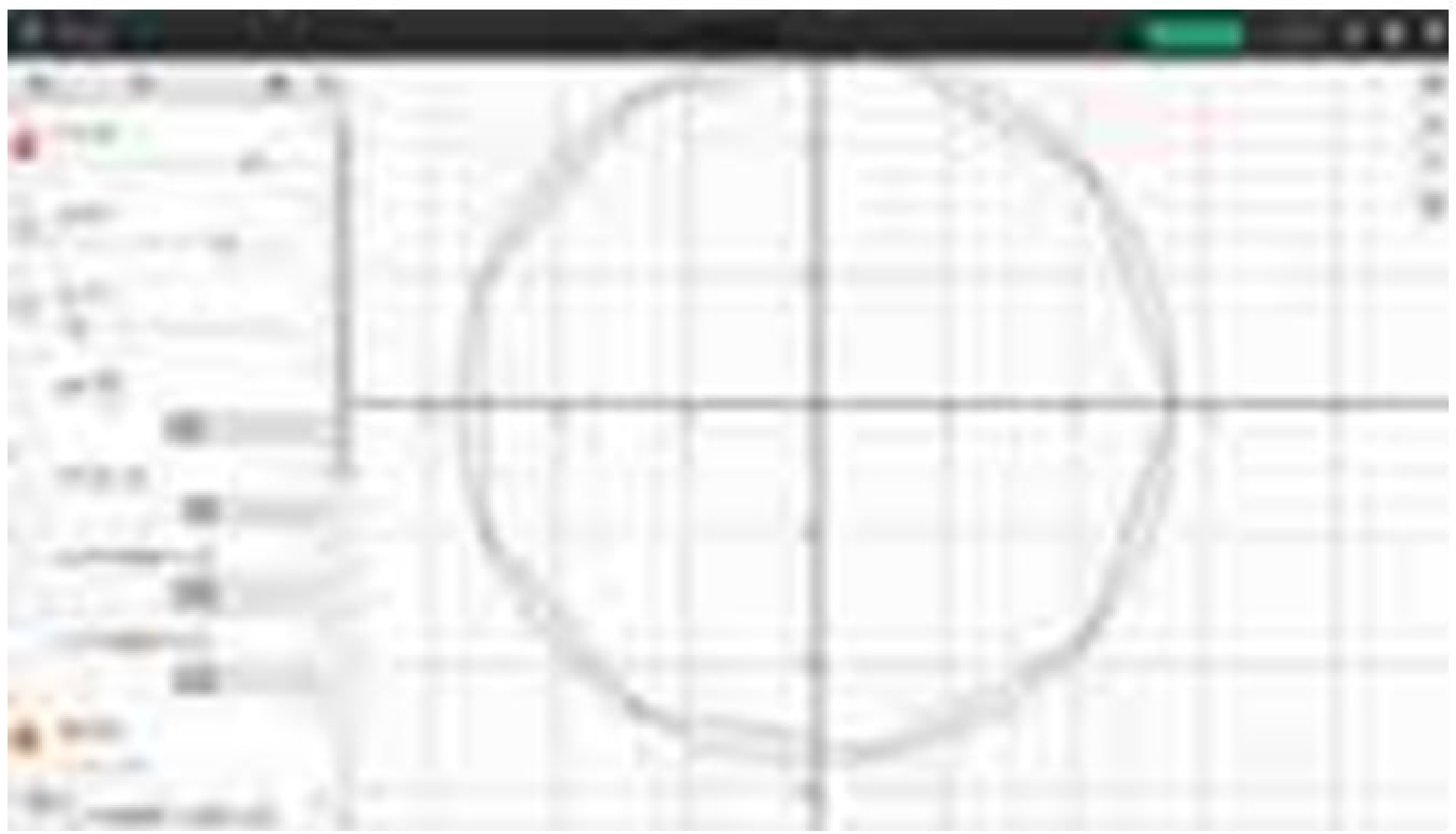
- Collaboration skills: Build consensus

Polygons are distinct and different shapes that we have learned to tell apart from a very young age. But is this true as the number of sides increases? As polygons get more sides, how we see them changes. When does a polygon stop being a polygon and become a circle? Does this point change from person to person?

Before technological innovation, we had to carefully draw these complicated polygons to investigate them. Now we can use a computer program that allows us to input the number of sides and the program draws the polygon for us.

1 **Without any experimentation, estimate the highest number of sides you think a shape could have and still not look like a circle? Take a guess if you are not sure.**

2 **Use one of the following two websites to quickly change the number of sides in a polygon:**
   - **Option 1: www.desmos.com/calculator/dfbrxkpsx5**
     Change n = to give greater values for the numbers of sides
   - **Option 2: www.geogebra.org/m/n4xePfQp**
     Change a = to give greater values for the numbers of sides



In this image, we can tell that the blue shape (a nonagon) is still clearly different from the red circle. Increase the number of sides carefully until you find the exact number at which you see the polygon 'disappearing' into a circle. Repeat a few times to make sure your final answer is as accurate as it can be. How certain are you that your answer is correct?

Now share your answer with the group, by collecting all the final answers in a list. Were there any differences? Would zooming in make a difference? As a class, discuss the possible reasons for any differences – perhaps personal interpretations or different technologies.

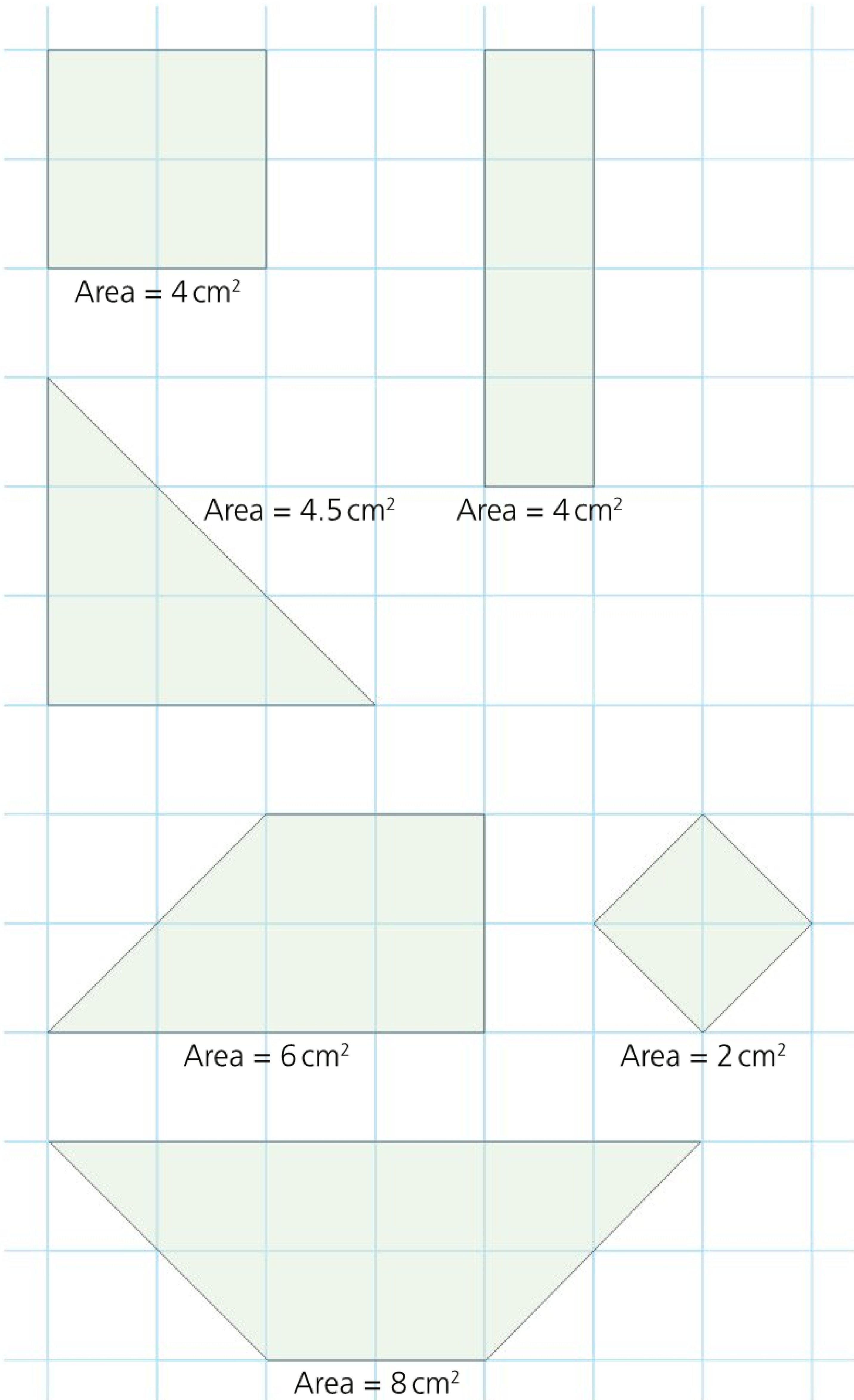
Now try to come to a final decision as to when a polygon stops being a polygon. You might want to watch this video together to confirm or discuss your decision: **www.youtube.com/watch?v=3Zs40ych2H0**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

What is inside these shapes?

In *Mathematics for the IB MYP 1* we saw that area is defined as the amount of space inside a two-dimensional shape.

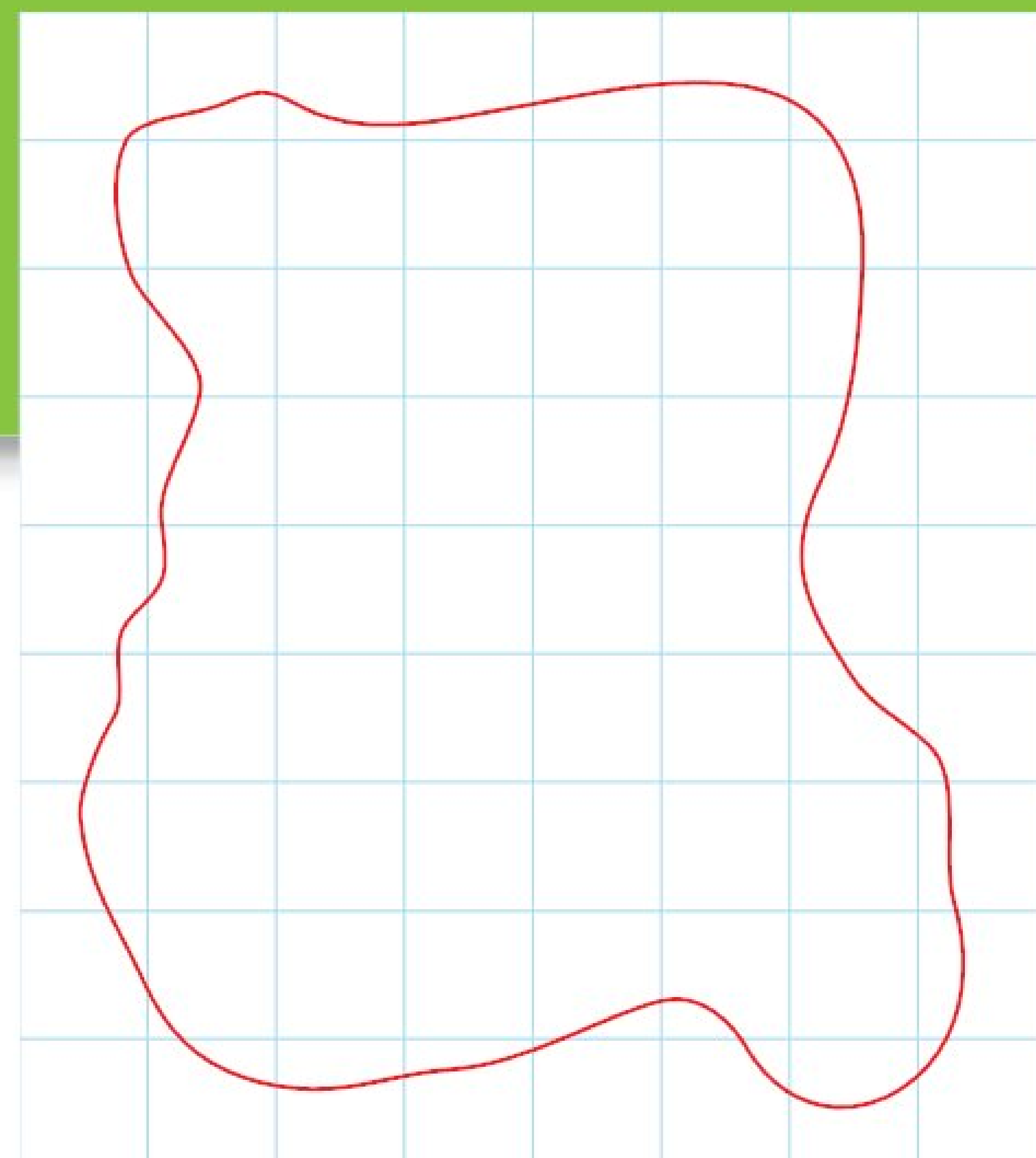


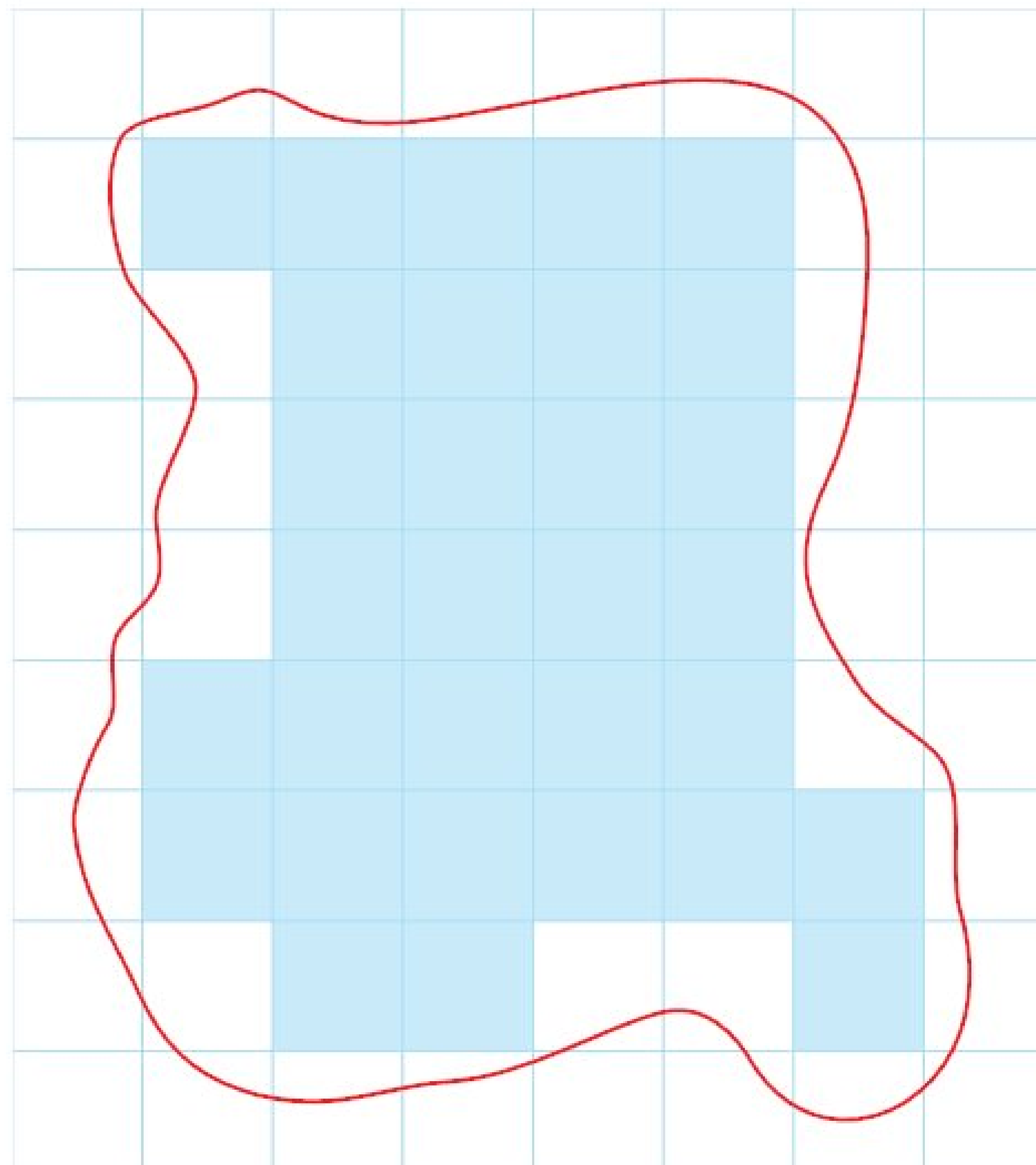
If the shapes are on 1 cm grid paper, we can easily count the boxes, or fractions of a box, within the shape to give the total area.

We can also use the grid paper to estimate the area of an irregular shape (look at the diagrams on the right).

The whole squares give an area of 31 cm^2 – 31 complete boxes. The remaining fractions of boxes can be estimated to be approximately 10 cm^2.

This gives a total area for the irregular shape of approximately 41 cm^2.





WHAT IF WE DON'T HAVE A GRID?

Real-life measurement rarely happens on grid paper, so we cannot rely on this method alone. Certain shapes have given formulas for their areas. This means there is a general rule which allows you to find the area within any similarly constructed shape.

Previously we have seen the following formula:

$$\text{area}_{\text{rectangle}} = \text{length} \times \text{width}$$

Which means that

$$\begin{aligned}\text{area}_{\text{square}} &= \text{length} \times \text{width}\\ &= \text{length}^2\end{aligned}$$

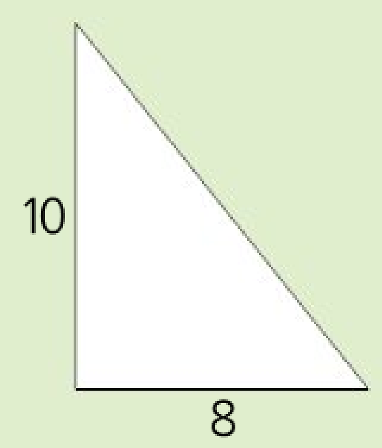
Also

$$\text{area}_{\text{triangle}} = \frac{\text{base} \times \text{height}}{2} \text{ or } \text{area}_{\text{triangle}} = \tfrac{1}{2}(b \times h)$$

PRACTICE EXERCISE

Using the formulas given above, find the areas of the following shapes.

1



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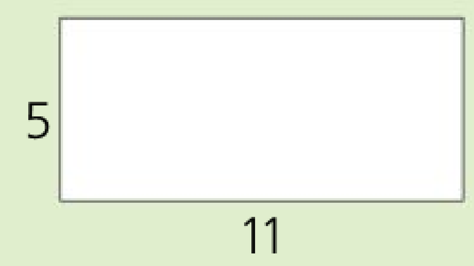

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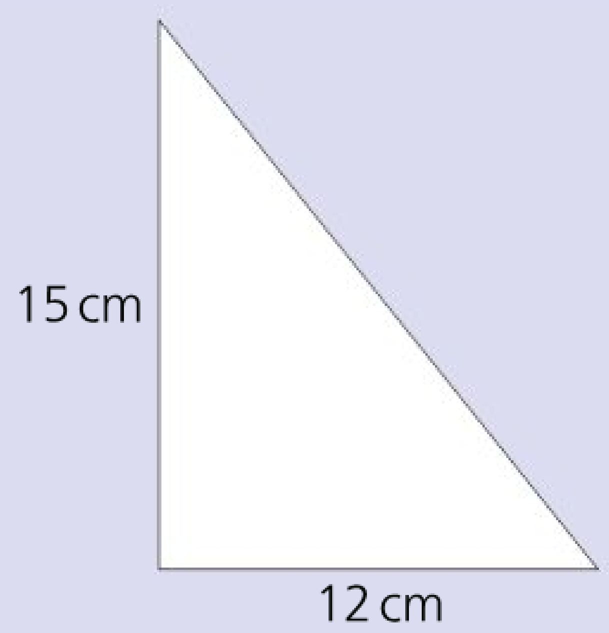

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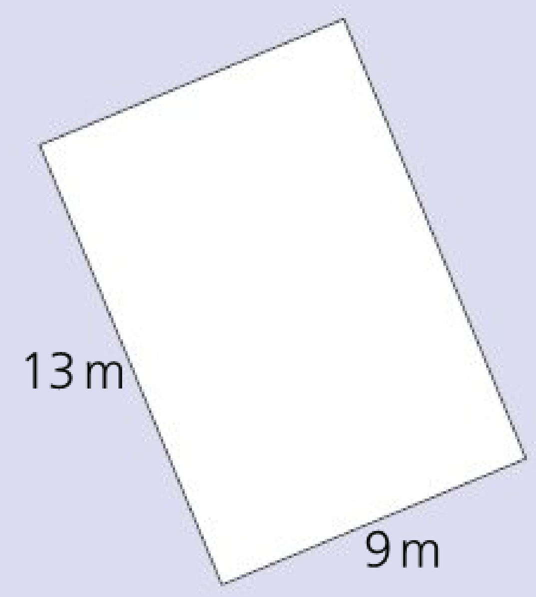

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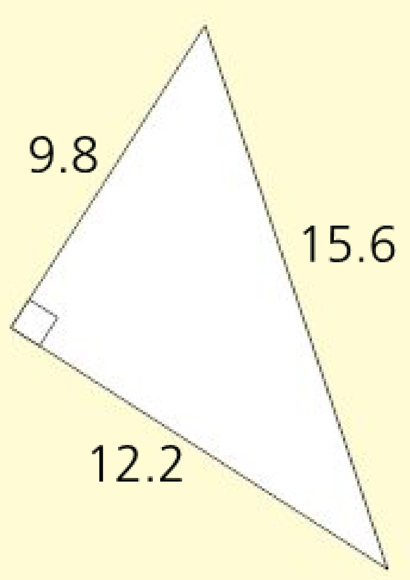

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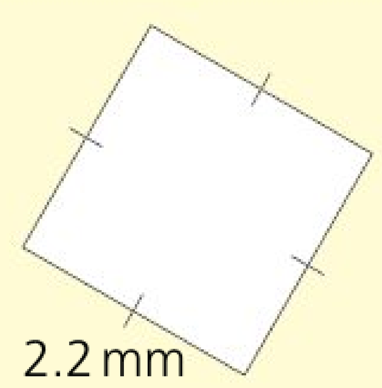
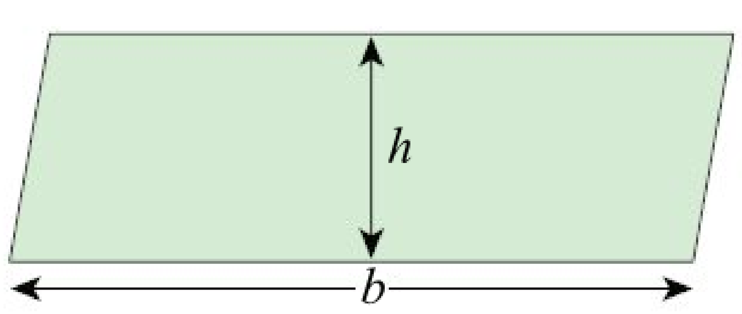


Other useful formulas for recognized shapes are:

area of a parallelogram

$A = b \times h$

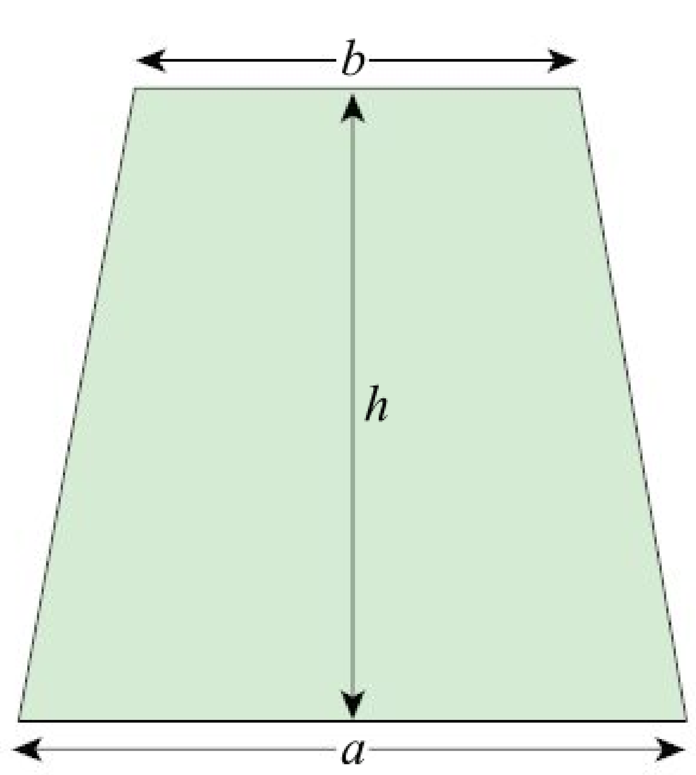


where b is the base and h is the (perpendicular) height.

area of a trapezium

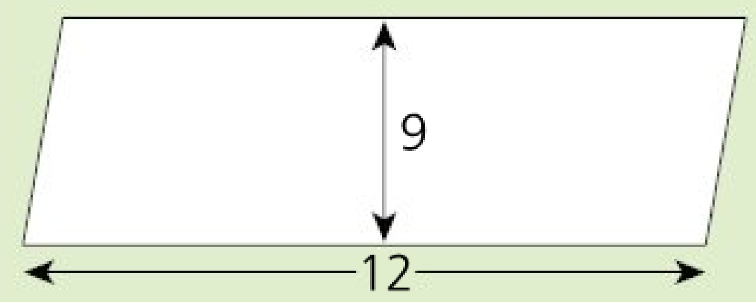
$A = \frac{1}{2}(a + b)h$

where a and b are the parallel sides and h is (perpendicular) height.



Examples

1 Find the area.



Solution

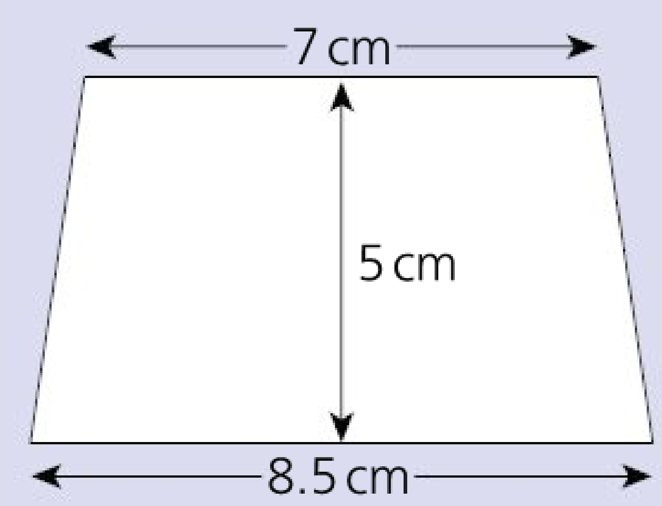
For a parallelogram:

$A = b \times h$

$A = 12 \times 9 = 108$

The area of the parallelogram is 108 units squared.

2 Find the area.



Solution

For a trapezium:

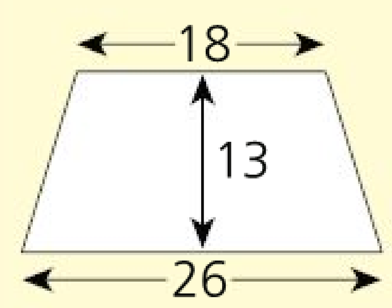
$A = \frac{1}{2}(a + b) \times h$

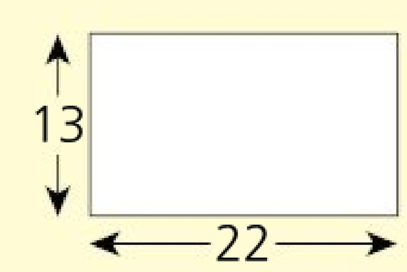
$A = \frac{1}{2}(7 + 8.5) \times 5$

$A = 7.75 \times 5 = 38.75$

The area of the trapezium is $38.75\,\text{cm}^2$.

3 Show that the areas of these shapes are the same.





Solution

area, $A = \frac{1}{2}(a + b) \times h$

$A = (0.5)(18 + 26) \times 13$

$A = (0.5)(44) \times 13$

$A = 22 \times 13 = 286$

area, $A = l \times w$

$A = 13 \times 22$

$A = 286$

The area of the trapezium **is** equal to that of the rectangle.

CIRCLE, SEMICIRCLES AND ELLIPSES

Previously, we have used the formula for the area of a circle:

$$A = \pi r^2$$

where r is the radius of the circle and π is the number pi.

Remember that π is an irrational number, whose decimal never ends and is found by the ratio of the circumference of a circle and its diameter. If you need a refresher on the meaning or origin of pi, π, look back to page 121 in *Mathematics for the IB MYP 1*.

! Take action

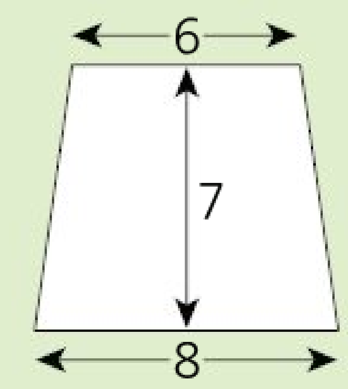
Celebrating π day

- On the 14th of March every year, mathematics fans celebrate Pi Day because the date is 3/14! In fact, in 2015 Pi Day was extra special because the date was 3/14/15 and some people had Pi Day parties which started exactly at 9.26 (and 53 seconds). Why was that?
- Why not plan a Pi Day celebration of some kind for your class or school? One fun activity is to have a 'Pi-off'. A good way to test your memory is to see how many digits of pi you can memorize. It won't help you in mathematics class, but it can be fun to test yourself and challenge others.

PRACTICE EXERCISE

Calculate the areas of these shapes.

1



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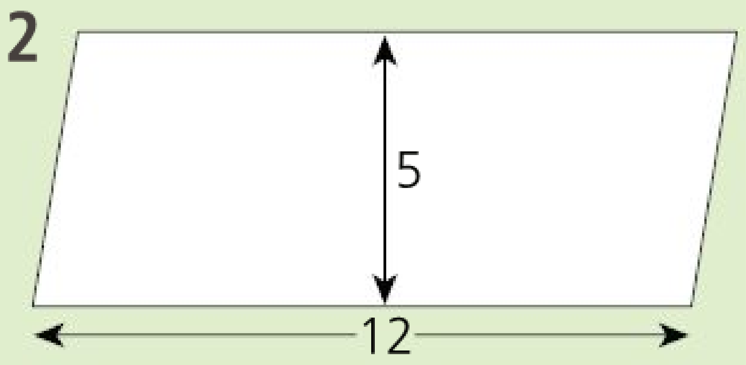

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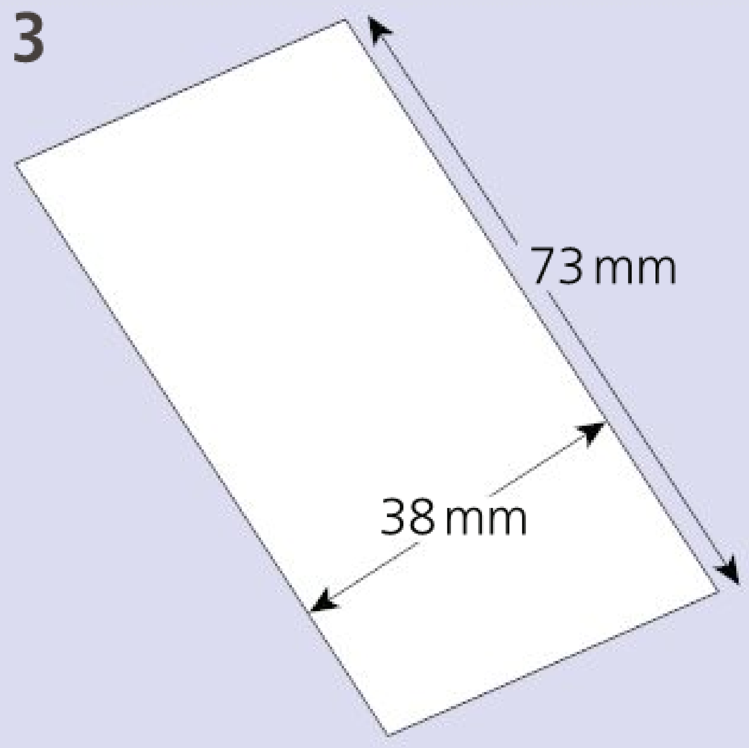

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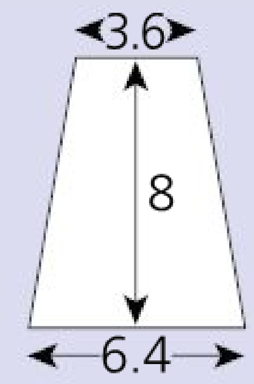

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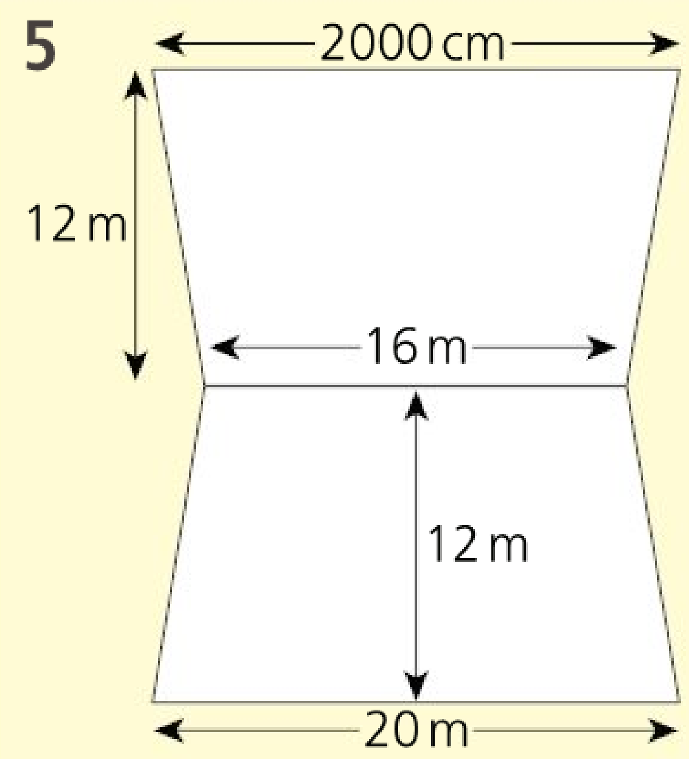

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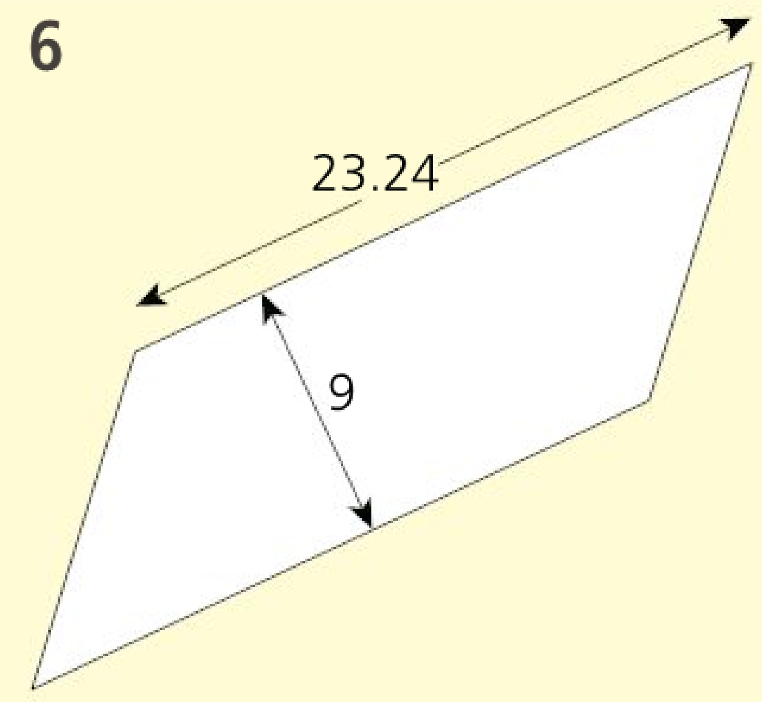
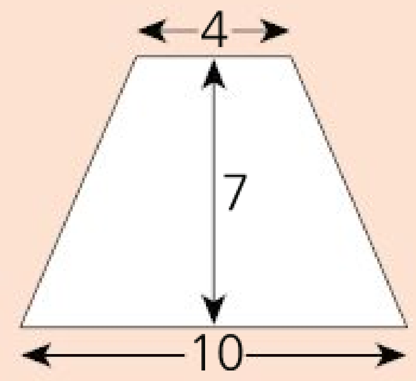
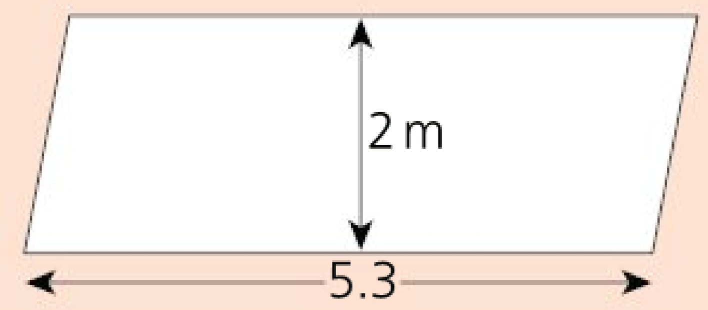


7 Find the lengths of the sides of a square with the same area as this trapezium.



8 Suggest possible values for a, b and h for a trapezium with the same area as this shape.



The number pi begins with the digits 3.141592 and continues without any recognizable or predictable pattern.

3	.	1	4	1	5	9	2
Hey	!	I	wish	I	could	calculate	pi

■ A fun way to remember the first seven digits of pi!

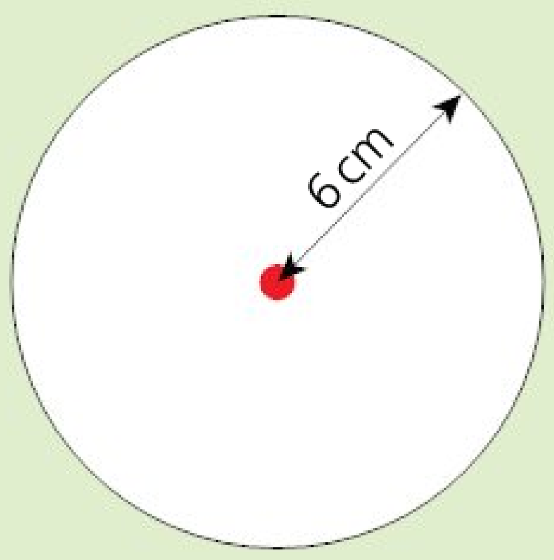
SO HOW DO WE USE π?

To find the area of a circle, semicircle or segment of a circle you need to know, understand and use pi.

Remember, the area of a circle is given by $A = \pi r^2$.

Examples

1 Find the area of this circle.



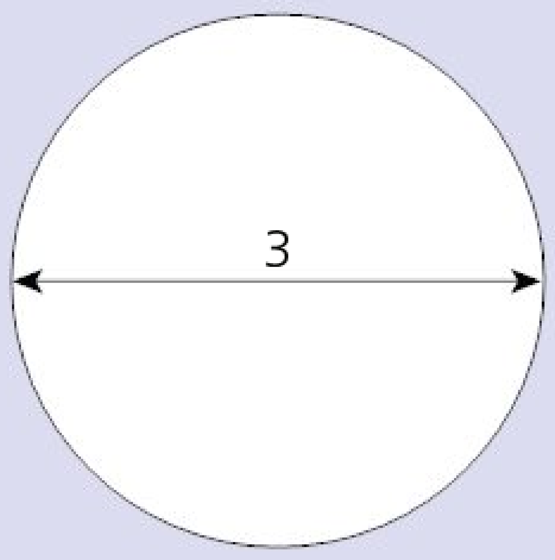
Solution

$A = \pi r^2$

$A = (3.141592)(6)^2$

$A = 113.1\,\text{cm}^2$ (correct to one decimal place)

2 Find the area of the circle.



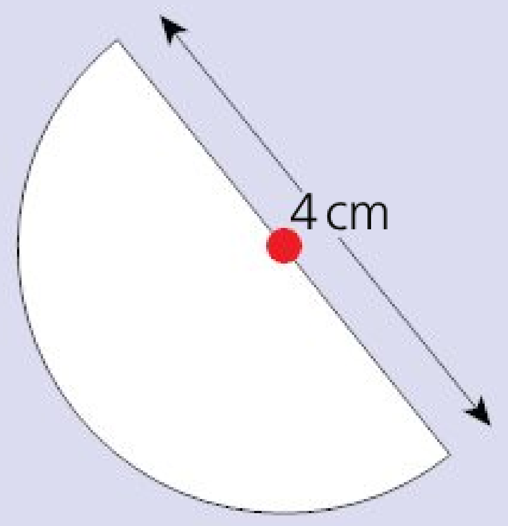
Solution

$A = \pi r^2$

$A = (3.141592)(1.5)^2$ as radius, r, is half the diameter, d.

$A = 7.1$ units squared (to one decimal place)

3 Find the area of the semicircle.



Solution

First find the area of the whole circle:

$A = \pi r^2$

$A = (3.141592)(2)^2$

$A = 12.56637\,\text{cm}^2$

If the area of the entire circle is approximately 12.56637, then a semicircle would have an area exactly half that. Therefore, the area of the semicircle is approximately $6.28\,\text{cm}^2$.

4 Find the radius of a circle with an area of $78.6\,\text{cm}^2$.

Solution

Using the formula $A = \pi r^2$

we know that $78.6 = \pi r^2$

Using algebra, we can rearrange:

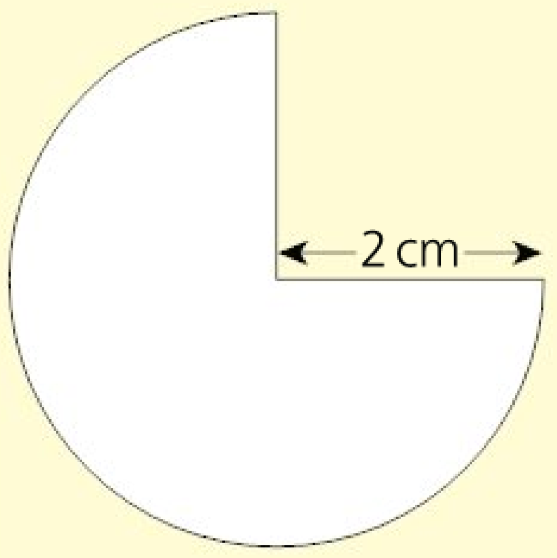
$$r^2 = \frac{78.6}{\pi}$$

$$r^2 = 25.002$$

$$r = \sqrt{25.002}$$

$$r \approx 5\,\text{cm}$$

5 Find the area of this shape.



Solution

$A = \pi r^2 \times \frac{3}{4}$

as the shape is three-quarters of a full circle.

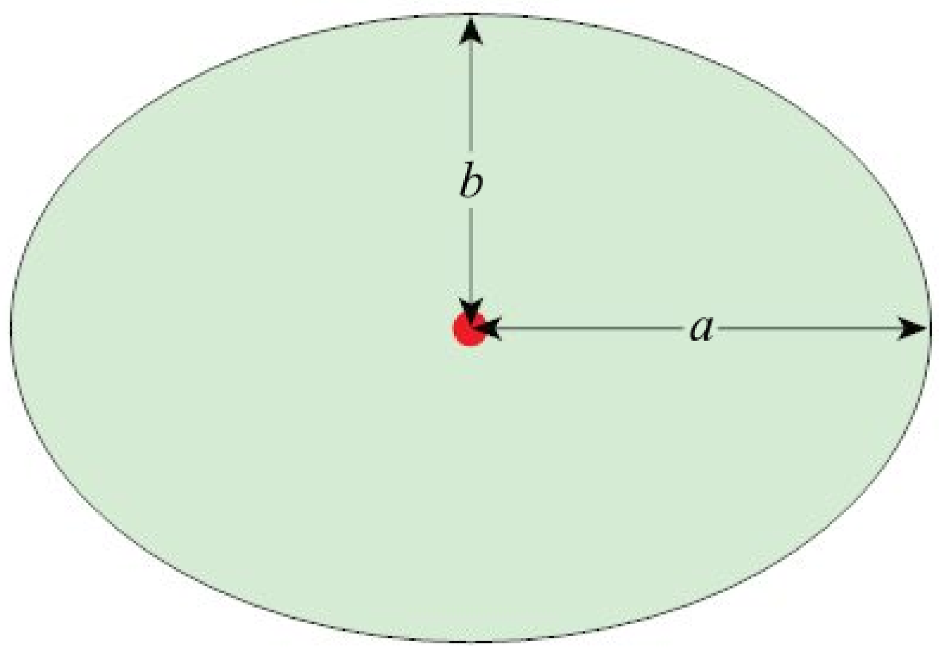
$A = \pi(2)^2 \times \frac{3}{4}$

$A = 9.42\,\text{cm}^2$

EXTENSION

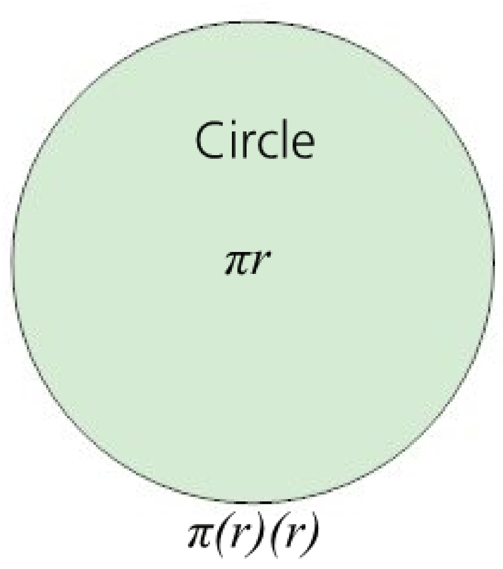
What about ellipses?

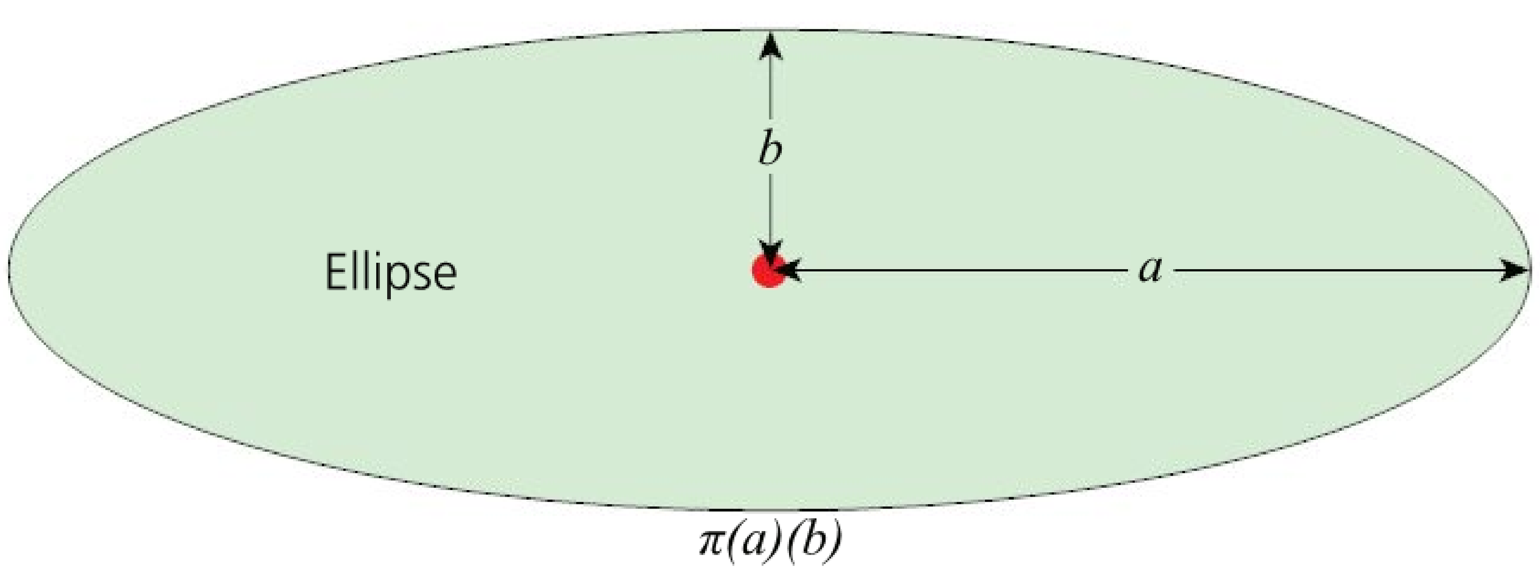
The radius in a circle is the same no matter where you measure it. If you take one point in the circle and start to stretch it to make it longer, you can make a shape known as an **ellipse**.



The lengths of a and b are different because an ellipse isn't a circle. These lengths are known as the **semi-axes** of the ellipse. The values of a and b are the longest and shortest distances from the centre to the perimeter (circumference).

The formula for the area of an ellipse can be found by replacing terms in the circle formula.





The formula for the area, A, of an ellipse is given by

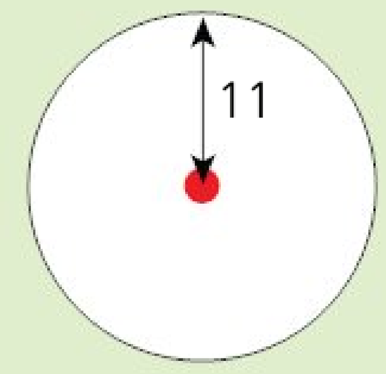
$$A = \pi ab$$

We often hear the word 'elliptical' in ordinary life, such as in the phrase 'elliptical orbit'. This phrase is used when discussing movements of bodies in space and can be heard in the dialogue for many science fiction movies. What do you know of that moves in an 'elliptical' orbit?

PRACTICE EXERCISE

Find the area of the following shapes.

1



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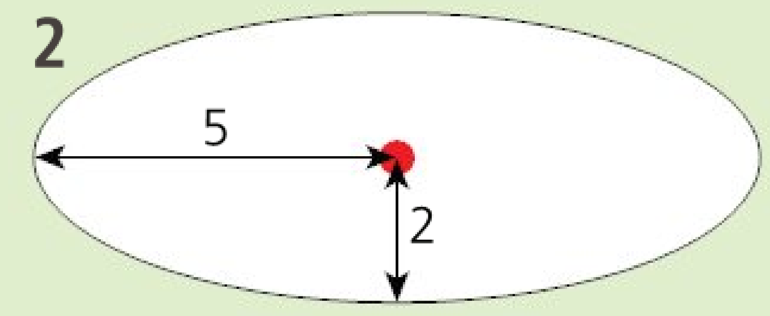
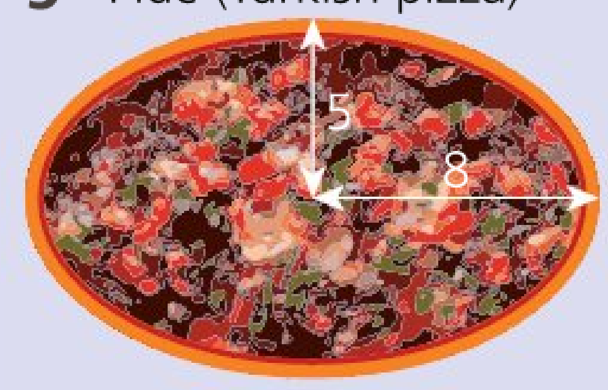
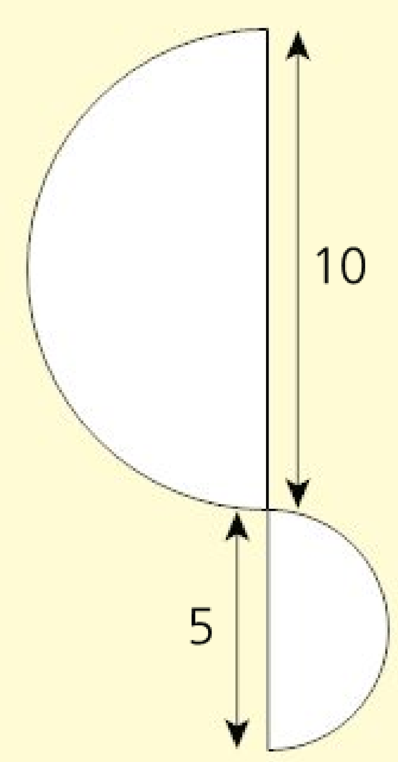


3 Pide (Turkish pizza)



4 Pizza of diameter 17 cm

5



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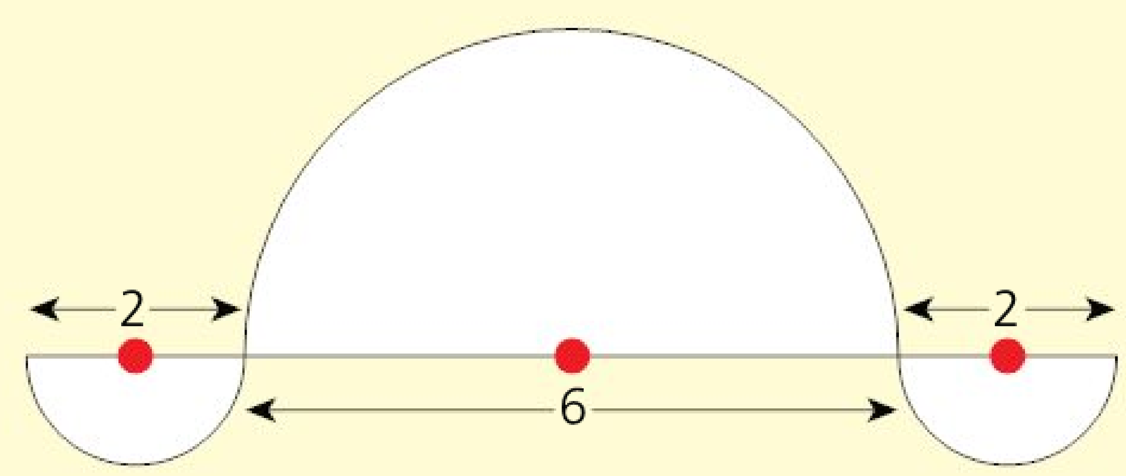


7

8

Is nature made from shapes or visa versa?

THINK-PAIR-SHARE

Why have humans created such myths and legends to explain this geometric phenomenon? Does the site seem unnatural to you? Why or why not? Do we need to understand the rock formation's origin to appreciate it?

www.ireland.com

The Giant's Causeway

Something this pretty couldn't be the result of a volcanic eruption 60 million years ago, could it?

Well, there are two lines of thought on that one. The first involves a certain giant by the name of Finn McCool (also known as Fionn mac Cumhaill).

Giant fights

Finn is having trouble with someone across the water. The Scottish giant Benandonner is threatening Ireland. An enraged Finn grabs chunks of the Antrim coast and throws them into the sea. The rock forms a path for Finn to follow and teach Benandonner a lesson.

Bad idea – Benandonner is terrifyingly massive. Finn beats a hasty retreat, followed by the giant, only to be saved by our hero's quick-thinking wife who disguised him as a baby. The angry Scot saw the baby and decided if the child was that big, the daddy must be really huge.

The science bit

'The Giant's Causeway is the aftermath of volcanic crashing, burning and cooling,' Eleanor [Killough, a guide at the Giant's Causeway visitor centre] explains. 'An epic 60-million-year-old legacy to lava. Over 40 000 basalt columns. Interlocked.

'It's no wonder this place is a UNESCO World Heritage Site because beyond the mindboggling beauty, the Causeway is our portal into Earth's most ancient past,' she concludes.

Picture-perfect scenes

Whatever you choose to believe, there's no disputing that the Causeway makes a pretty picture. Thousands of tourists click their cameras here every year, and when the Olympic Torch visited Northern Ireland, it was a photo opportunity not to be missed.

Source: www.ireland.com

Look around you now – what shapes can you see? How many different regular polygons can you find? How many irregular polygons can you find? Can you name them all? How do their sizes and **orientations** change?

MEET A MATHEMATICIAN: MARCUS DU SAUTOY (1965–PRESENT)

Learner Profile: Inquirer, Reflective

> 'The power of mathematics is often to change one thing into another, to change geometry into language.'
>
> *Marcus du Sautoy*

Mathematician, Professor, TV presenter, tour guide and author, Marcus du Sautoy is known for his work in popularizing mathematics and engaging audiences. His 2011 documentary for the BBC, entitled 'The Code', shows mathematics all around us. Professor du Sautoy takes us on a journey from honeycombs to soap bubbles to see how the hidden code in the world around us is actually geometry in action. Why does a honeycomb always consist of hexagon shapes? How do salt crystals form as perfect cubes? Why do bubbles share sides? It is a fascinating journey, in which we are invited to inquire into and reflect on the patterns that shape the world around us.

Search for: **The Code BBC documentary**

In 2009, Professor du Sautoy was awarded the Royal Society's Faraday Prize, the UK's premier award for excellence in communicating science. He received an OBE for services to science in the 2010 New Year's Honours List. In 2014 he was a Keynote Speaker for the IB Conference in Singapore.

ACTIVITY: What are the mathematics of apps like Snapchat?

ATL

- Affective skills: Practise focus and concentration

Snapchat is a popular social media platform that allows users to share photographs for a short period of time. One of the most popular features is the ability to add objects to, or change, an image of your face. The software identifies key features of your face and changes them dramatically using 'Lenses' or 'Filters'.

How does this work? How can Snapchat and other similar apps tell where your nose, chin or hair is? Watch this video to learn more: **www.youtube.com/watch?v=Pc2aJxnmzh0**

Now, let's consider the 'mask' that social media apps **superimpose** onto a face. You will need a ruler and a calculator to continue this activity.

Part 1 – Making a mask

1 **Measure the total perimeter of the Snapchat mask, or a mask from a similar app.**

2 **Find the area of the lips, according to this mask.**

3 **Suggest a reason why it might be difficult to find the area of the forehead only (without any eyebrows) from the image.**

4 **Using your measurements, find a total area for the mask. Be patient when doing this. There are lots of smaller sections within the mask and it is important that you count them all. If you have a copy or a sketch of the mask, you might want to shade in any shapes you have included, so you don't lose count!**

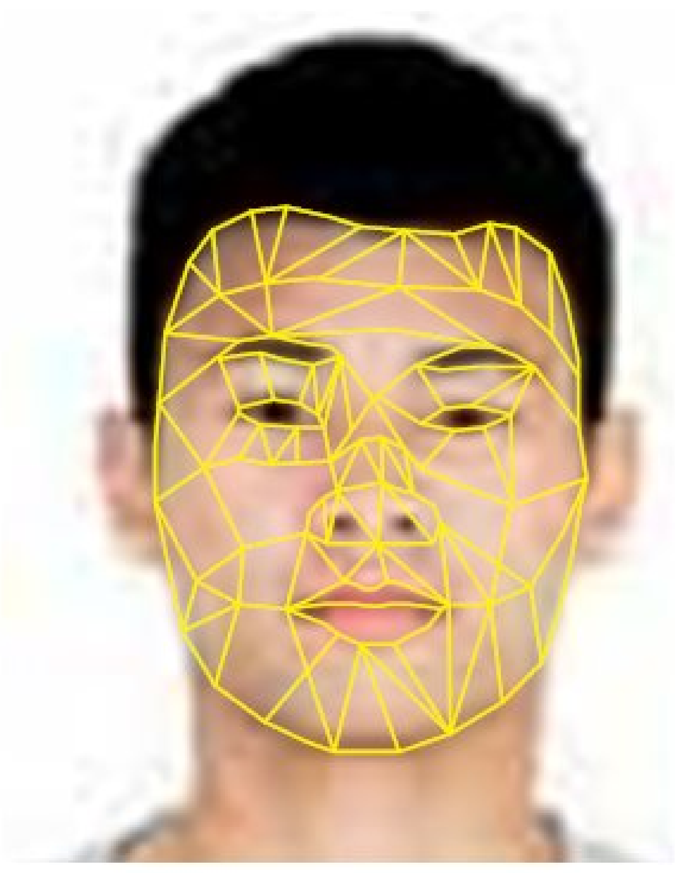
5 **See how close you are to calculations carried out by other members of the class. Comment on the accuracy of your answer.**

Part 2 – Applying a mask

You will need a printed photograph of your face to continue with this task.

6 **As carefully as you can, draw the mask perimeter around your face.**

7 **Using what you have learned about facial recognition, or estimating the triangles on the mask you made in Part 1, draw the mask onto the image of your face.**



8 **Calculate the area of your own mask.**

Part 3 – Creating a new effect

9 **A new Snapchat effect takes parts of the mask and makes them much larger than other parts of the face, for example the eyes get exactly two times larger and the mouth gets three times larger. It moves the rest of the face outwards and creates a spookily unreal effect.**

Calculate the new area of your mask, once these changes have been made.

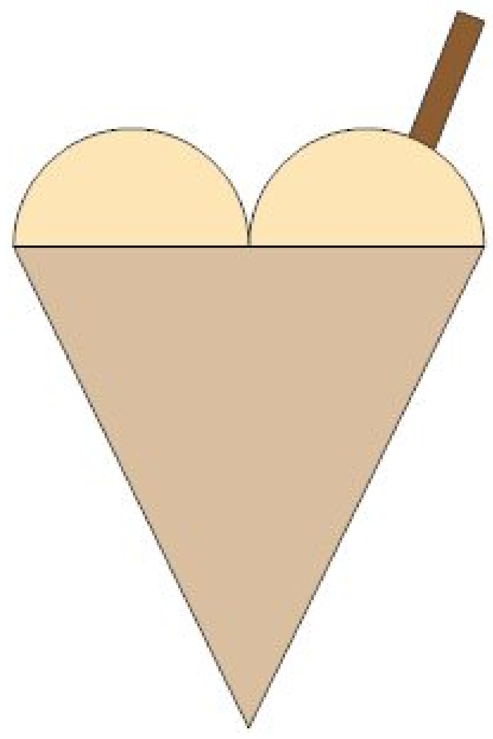
10 **Now cut up your mask, so that you can remove the eyes and the mouth. Copy the eyes and mouth, redrawing them with the correct enlargements (make the eyes twice as big and the mouth three times as big). Place them back in your mask.**

You have just created your own Snapchat-type effect outside of the virtual world!

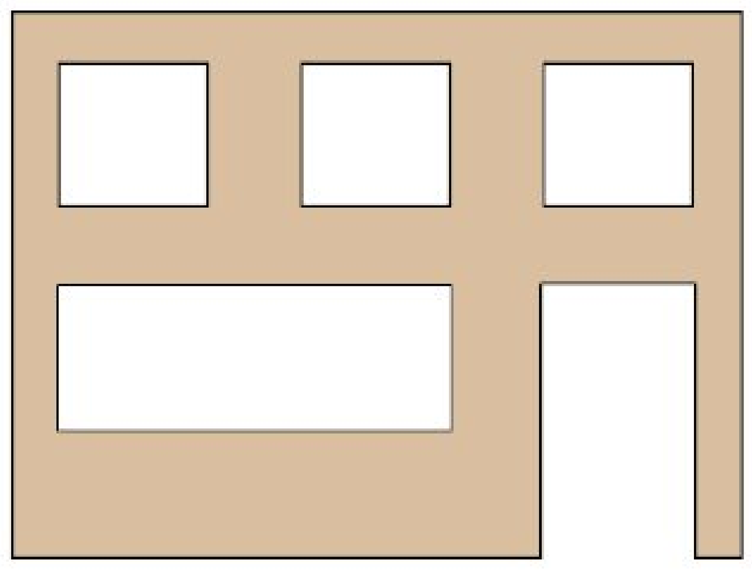
Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

Example A



Example B

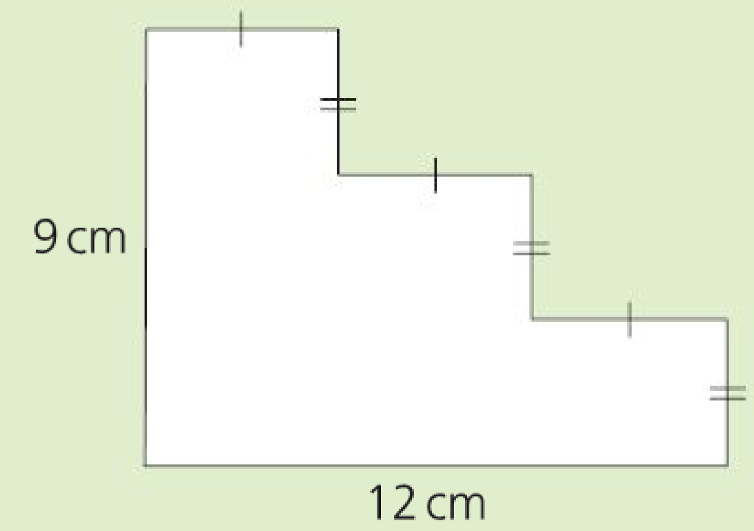


A Snapchat mask is an excellent example of a composite shape – a shape which is made up of smaller, recognizable polygons or elements.

Many shapes we see around us, in nature and human-made, are composites of shapes. You may remember from *Mathematics for the IB MYP 1* that we broke complicated composite shapes into smaller and simpler elements to help us with calculations. Sometimes the area of a shape is found by adding the smaller parts (Example A) and other times it can be found by subtracting them, to find remainders or shaded sections (Example B).

Example

Calculate the area and perimeter of the composite figure in the diagram.



Solution

area, A, of rectangle A

$A = 4\,\text{cm} \times 3\,\text{cm} = 12\,\text{cm}^2$

area, A, of rectangle B

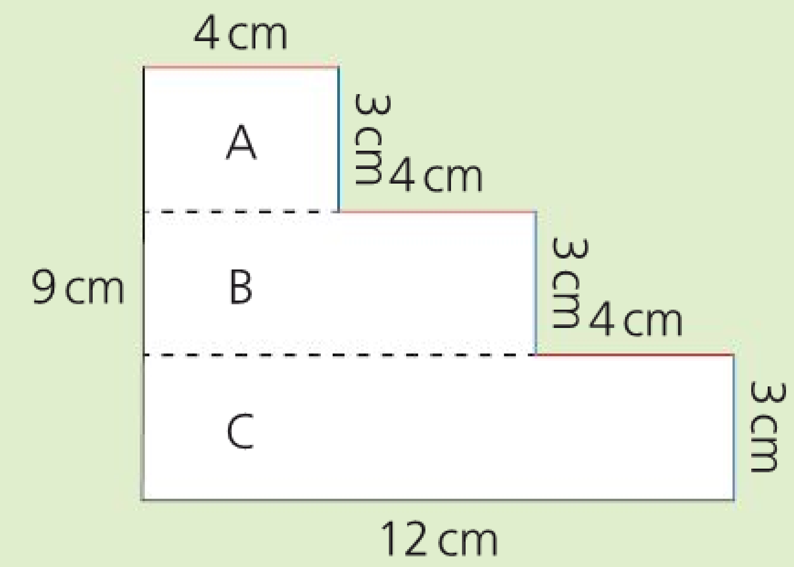
$A = 8\,\text{cm} \times 3\,\text{cm} = 24\,\text{cm}^2$

area, A, of rectangle C

$A = 12\,\text{cm} \times 3\,\text{cm} = 36\,\text{cm}^2$

Total area:

$12\,\text{cm}^2 + 24\,\text{cm}^2 + 36\,\text{cm}^2 = 72\,\text{cm}^2$



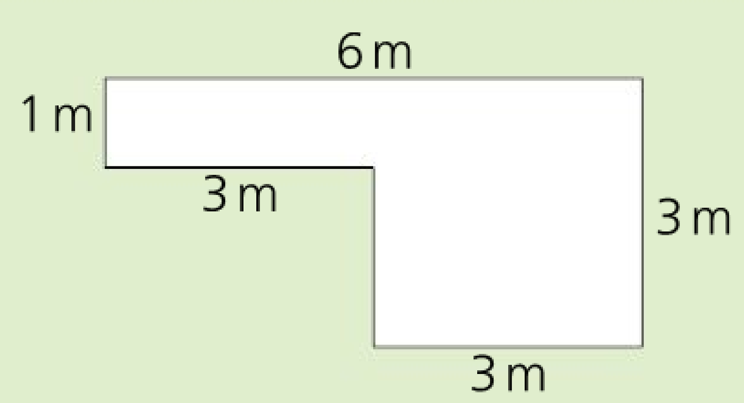
WHY DO WE MEASURE SHAPES IN OUR EVERYDAY LIVES?

Measurement of shapes is important for a variety of job roles – engineer, quantity surveyor, artist and designer, for example. It is also important in international relations, in mapmaking for example.

PRACTICE EXERCISE

Calculate the areas of the following shapes.

1



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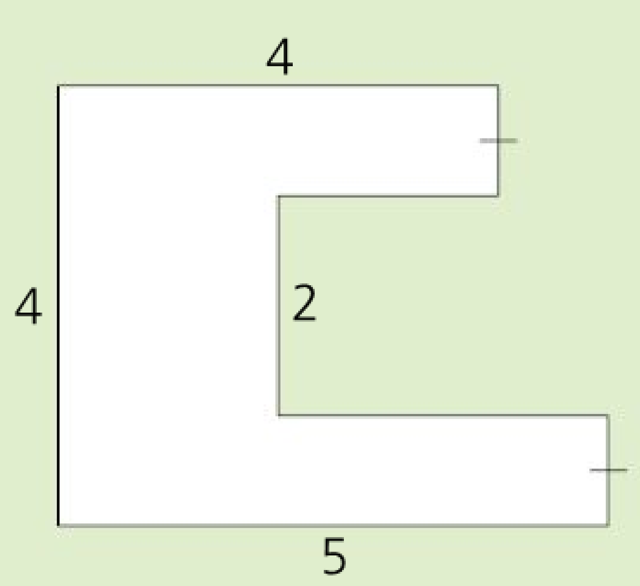

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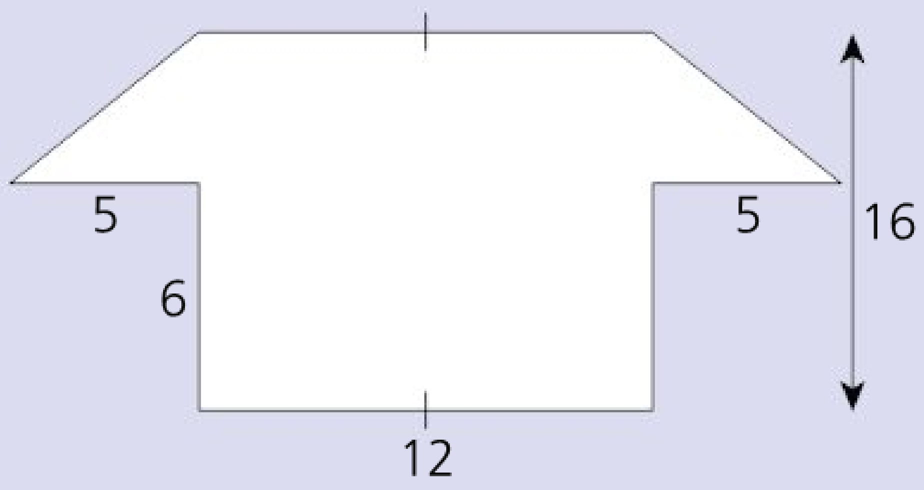

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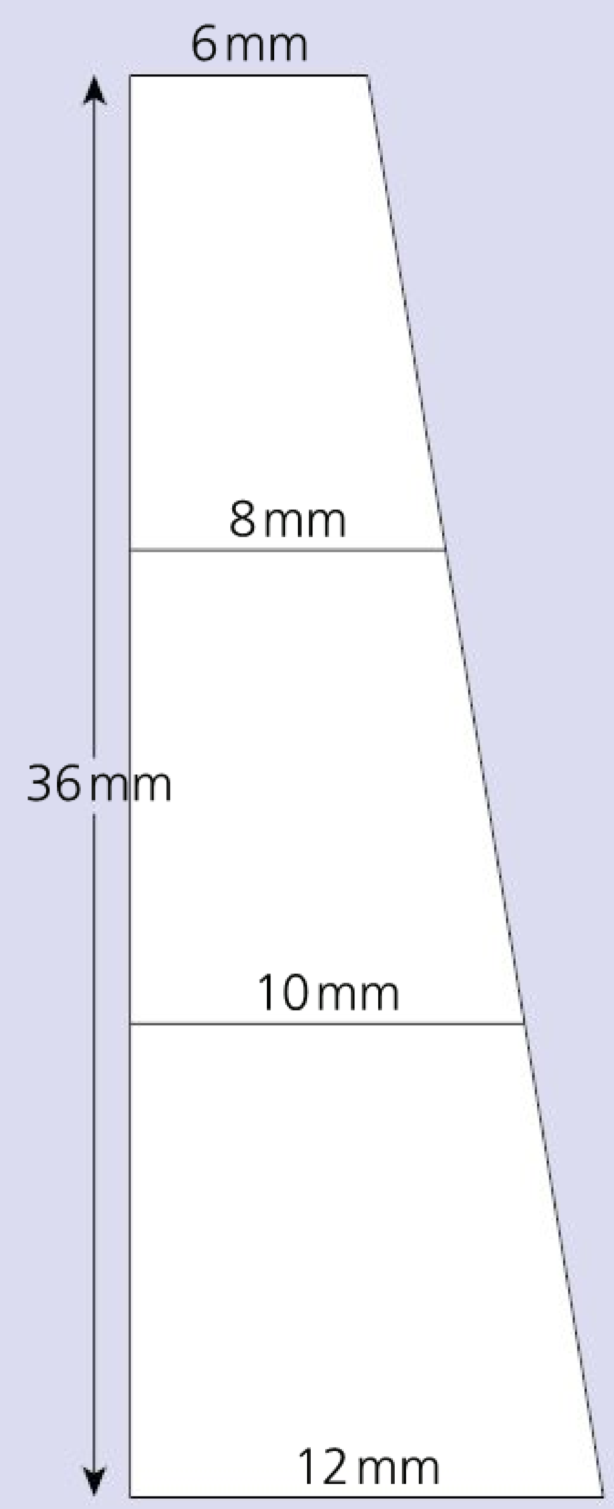

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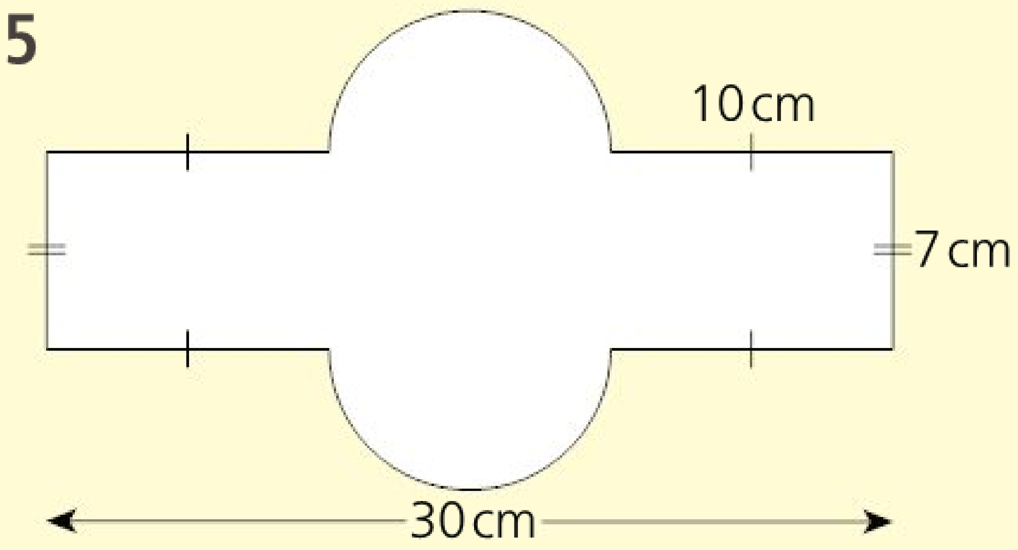

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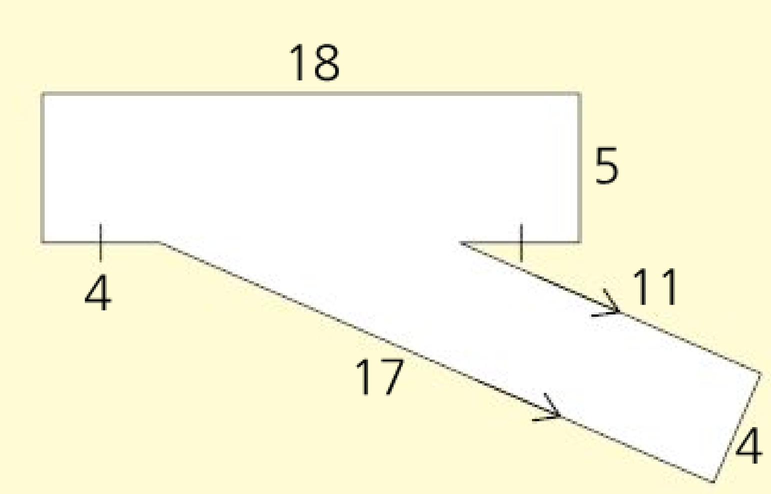

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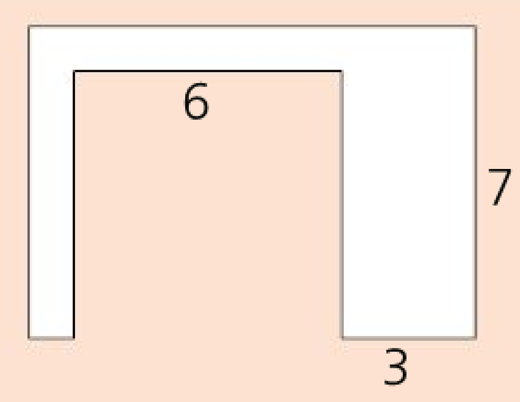


How can logic help us map 2D to 3D?

SEE-THINK-WONDER

If you have the chance, watch the movie *Flatland* and consider what it means for us in the third dimension. Can we visualize a fourth dimension? What is a **tesseract**?

MEET A MATHEMATICAL BEING: HEX

Learner Profile: Inquirer

Flatland: The Movie

Imagine that you lived your entire life in just two dimensions. You would never have seen a sphere or a cube and probably couldn't even imagine what they might look like. This idea is the basis of a science fiction novel called *Flatland*, originally written in 1884 by Edwin A. Abbott. In 2007, the novel was adapted into a short, animated movie.

The story begins with a family in Flatland, a land of only two dimensions where the more sides you have, the more powerful you are. It focuses on the character of Arthur Square and his granddaughter Hex. Now, Hex is a hexagon who wonders, if there is a zero dimension (a point) and a world of one dimension (a line) and their own two-dimensional Flatland, why wouldn't a third dimension be possible – beyond what they could see?

> 'Based on Edwin Abbott's book 'Flatland', this is an animated film about geometric characters living in a two-dimensional world. When a young girl named 'Hex' decides to 'think outside the box' (in a world where such thought is forbidden), her life becomes in danger and it is up to her grandfather to save her life.'
>
> *By JWW on IMDb. Source:* **www.imdb.com**

SO HOW MIGHT WE GET FROM 2D TO 3D SPACE?

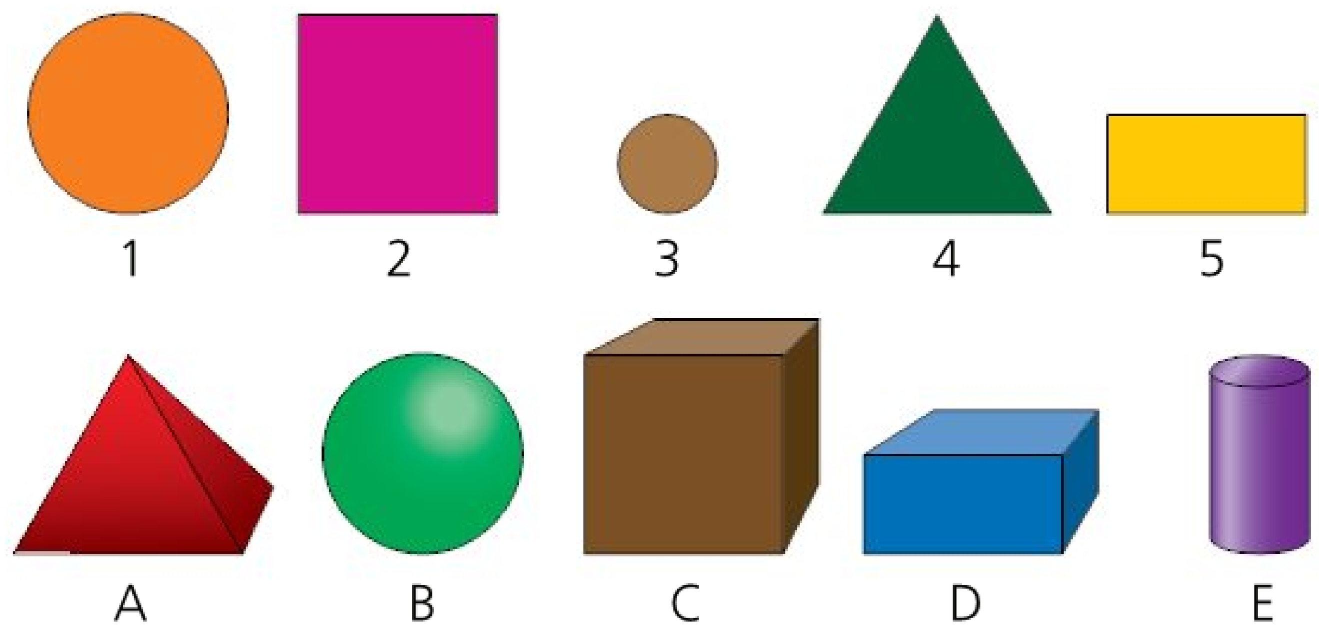
If we want to take two-dimensional shapes into the third dimension, we need to take them 'upwards' or 'downwards' into the other dimension. This might mean dragging a flat shape in a certain direction, or we might rotate the shape around an axis.

■ Computer design in three dimensions

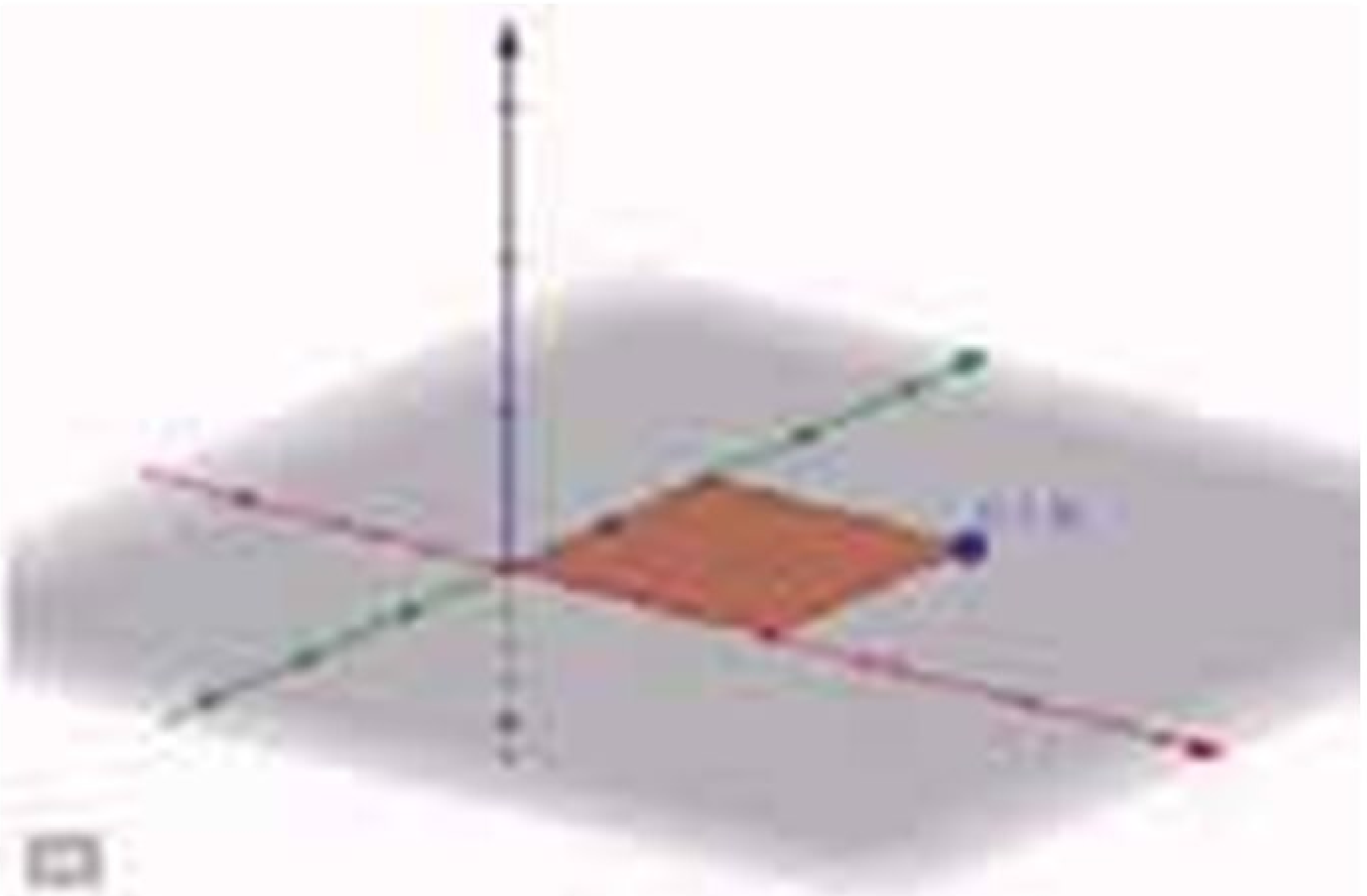
For example, watch as this teacher uses a program called GeoGebra to animate the move from 1D to 2D and then to 3D to 'grow' a cube.

Next,the little blue ball moves parallel to the green axis, giving the line a second dimension.

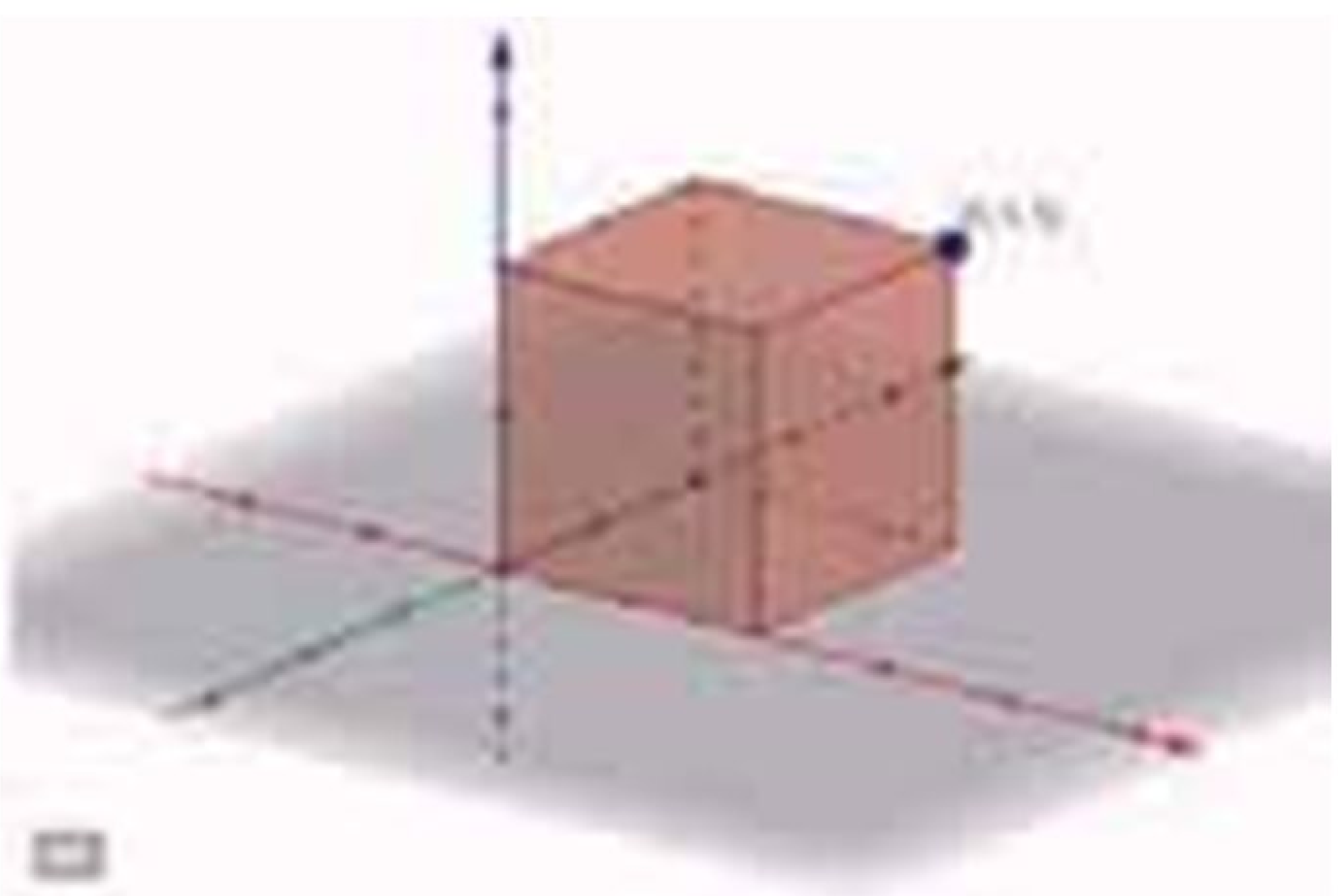
PRACTICE EXERCISE



Can you match the 2D shape with a related 3D one?



Lastly, the blue ball moves parallel to the blue axis and into the third dimension.

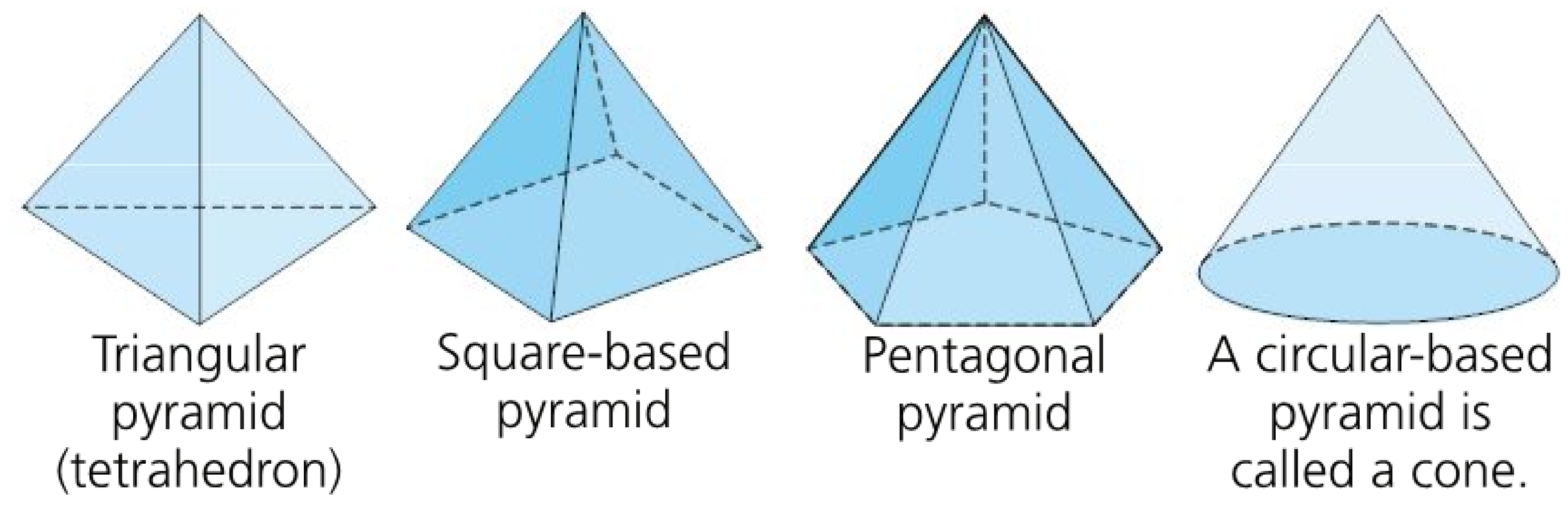


A circle dragged in the third dimension makes a cylinder, while a circle rotated around on itself makes a sphere.

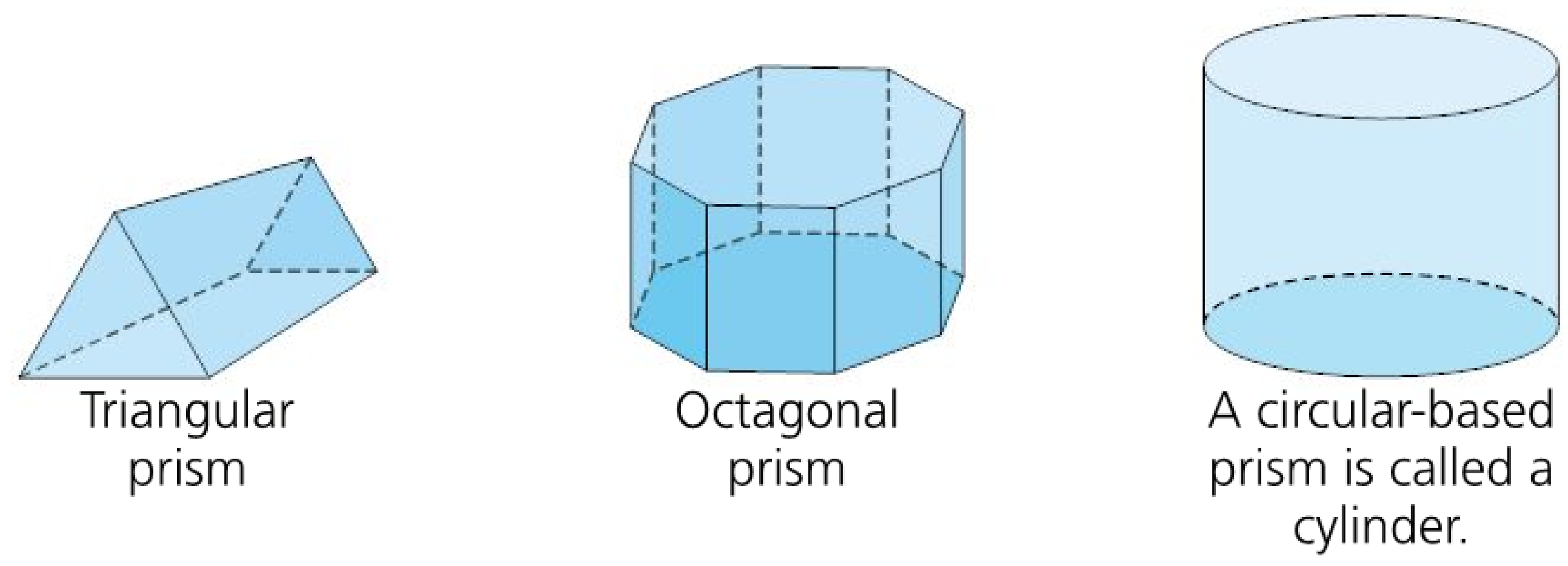
You will be familiar with several 3D shapes already such as a cube, cuboid, sphere and others.

Other examples of 3D shapes are pyramids and prisms. Pyramids have sloping edges which meet at a point, while prisms have parallel sides (and a constant **cross-section**).

Examples of pyramids



Examples of prisms



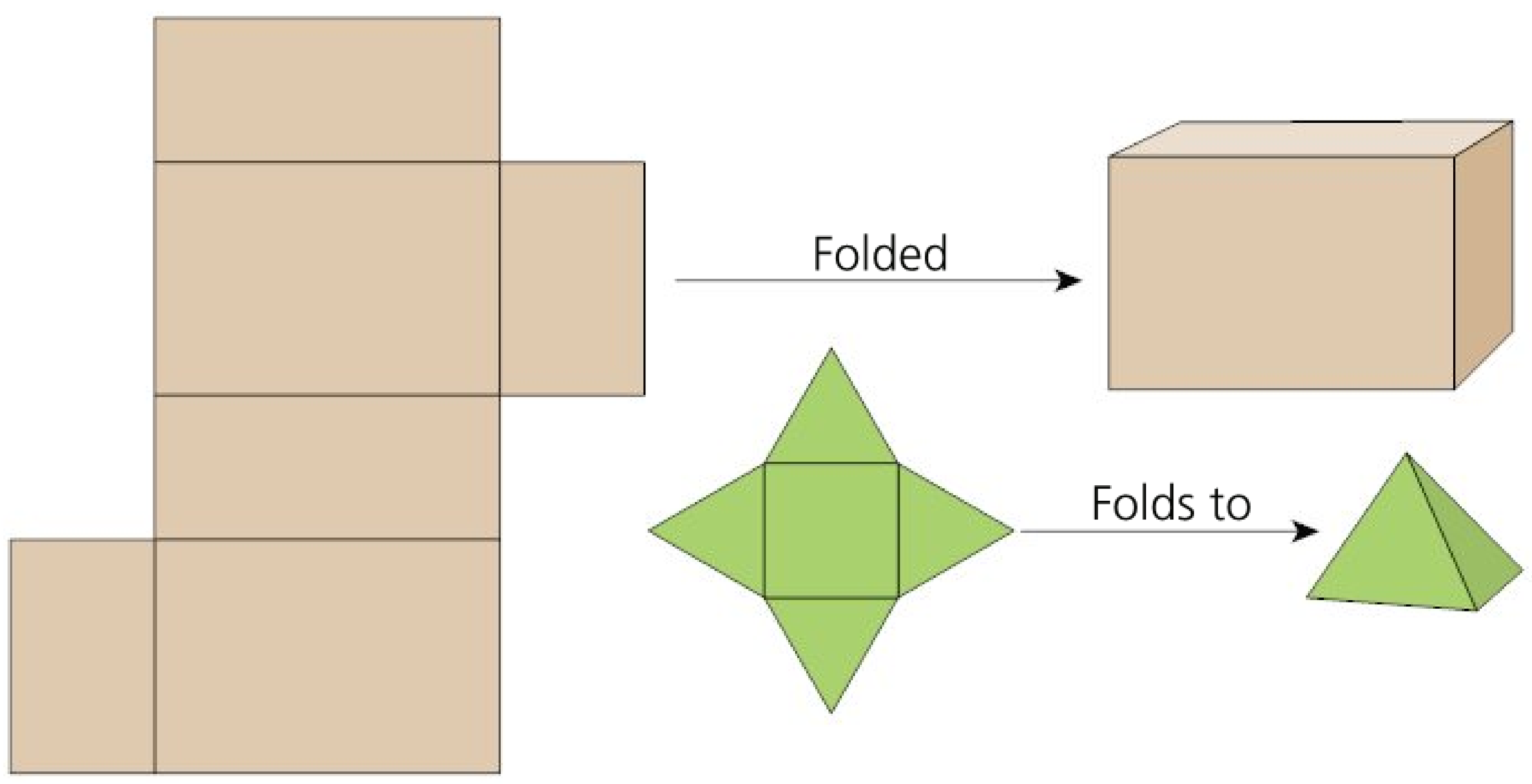
WHAT IS A 3D PRINTER?

A 3D printer is a technological innovation that allows us to use computer-aided design (CAD) in two dimensions to create and print out new designs in three dimensions, using a variety of materials. These printers often print in plastic, but there are exciting developments in printing in the medical fields, such as in making lenses for eyes, printed teeth and biological valves. Food technology is experimenting with printed meat and you can even invest in a pancake 3D printer to create any design you like. The future is incredible!

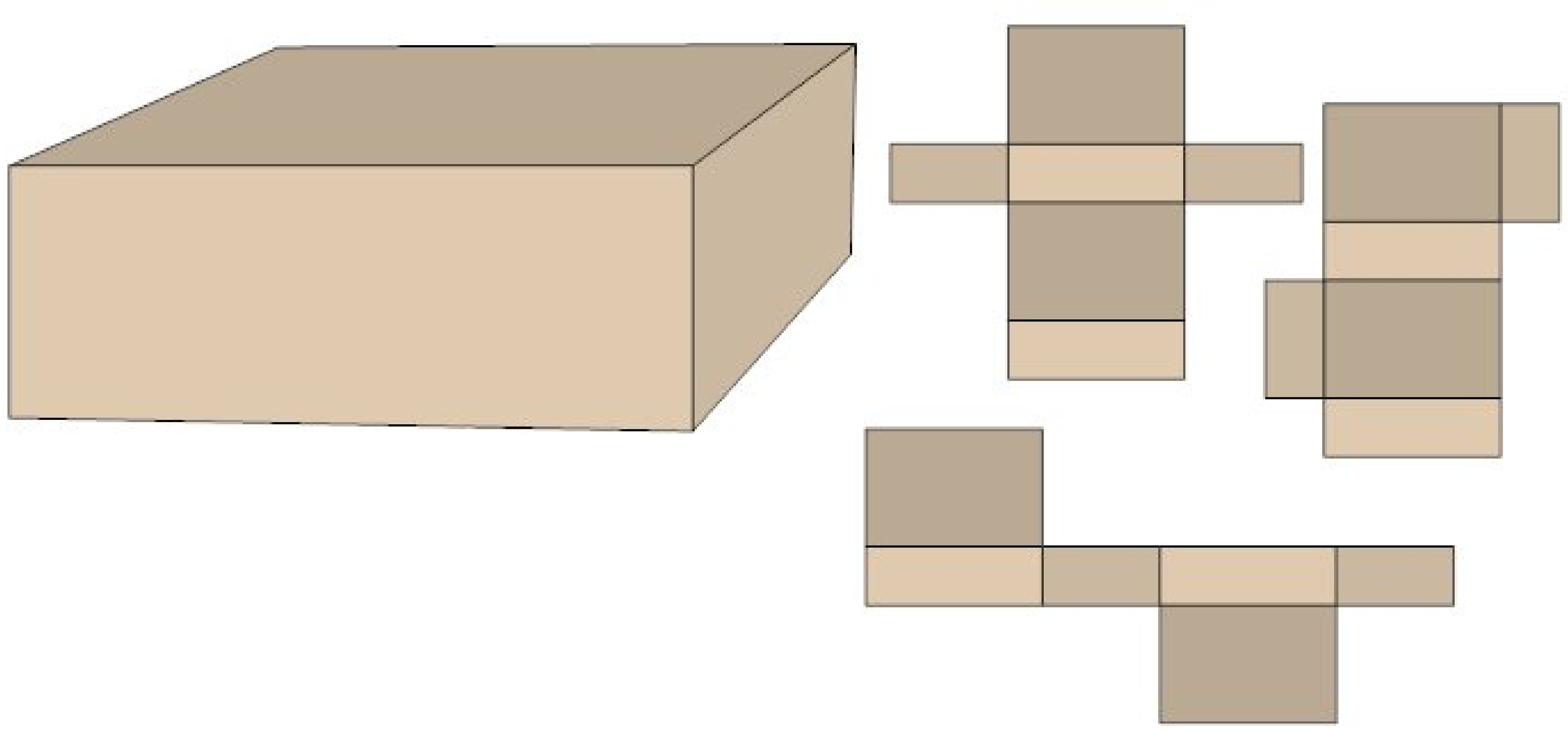
What general rules can we find for objects?

NETS AND SURFACE AREA

Nets show us the flat form of a three-dimensional object's surfaces. When you fold up a net, it gives a recognizable solid.



Similarly, a solid can be unfolded to give the net, like this.

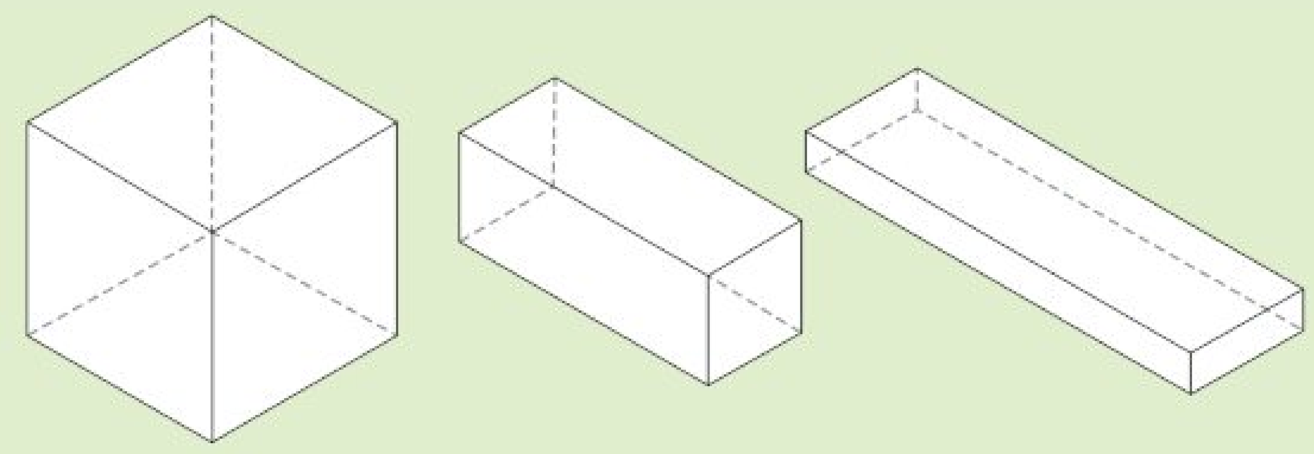


The net is a quick way for us to visualize the surface that 'wraps around' the shape, which we call total surface area.

PRACTICE EXERCISE

For the following shapes, draw the nets.

1



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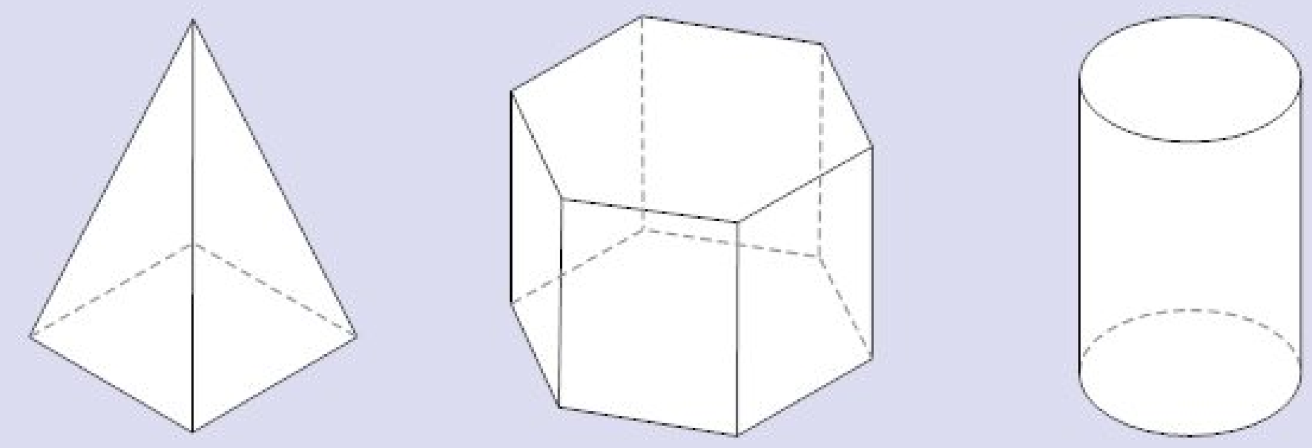

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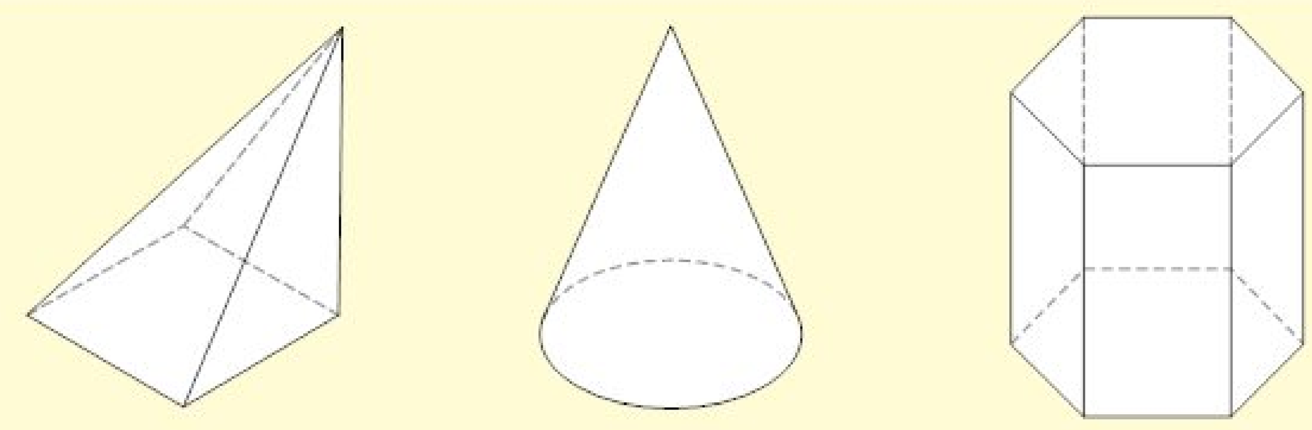
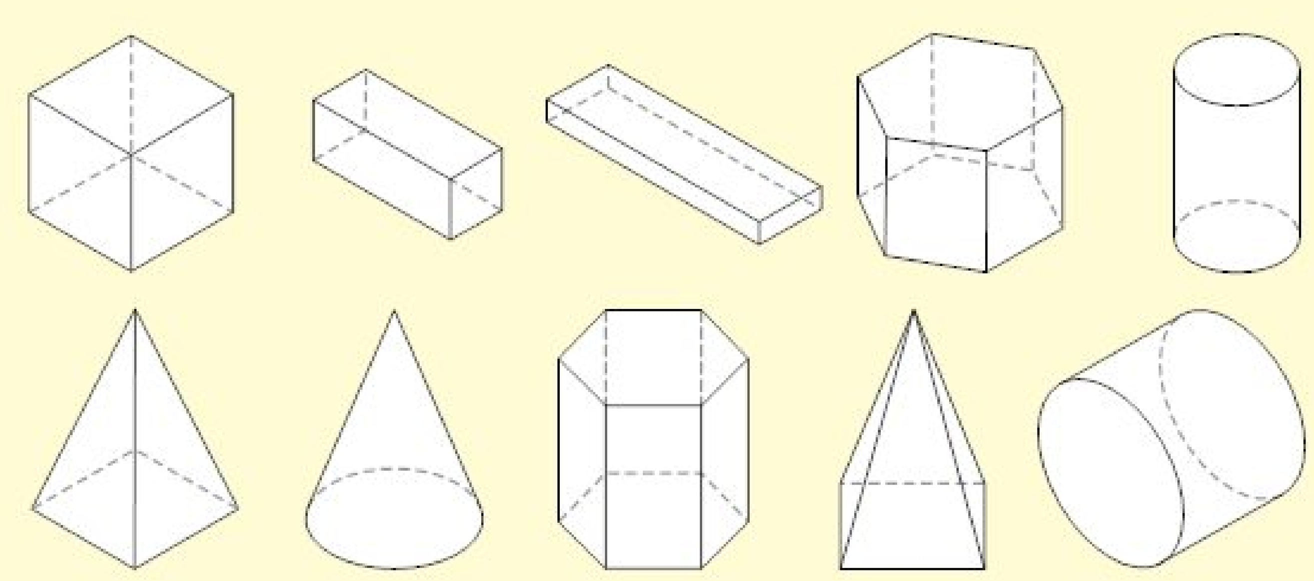


4 **a** Find the dimensions of each shape shown below by measurement.

b How much wire would be required to make each shape?

c Draw the net for each shape.

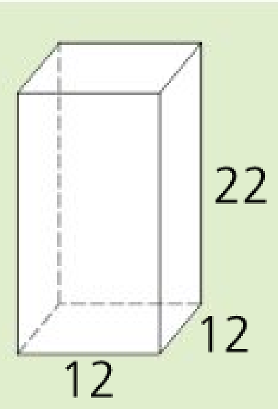
d Hence, using the net or otherwise, find the total surface area of each shape.



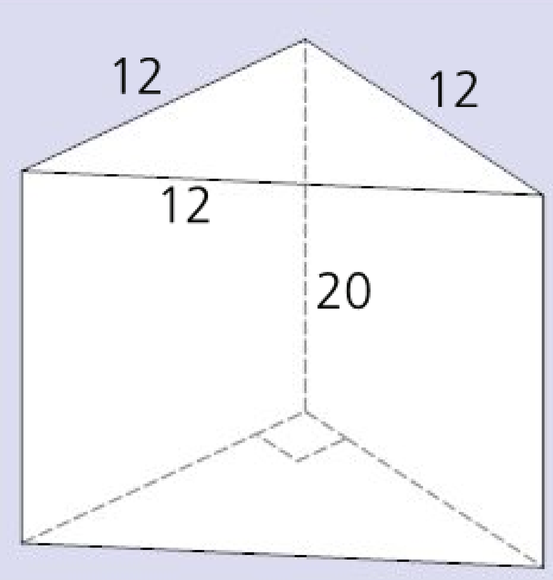
PRACTICE EXERCISE

Find the **surface area** for the given solids, using your knowledge of composite shapes.

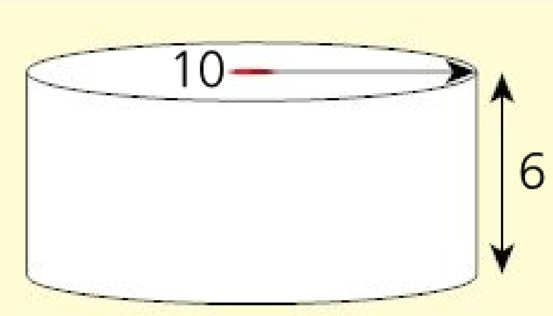
1



2



3



4 Given a cube of surface area of 600 cm², how can we find the length of each edge?

How can we have fun with shapes?

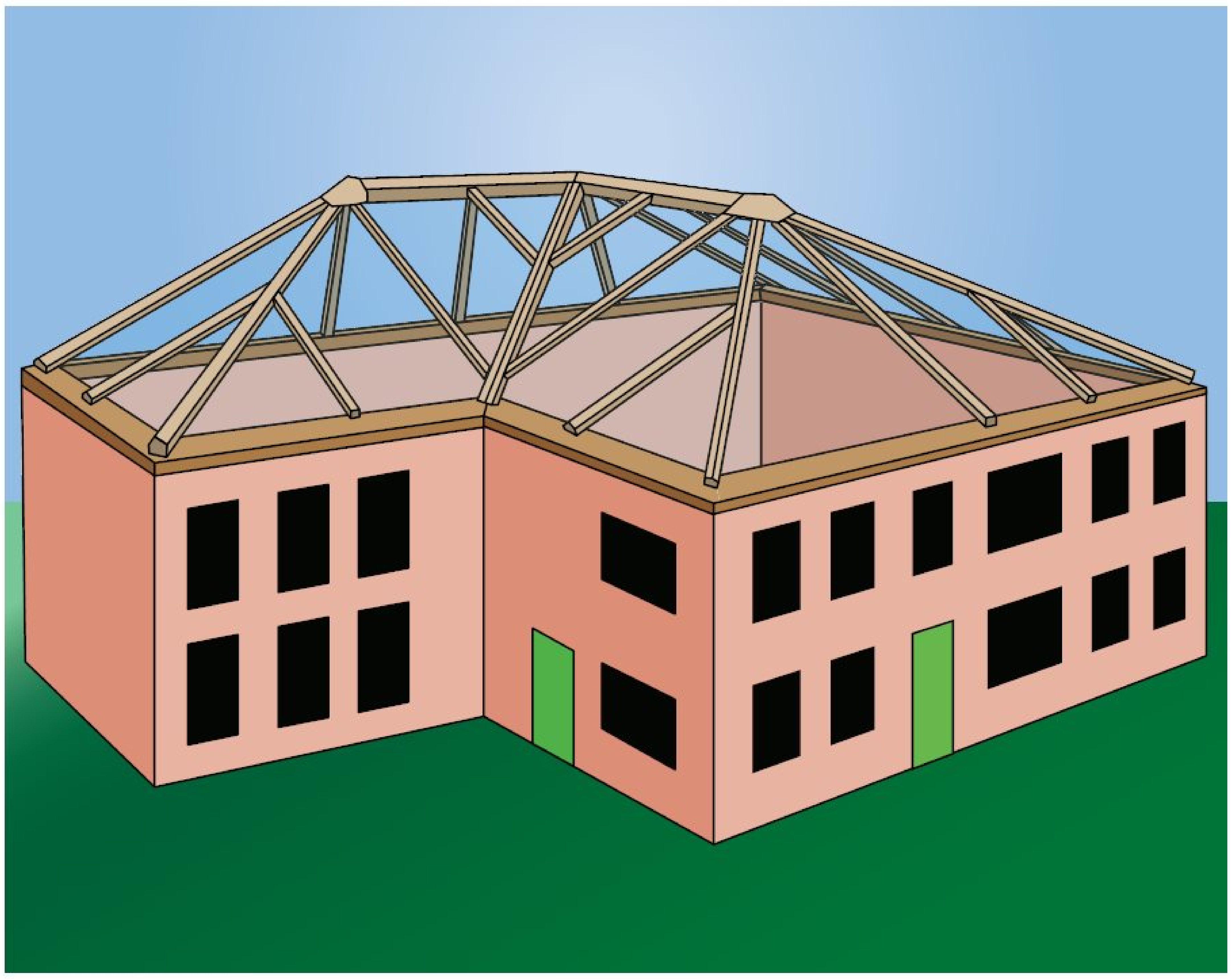
SEE-THINK-WONDER

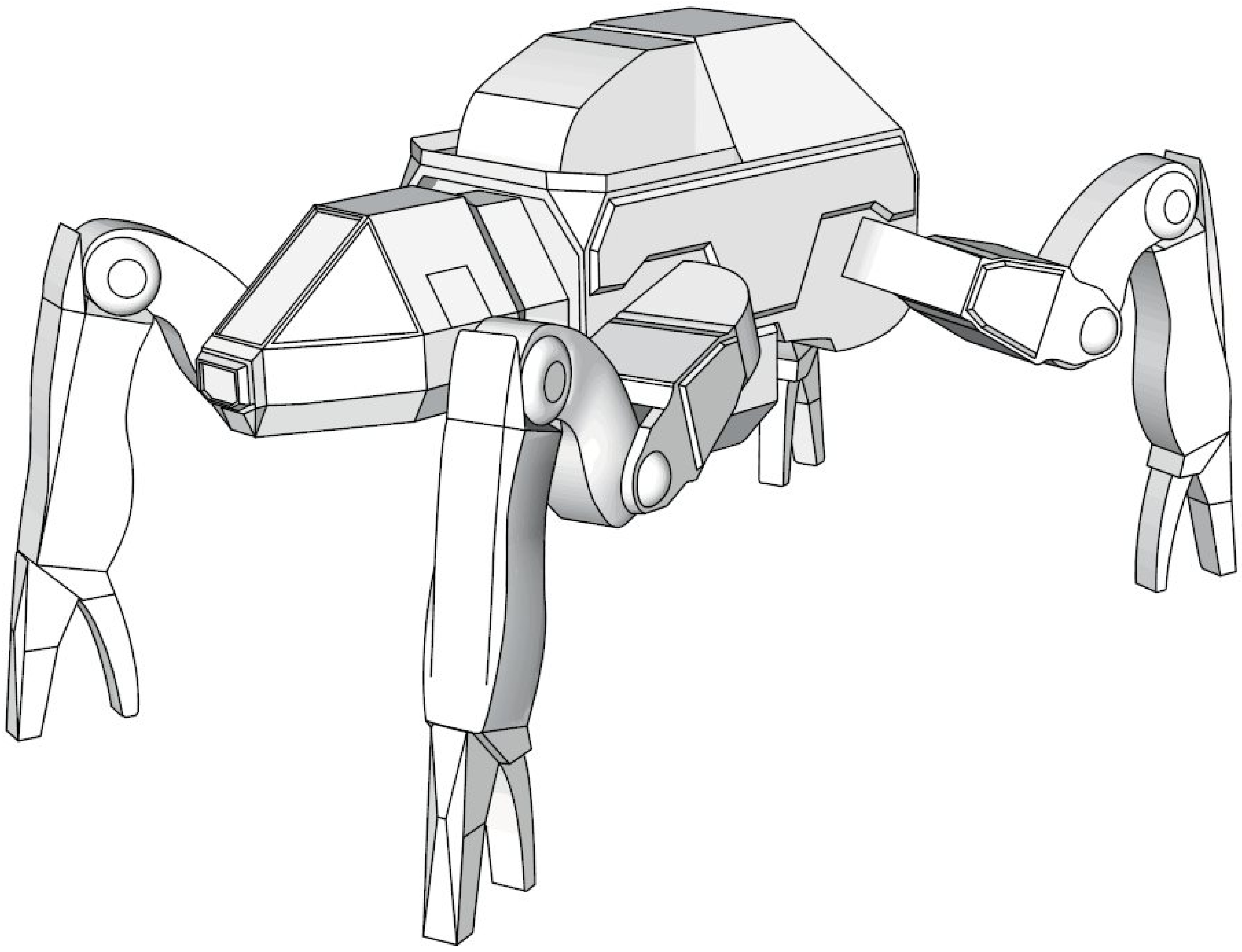
Shape parade

The following images all show geometry and shapes being used in interesting and fun ways. For each of the images, consider the following questions:

- **What do you see?**
  - **What three-dimensional shapes can you see in the image?**
  - **What two-dimensional shapes can you see?**
- **What do you think about that?**
- **What does it make you wonder?**

For each image, complete the sentence: 'I see … I think … I wonder …'.





So-Shan Au liked

Nigel Auchterlounie @spleenal · 12h

Take photo of random stuff.

Trace photo.

Draw Mega City One.

21 1,427 2'638

Think back to the Discuss box on page 55. We considered how spatial objects crowd into our consciousness and fade from view; that if we look properly, we will see them all around us and that they are hiding, but hiding in plain sight.

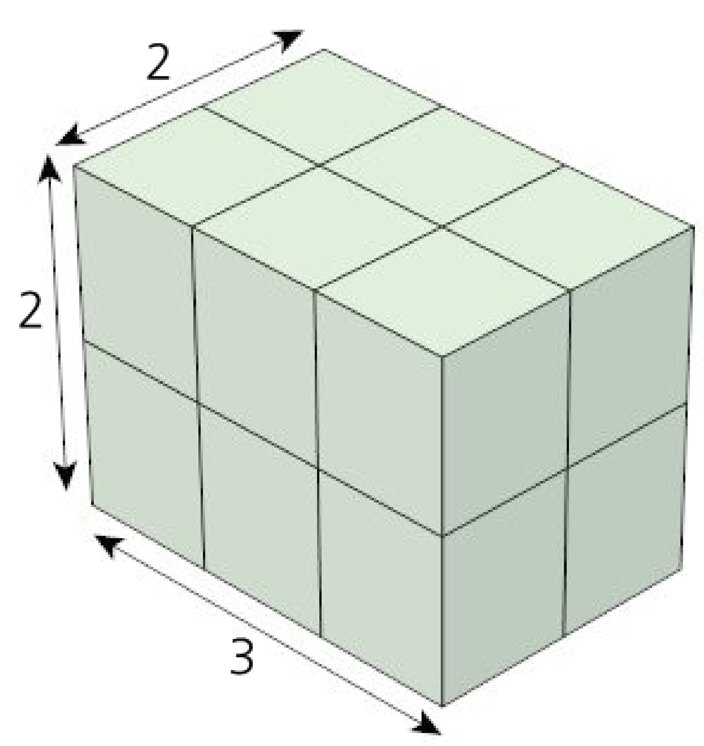
Now you have thought about the 'Shape parade', does this idea about how we see shapes (but don't always register them) make more sense to you?

How do we measure what is 'inside' a shape?

Volume is defined as the amount of space an object occupies or contains. This means how large it is, and how much space it takes up in the three dimensions.

HOW TO FIND VOLUME

When we first started to find area, we used a grid paper method to count individual squares. We can do the same with volume but using 3D boxes, or counting 3D cubes.



Here we can see that the total volume of this cuboid is 12 cm^3. You can see this either as three slices of four or two layers of six, or any other way that you can divide this up. The total number of boxes is found by $2 \times 2 \times 3$ boxes.

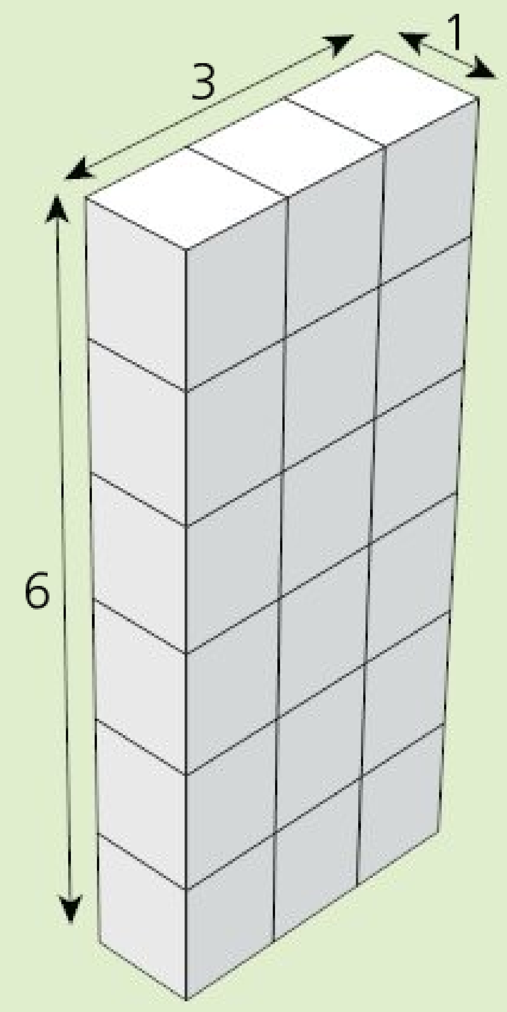
How can we calculate the volume when we don't have these unit boxes or blocks?

Think back to how we 'grew' a cube (pages 69–70). First, we saw a square. This was dragged upwards, almost as if it was a square on top of a square on top of yet another square, until it reached the height (or depth) h of the third dimension.

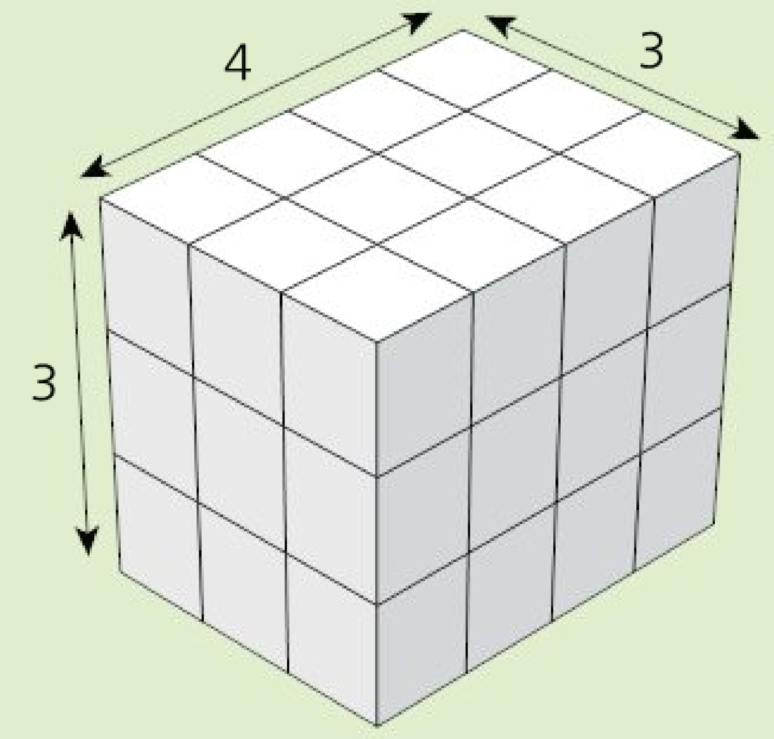
PRACTICE EXERCISE

Find the volume of each of the following shapes.

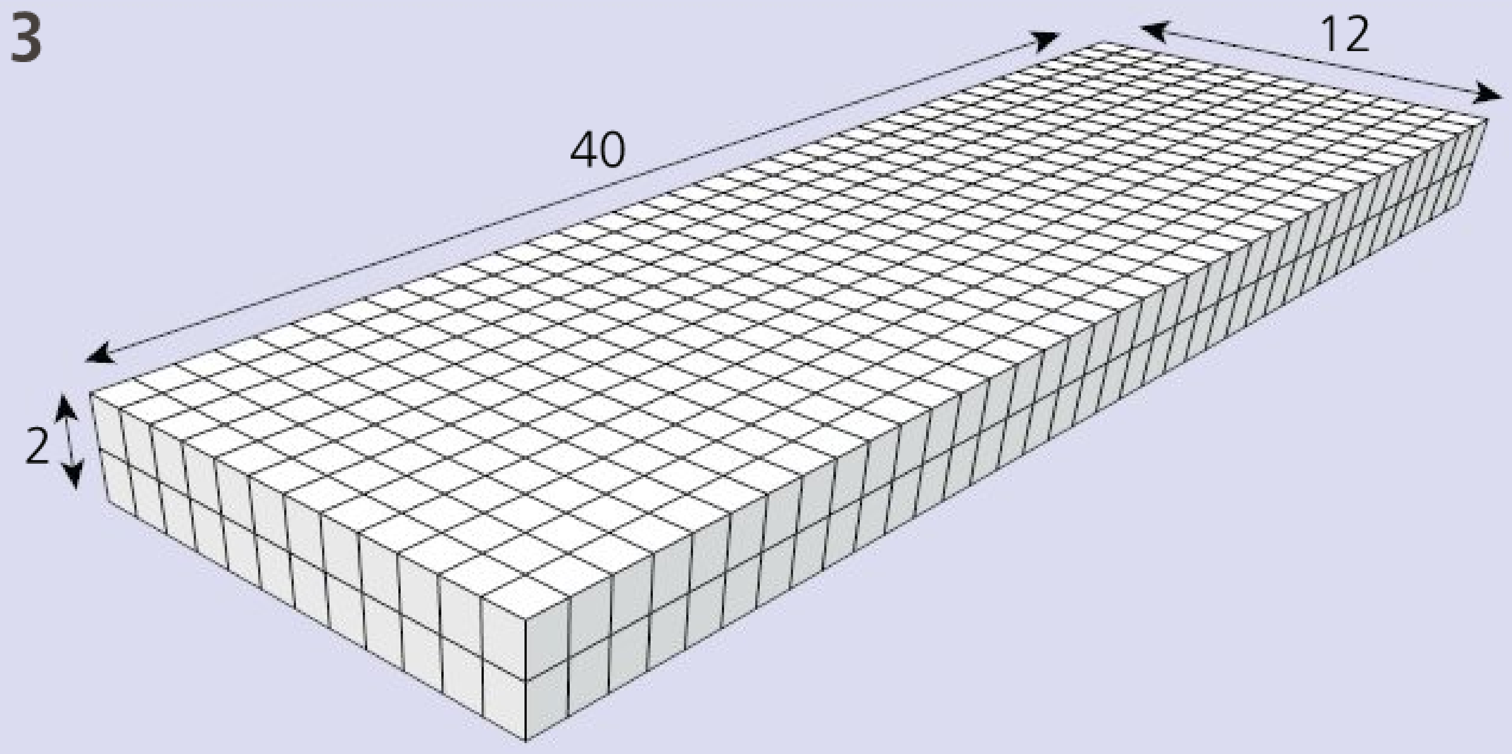
1



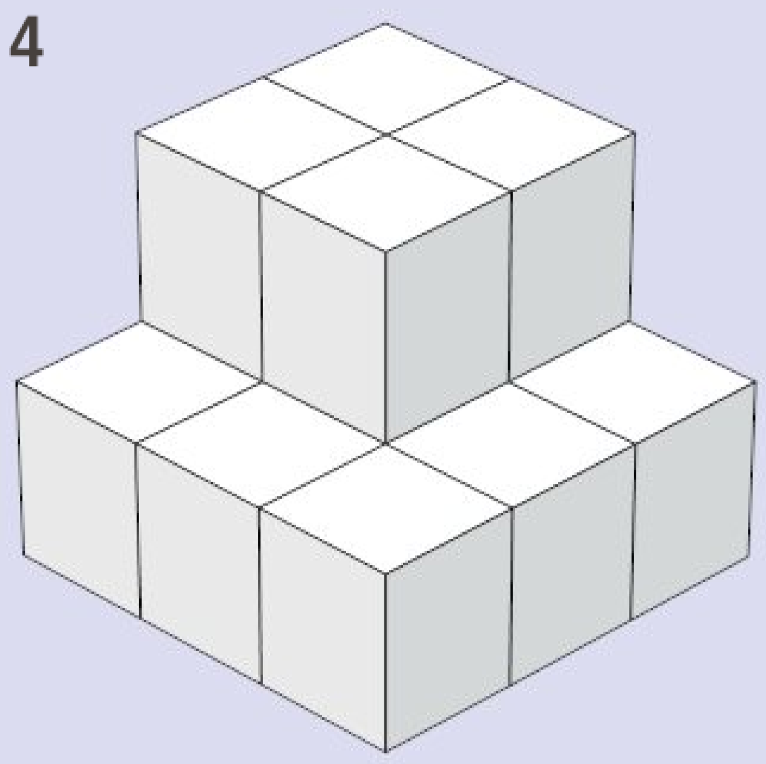
2



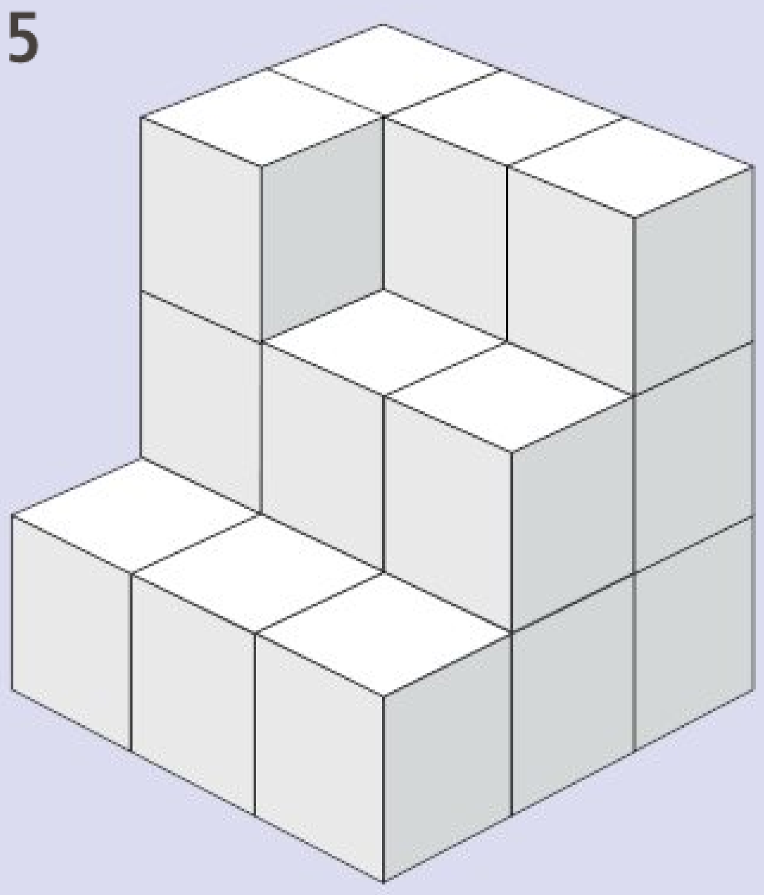
3



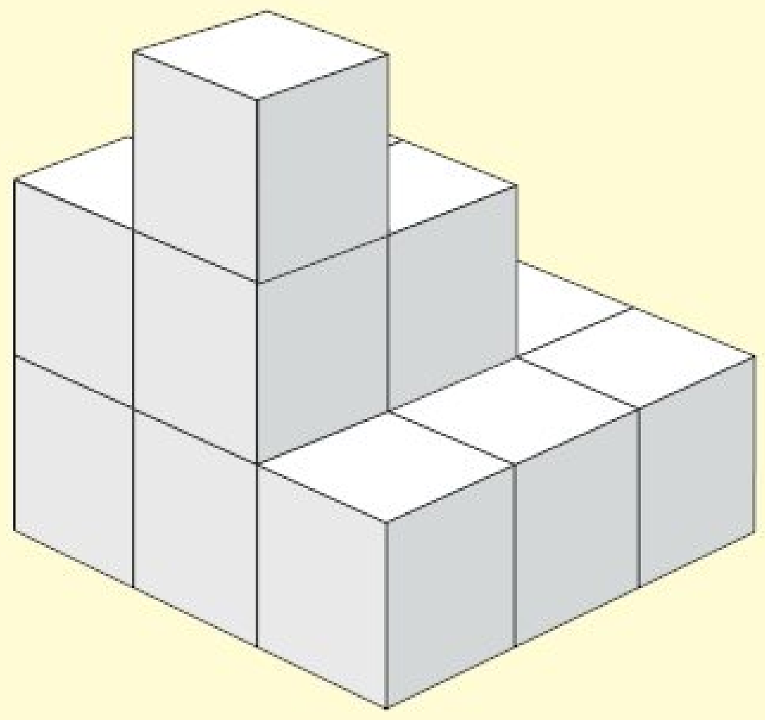
4



5



6 State one reason why it is hard to know the correct volume of this shape.



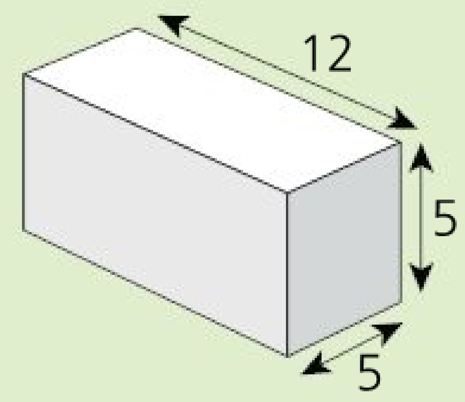
7 Give two possible values for the volume of the shape in question 6, explaining the reason for the difference in your answers.

We have seen how a cube 'grows', dimension by dimension. It helps us to understand the introduction of the depth (or height) into our previously familiar formulas.

Area of a square = $(\text{length})^2$	Volume of a cuboid = $(\text{length})^3$
Area of a circle = πr^2	Volume of a cylinder = $\pi r^2 h$
Area of a rectangle = length × width	Volume of a cuboid = length × width × height

Examples

1 Calculate the volume of this cuboid.



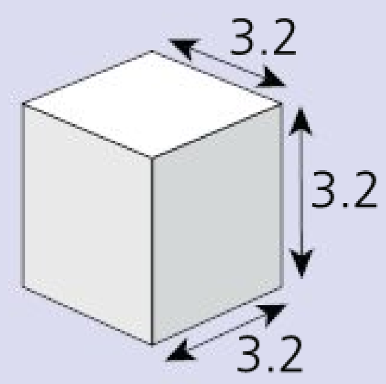
Solution

volume of a cuboid = $l \times w \times h$

$= 5 \times 5 \times 12$

$= 300\,\text{cm}^3$

2 Calculate the volume of this cube.



Solution

volume of a cube
$= l \times w \times h$

$= (3.2) \times (3.2) \times (3.2)$ or $(3.2)^3$

$= 32.8\,\text{mm}^3$

3 Which has a greater volume, a cube of length 10 cm or a cuboid with sides of 12, 8 and 10 cm?

Solution

volume of cube = 10^3

$= 1000\,\text{cm}^3$

volume of cuboid = $12 \times 8 \times 10$

$= 960\,\text{cm}^3$

The cube has the larger volume.

EXTENSION

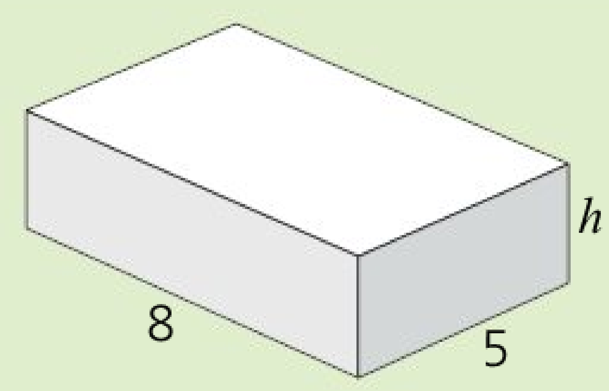
There are other prisms and pyramids which we have seen before, but that are not listed in the table opposite. Can you remember them? Can you find their formulas by researching them?

For more practice on calculating volumes, see the digital T&L resource.

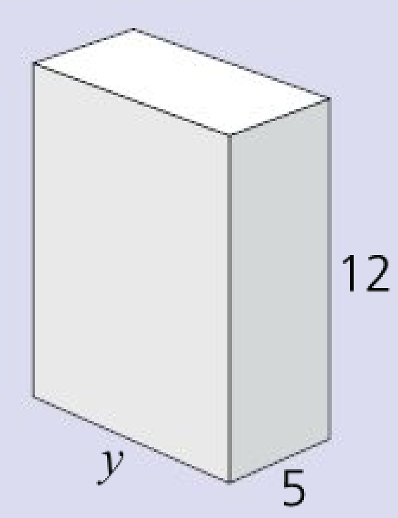
PRACTICE EXERCISE

Write an expression for the volume of these shapes.

1



2

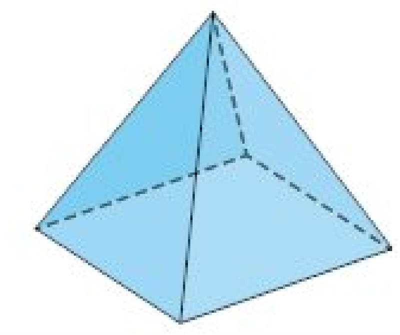
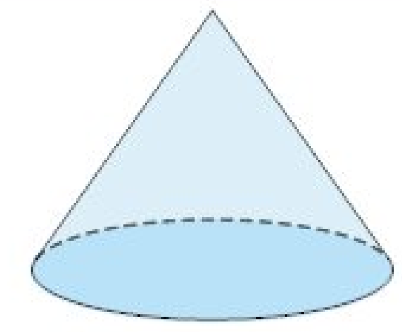
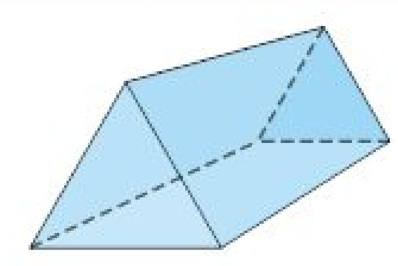


3 A cube of length 11 cm is placed inside a rectangular box (cuboid) which has a length of 20 cm, height of 10 cm and width of 13 cm.

What is the difference in their volumes?

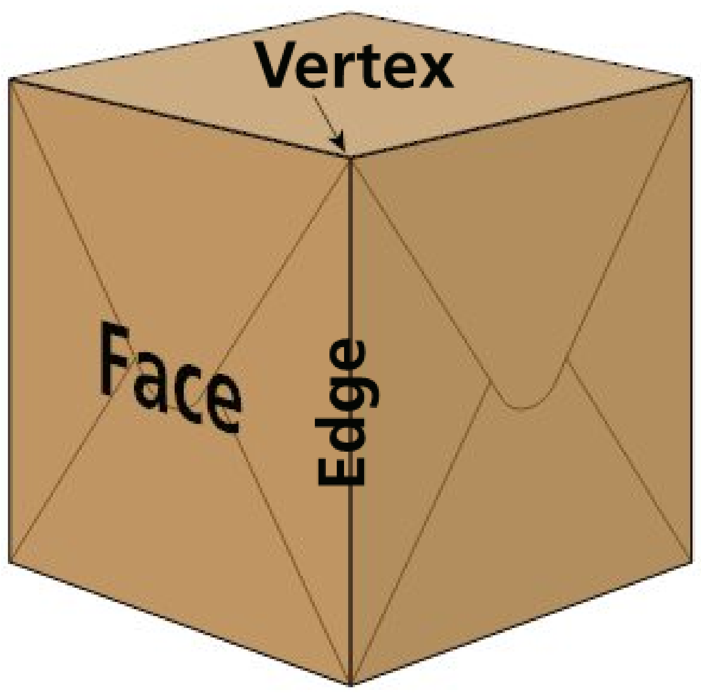
Can the lid be closed on the box? Why or why not?

Previously, we looked at examples of prisms and pyramids (page 70). Do these also have formulas for volume?

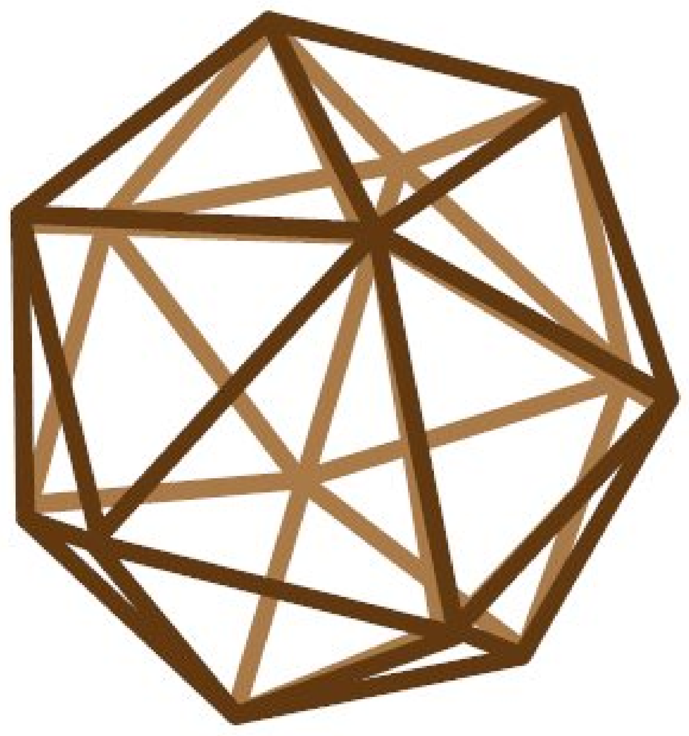
Shape	Example	Formula
square-based pyramid		$\frac{l \times w \times h}{3}$
cone		$\frac{\pi r^2 h}{3}$
triangular based prism		$\frac{1}{2} b \times h \times l$

ACTIVITY: Vertex Investigation

In this investigation, your task is to explore the patterns or relationships between the properties shown here.



On a 3D shape, a vertex is a corner. A vertex is made where edges meet, so we can say that an edge is a line segment that joins vertices. A face is a surface on a shape, defined by the edges.



Copy and complete the table below for your own selection of shapes.

Solid	Faces (f)	Vertices (v)	Edges (e)

Describe any pattern(s) you can see.

State the pattern as a general rule.

Justify your general rule by testing it on a new shape that is not shown in your completed table. Also, prove the rule by drawing.

Find solids which do **not** follow this general rule. You might like to look back through the chapter to remind yourself of different solids.

To verify your findings and learn more, search online for **Euler's Polyhedral Formula**.

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns.

Do we need to understand shapes to innovate?

SUMMATIVE ASSESSMENT

Use these problems to apply and extend your learning in this chapter.

Part 1: 2D tangram

A tangram is an ancient Chinese geometrical puzzle consisting of a square cut into seven polygons, which can be arranged to make various other shapes for fun. See how the tangram in the picture has been used to make animal shapes. How many animals can you identify?

1 State the name of each polygon. You can refer to them by their colours: purple, jade green, olive green, orange, yellow, blue and pink.

2 Given that the area of the tangram square is $9\,cm^2$, can you work out the area of each coloured polygon?

3 Using logic or measurement, state a value for each of the angles in every coloured polygon.

4 State the ratio of the areas for these pairs of polygons:

   a purple to jade green

   b olive green to pink

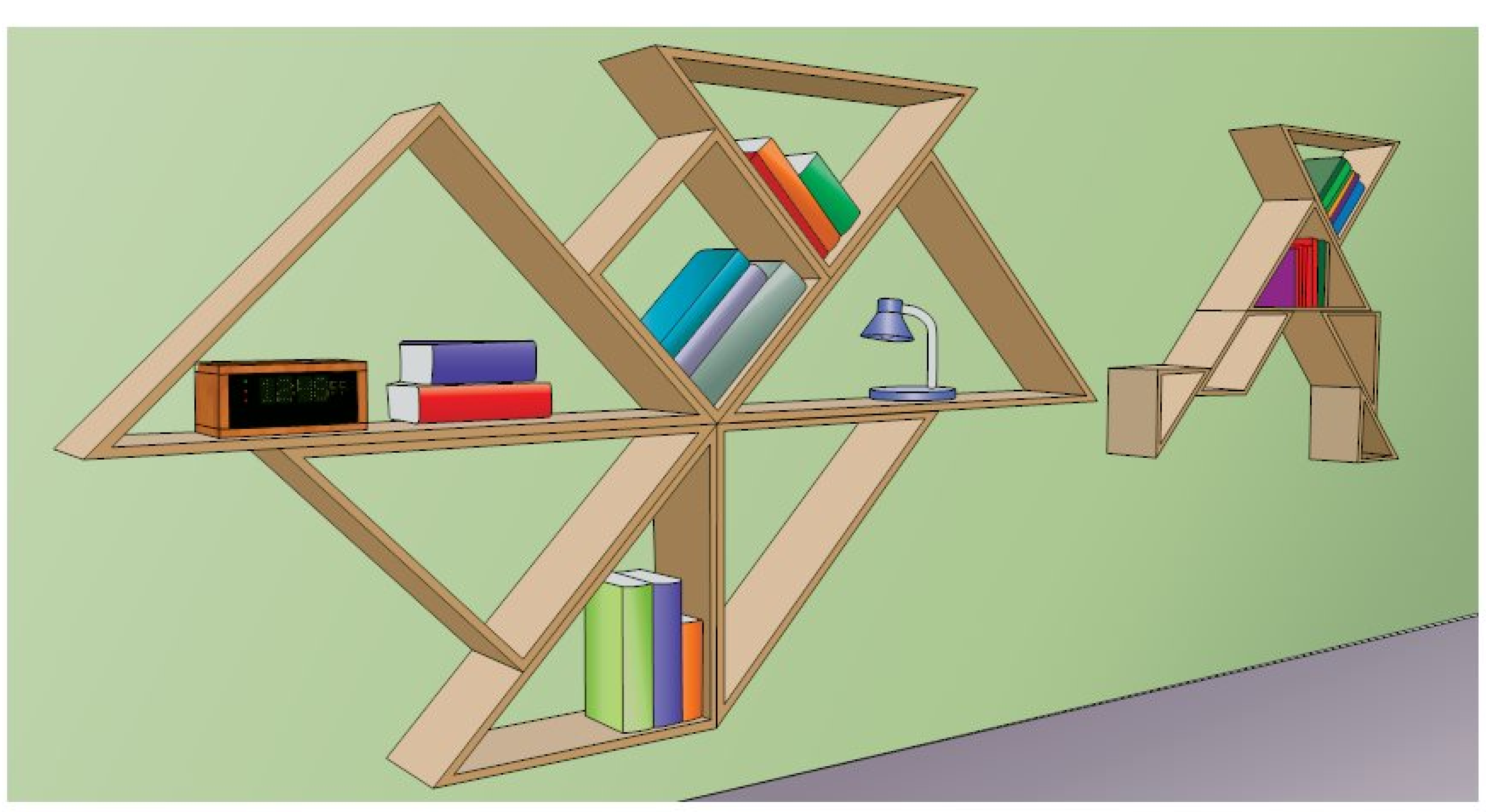
   c purple to blue.

5 State the ratio of the areas of all the coloured polygons in the tangram – purple : jade green : olive green : orange : yellow : blue : pink.

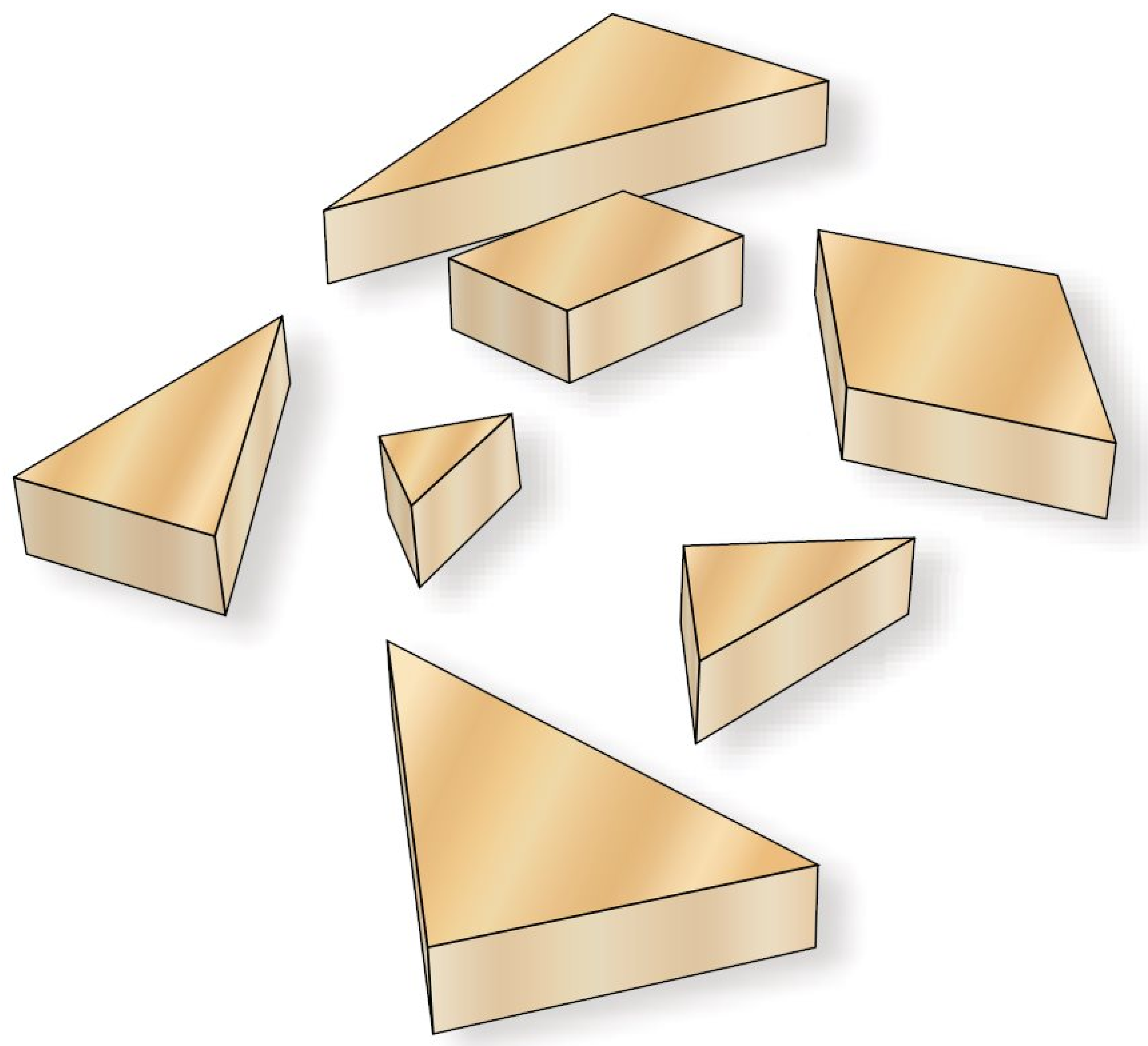
◆ Assessment opportunities

- ◆ In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

Part 2: Creating a 3D tangram



Now take the idea of tangrams into three dimensions by creating your own puzzle. Your task is to create a detailed design for a 3D tangram. This design could then be followed to build the puzzle (by hand or by printing using a 3D printer).



Your design must include the detailed descriptions of

- **the shapes you will use in the tangram**
- **the dimensions of each shape**
- **the properties of the shapes**
- **whether the shapes will be hollow or filled**
- **the material and colours for the individual shapes (and why you chose them)**
- **sketches of the tangram components**
- **comments on the accuracy of your design solution**
- **comments on your design in the context of the real-life situation.**

◆ Assessment opportunities

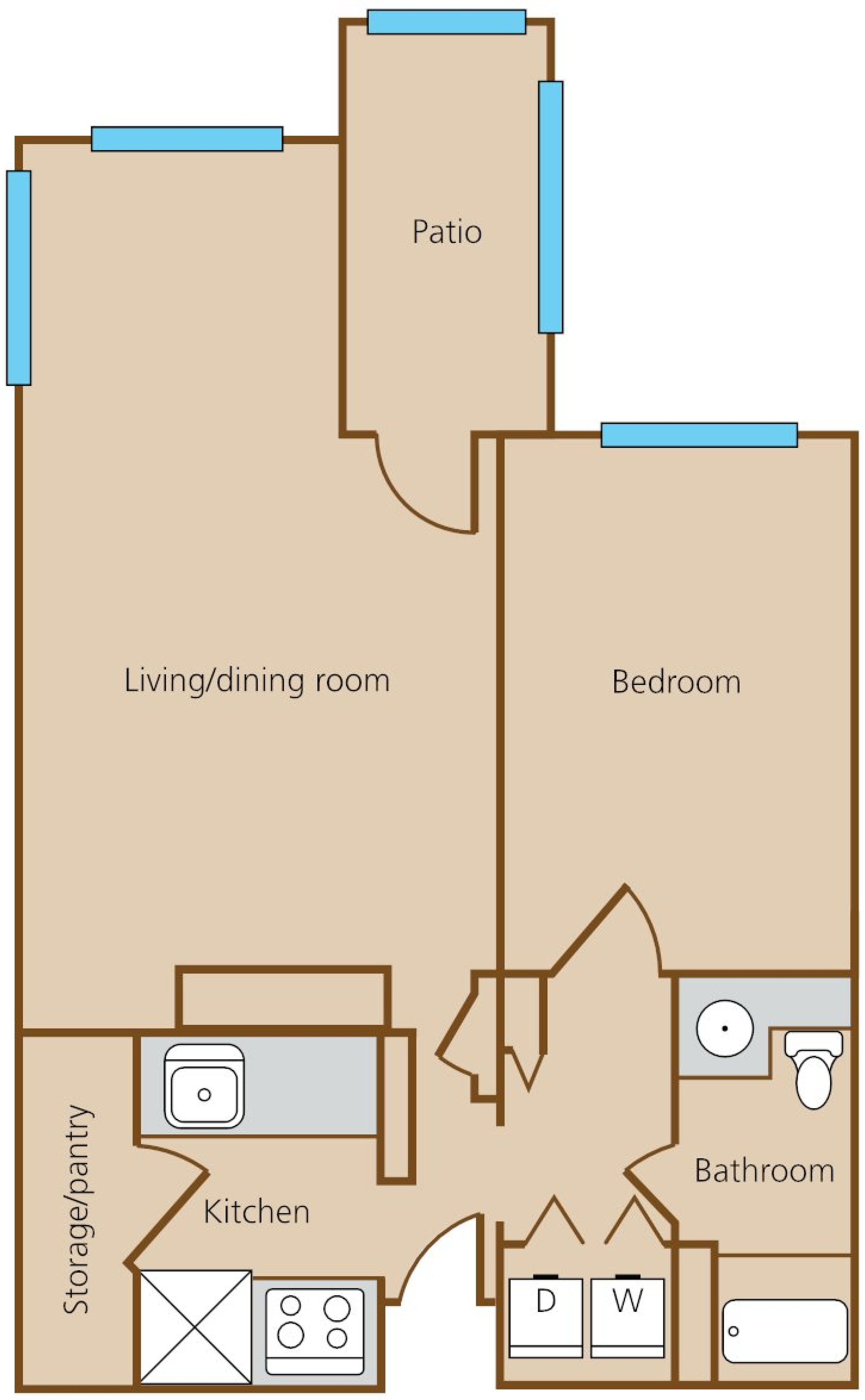
- ◆ In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Hint

3D-print fails are often due to human error (such as through a misunderstanding about geometry and shape). Make sure you consider all angles, faces, edges and vertices when designing your 3D tangram.

SUMMATIVE ASSESSMENT

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.



This image shows the apartment or living space at a university dorm.

The image is drawn to a scale of 1 cm : 2 m.

Hint

As this is a Criterion D assessment, don't forget to comment on the accuracy of your solutions and whether they make sense in the context of the authentic real-life situation.

Links to: Design

Can you design an apartment with the **same** perimeter but a **better** area for living space?

Create either a 3D model or a 2D representation of the floor plan.

1 How many rooms are in the apartment? How can you tell?

2 What do the curved shapes in each room represent?

3 What might W and D represent?

4 Find the perimeter of the entire apartment.

5 Find the area of the bedroom.

6 Find the area of the living room.

7 Find the perimeter of the living room.

8 What is the total floor space of the apartment?

9 What is the ratio of outdoor living space to indoor living space?

10 If the walls are 2 m high, what is the volume of the apartment?

11 The area taken up by a door's opening and closing must be kept free.

a How much space is lost due to the doors?

b What is the actual area of the bedroom?

c What is the real area of the apartment, removing any space that cannot be used?

Reflection

Use this table to reflect on your own learning in this chapter.

Questions we asked	Answers we found	Any further questions now?			
Factual: What do we know about shapes? How can we have fun with shapes? How do we measure what is 'inside' a shape? What are the mathematics of Snapchat? What is a 3D printer?					
Conceptual: What is inside these shapes? How does the measurement of shapes appear in our everyday lives? How can logic help us map 2D to 3D? What general rules can we find for spatial objects?					
Debatable: Is nature made from shapes or do shapes come from nature? Do we need to understand shapes to innovate?					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Creative-thinking skills					
Affective skills					
Social skills					
Self-management skills					
Collaboration skills					
Learner Profile attribute(s)	Reflect on the importance of being reflective and an inquirer for your learning in this chapter.				
Reflective					
Inquirer					

4 Where do conclusions come from?

Relationships between variables form **patterns** which often justify important **logical** conclusions.

CONSIDER THESE QUESTIONS:

Factual: How do we keep track of how far we've come? What are positive and negative correlations? Can one positive relationship be 'more positive' than another? What steps are needed to draw a box plot?

Conceptual: What is a mathematical echo? Why does the average person use *average*? How can outliers affect range? How can we visually represent spread?

Debatable: Which is the best measure of central tendency? Is it a coincidence or a correlation? Will statisticians become obsolete?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

IN THIS CHAPTER, WE WILL …

- **Find out** how to describe patterns in relationships using mathematical language.
- **Explore** the different ways we can represent the middle and the spread of data sets.
- **Take action** by promoting healthy habits using data to justify the need for these.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how to perform basic calculations using the order of operations
- how to read and interpret traditional graphs.

These Approaches to Learning (ATL) skills will be useful …

- Critical-thinking skills
- Communication skills
- Information literacy skills

Assessment opportunities in this chapter:

- **Criterion A:** Knowing and understanding
- **Criterion B:** Investigating patterns
- **Criterion C:** Communicating
- **Criterion D:** Applying mathematics in real-world contexts

We will reflect on this Learner Profile attribute …

- **Thinker** – We use critical- and creative-thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

KEY WORDS

average	intuitive	running total
consistent	numerical	sporadically
fluctuate	random	subjective

Many countries use standardized tests in Mathematics and English Language across all their schools, in order to assess students in a consistent way. Some countries use these tests to ensure students are all learning the same material, while others go as far as rating their schools and paying teachers differently based on how their students perform in these tests compared to average results. Does this seem like a logical way to rate a school or a teacher? To gain perspective, let's review what we've learned about measuring the middle value, the range and outliers.

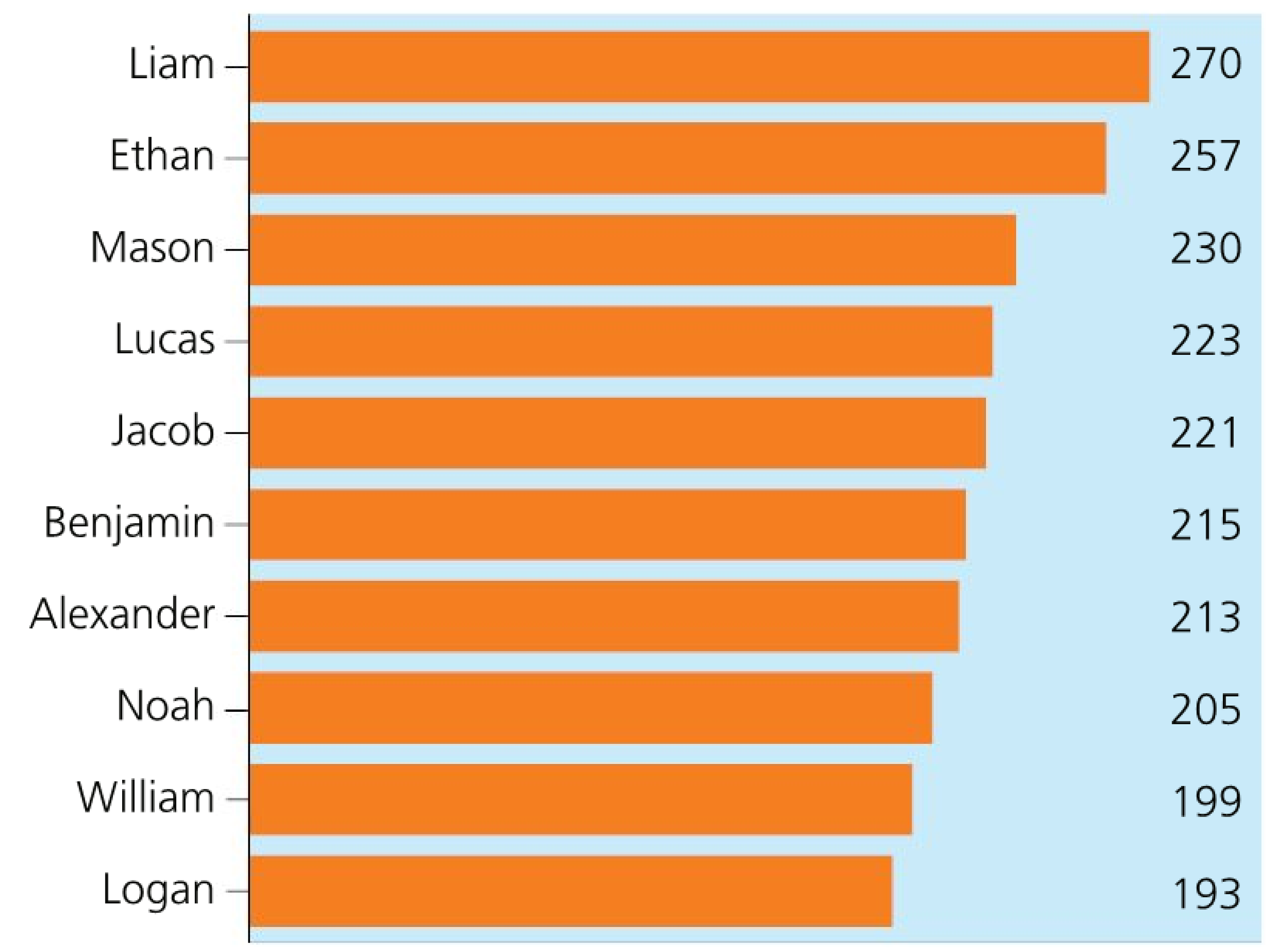
What is a mathematical echo?

Uni-variate data consists of a single variable, such as a favourite colour, a test grade, or the height of a tree. One way to generalize these data is to determine their **centre** – the value that the data surround equally. There are three ways to measure this **central tendency**. Let's examine the first.

THINK-PAIR-SHARE

Write down the most frequently occurring value in each data set.

1 Newborn baby boy names in Alberta



Source: www.edmontonjournal.com

2 The number of times your most recent Instagram posts were viewed in their first hour

Views	Frequency
20–29	5
30–39	12
40–49	38
50–59	71
60–69	45

PRACTICE EXERCISE

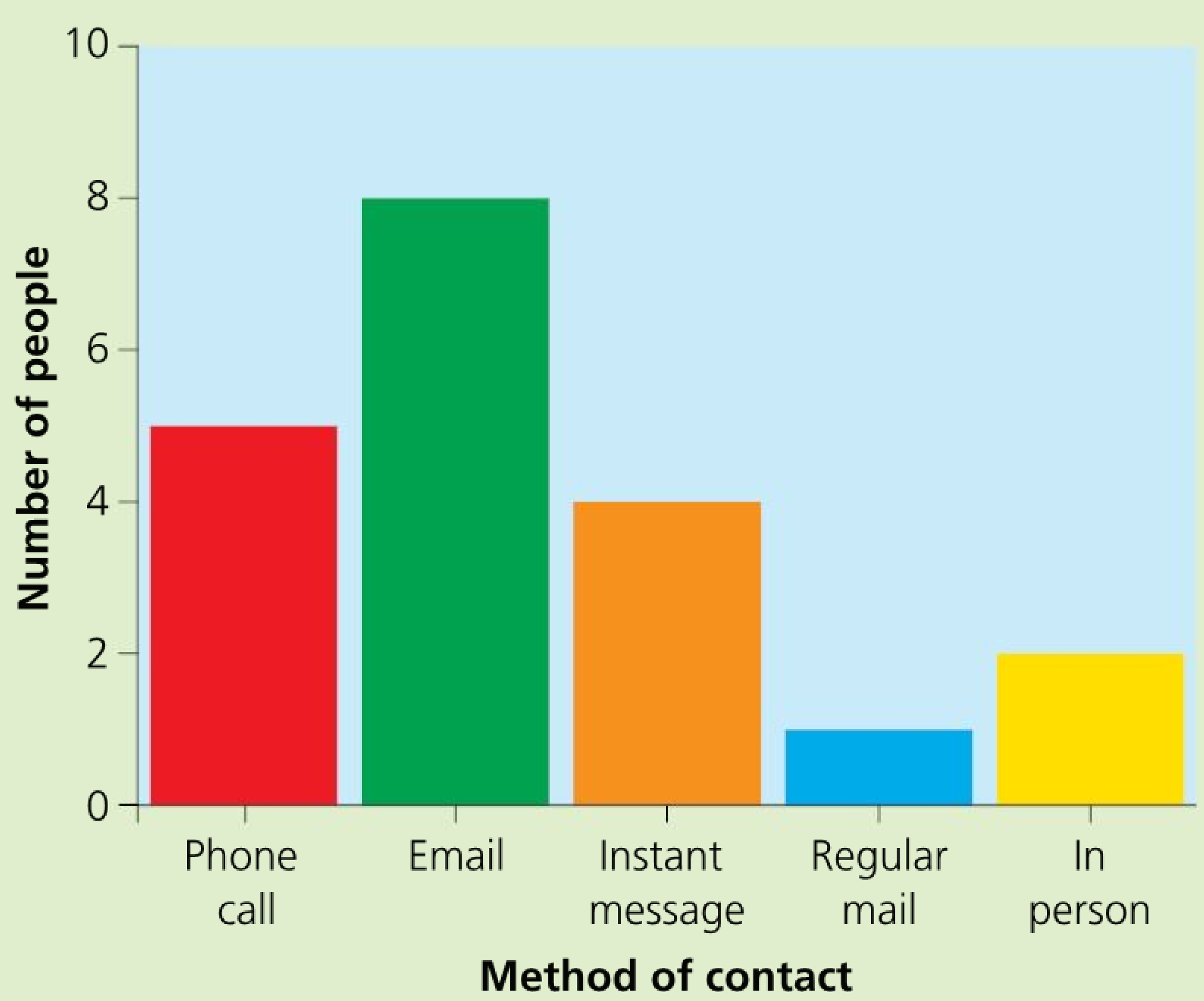
Find the mode for each data set.

1 a 10 20 20 20 30 30 40 50

b Tennis serves (mph)

Stem	Leaf
8	6
9	1 3 3 6 8
10	2 7 9
11	1 4 7

c What is your preferred method of contact?



2 a 14 12 20 11 15 14 10 12 14

b Height of students in M2 tutor group

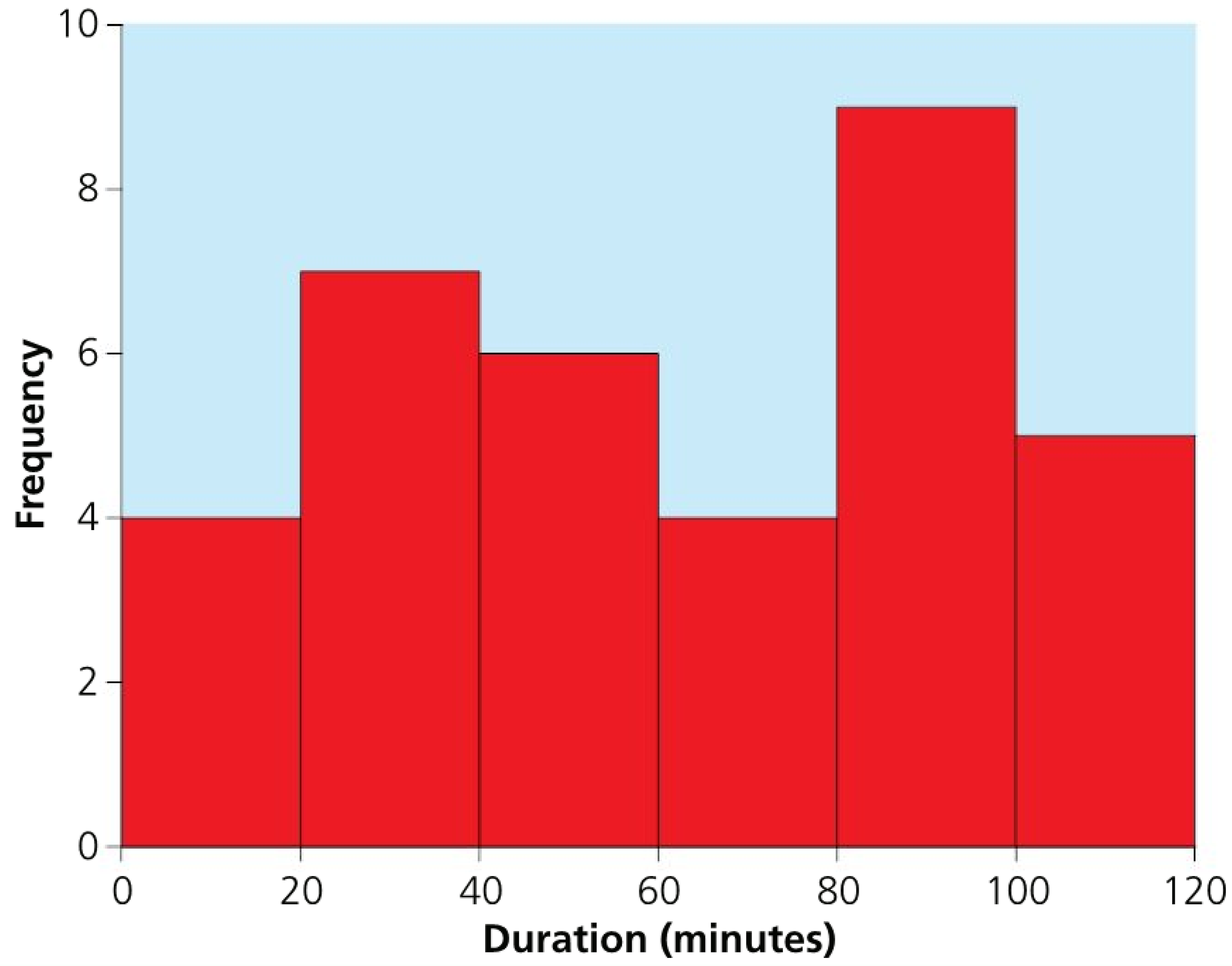
Height (cm)	Frequency
135–140	2
140–145	4
145–150	7
150–155	5
155–160	1

3 Tube timetable

No. 43
Downtown from NE 45th and University Way

AM		PM	
12	:06 :30 :55	**12**	:11 :25 :38 :53
1	:25	**1**	:08 :23 :37 :52
2	:25	**2**	:07 :22: 36 : 51
3		**3**	:06 :16 :26 :36 :45 :55
4		**4**	:04 :15 :25 :35 :45 :55
5	:25 :43	**5**	:05 :15 :26 :38 :51 :55
6	:07 :19 :34 :50	**6**	:06 :14 :19 :22 :32 :47 :56
7	:04 :17 :32 :47 :54	**7**	:04 :17 :30 :42
8	:01 :16 :32 :47	**8**	:01 :08 :32
9	:01 :11 :16 :31 :46	**9**	:02 :17 :32 :47
10	:01 :16 :30 :44 :59	**10**	:02 :18 :32 :51
11	:14 :29 :44 :58	**11**	:16 :45

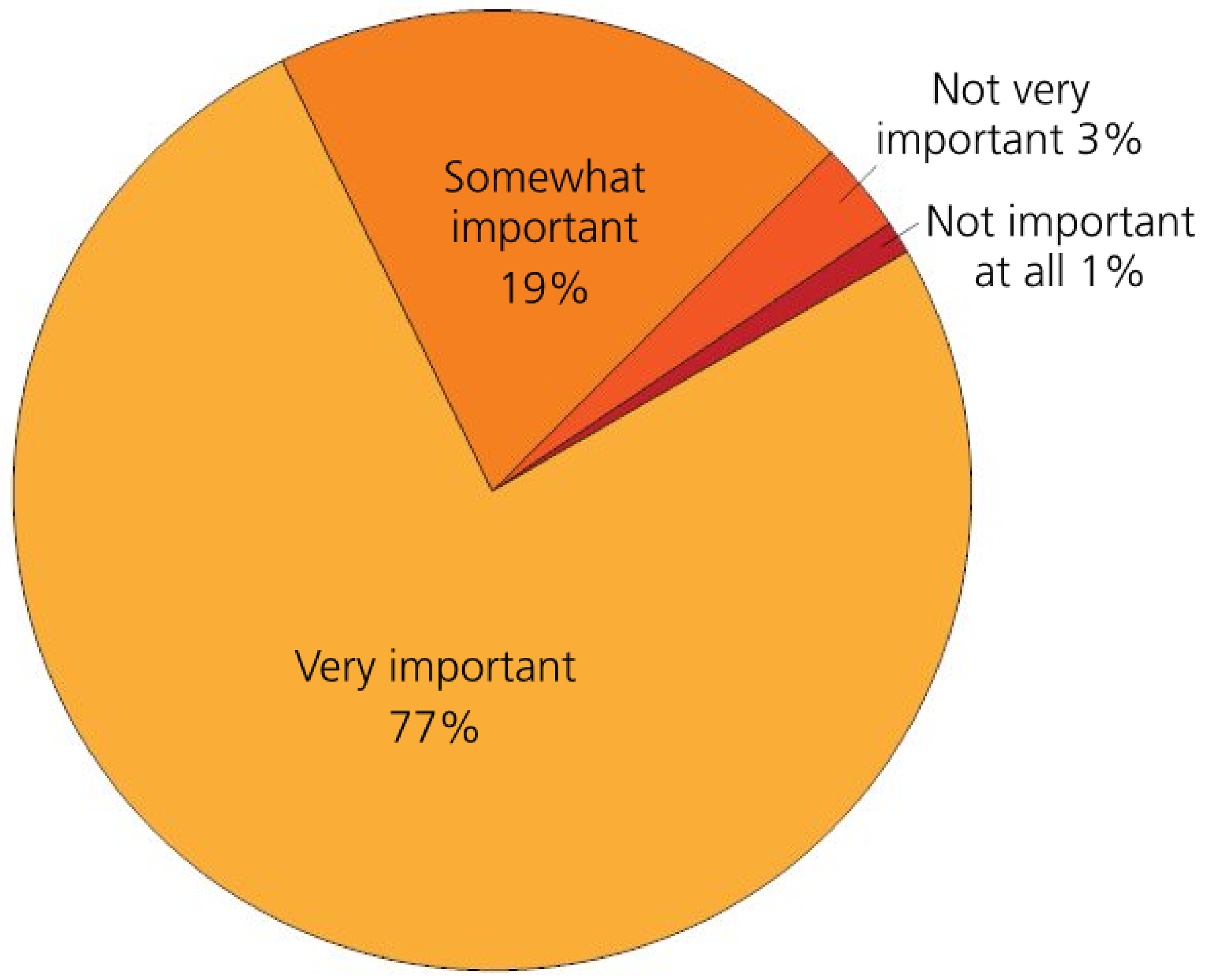
4 Number of minutes spent in car today



5 The number of regular characters in some BBC comedy shows

4 9 12 8 10 12 6 4 8 15 8

6 European views of national identity. How important is being able to speak our national language?

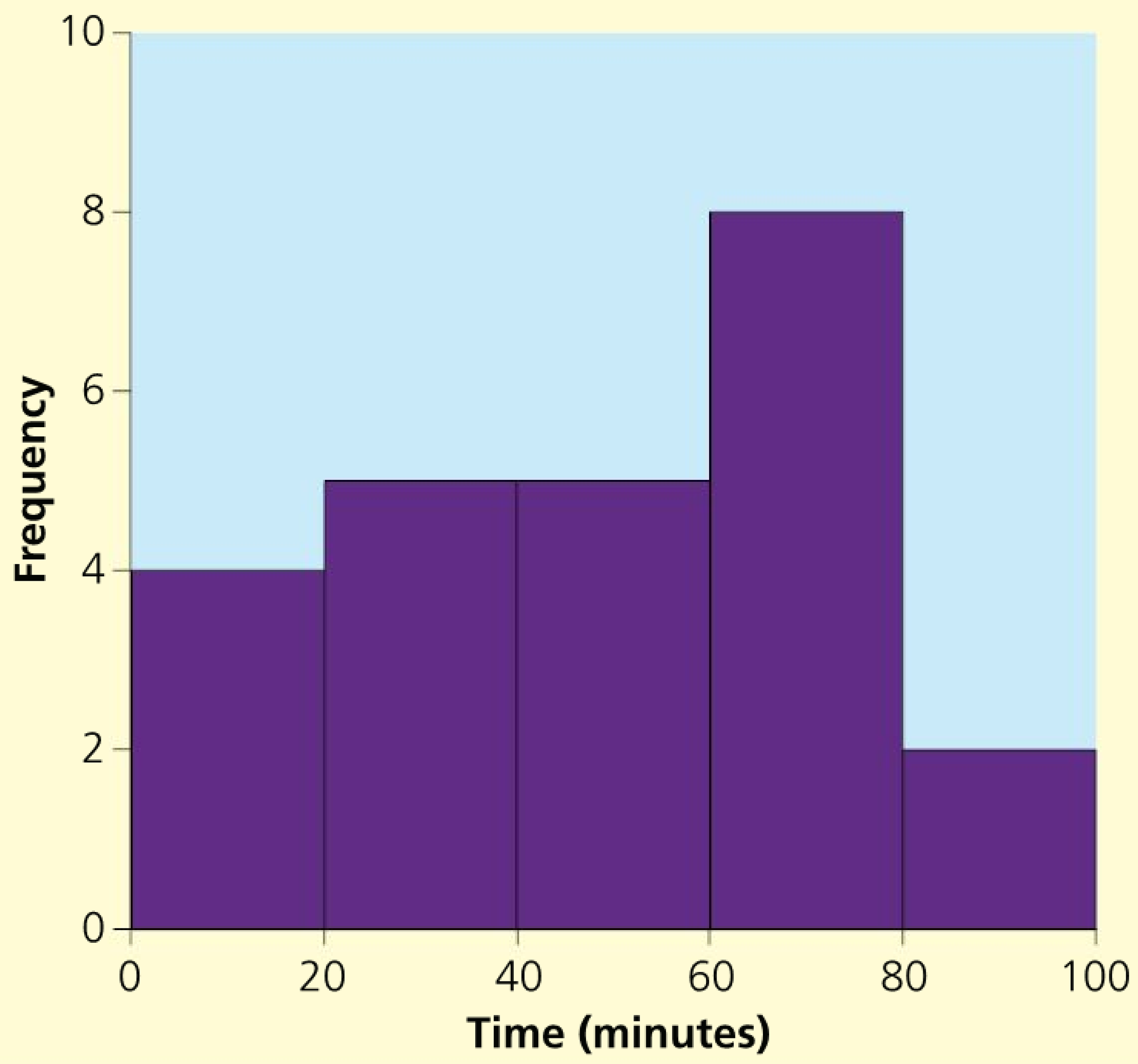


Source: Pew Research Centre

In each instance, you have determined the **mode** – the most frequently occurring value. Of the three measures of central tendency we will discuss, this is the only one that can be applied to both qualitative (non-numerical) and quantitative (numerical) data.

In pairs, describe a time when you have personally used mode in your day-to-day life, and then share with the rest of the class.

3 The amount of time MYP2 students spend reading on a typical day



4 The word cloud shows popularity of books read by M2s.

When should the centre be the middle?

MEDIAN

Links to: Languages

Look up the definitions for these words: mediator, medial, medium, mediocre, median. What do they all have in common?

When looking for a way to generalize numerical data by determining their 'centre', it is only natural to interchange that word – centre – with 'middle'. Even when mathematicians discuss the three measures of central tendency, 'median' often seems to be in the middle of the list!

Example

Try these questions yourself before looking at the solutions.

Find the middle number in each data set.

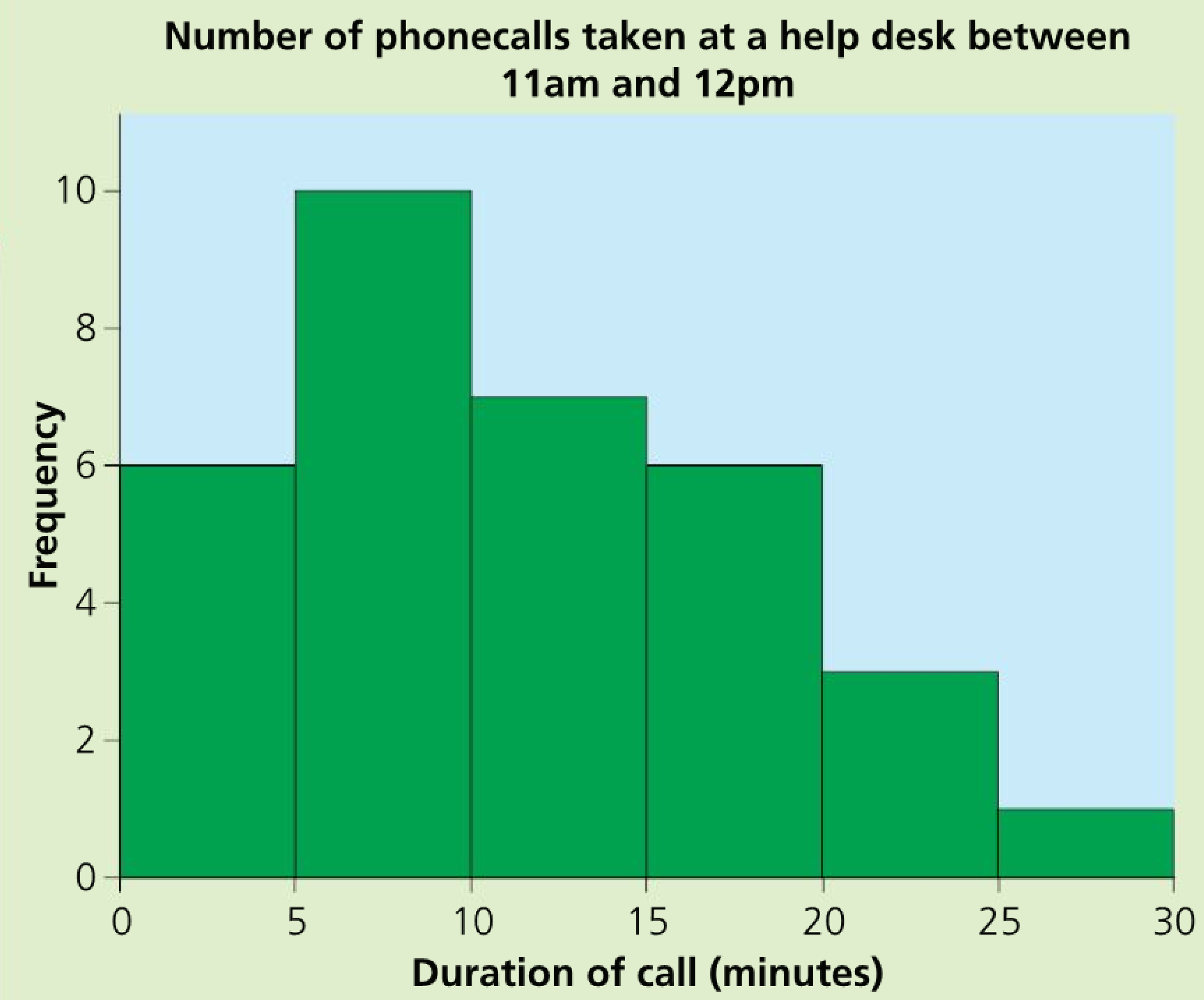
1 The data set is the number of items people buy at a grocery store.

4 24 8 13 7 2 38 26 16 29 11

2 The table shows the frequency of car journeys made versus distance between home and school.

Distance from school (km)	Frequency of car journeys
0–5	4
5–10	12
10–15	10
15–20	3

3 The histogram shows the number of phone calls taken at a help desk, 11am–12pm.



4 The stem-and-leaf diagram shows the waiting times (in minutes) at a doctor's office.

Stem	Leaf
0	8
1	0 4 7 7 8
2	1 5 5 9
3	2 3
4	5

Solutions

1 First we rewrite the data points in numerical order:

2 4 7 8 11 13 16 24 26 29 38

As there are 11 numbers, we find the sixth, so that we will have five on either side. (We can determine that the middle number is the sixth data point either by adding one to the number of data points and dividing by two, or by crossing off one datum from each side until we are only left with the middle number.)

2 4 7 8 11 13 16 24 26 29 38

So we know that half the people at the store buy more than 13 items at one time, and half buy fewer.

2 If we add up all the numbers in the frequency table, we see that there are 4 + 12 + 10 + 3 = 29 data points. To find the median, we need the fifteenth student, since 29 + 1 ÷ 2 = 15. Looking at the table again, we know that the first 4 students travel between 0–5 km, and the next 12 travel between 5–10 km. The fifteenth student will fall in this interval, as 4 + 12 = 16. Another way to look at it is to start counting to 15 at the first interval. When you reach 4 move to the next interval.

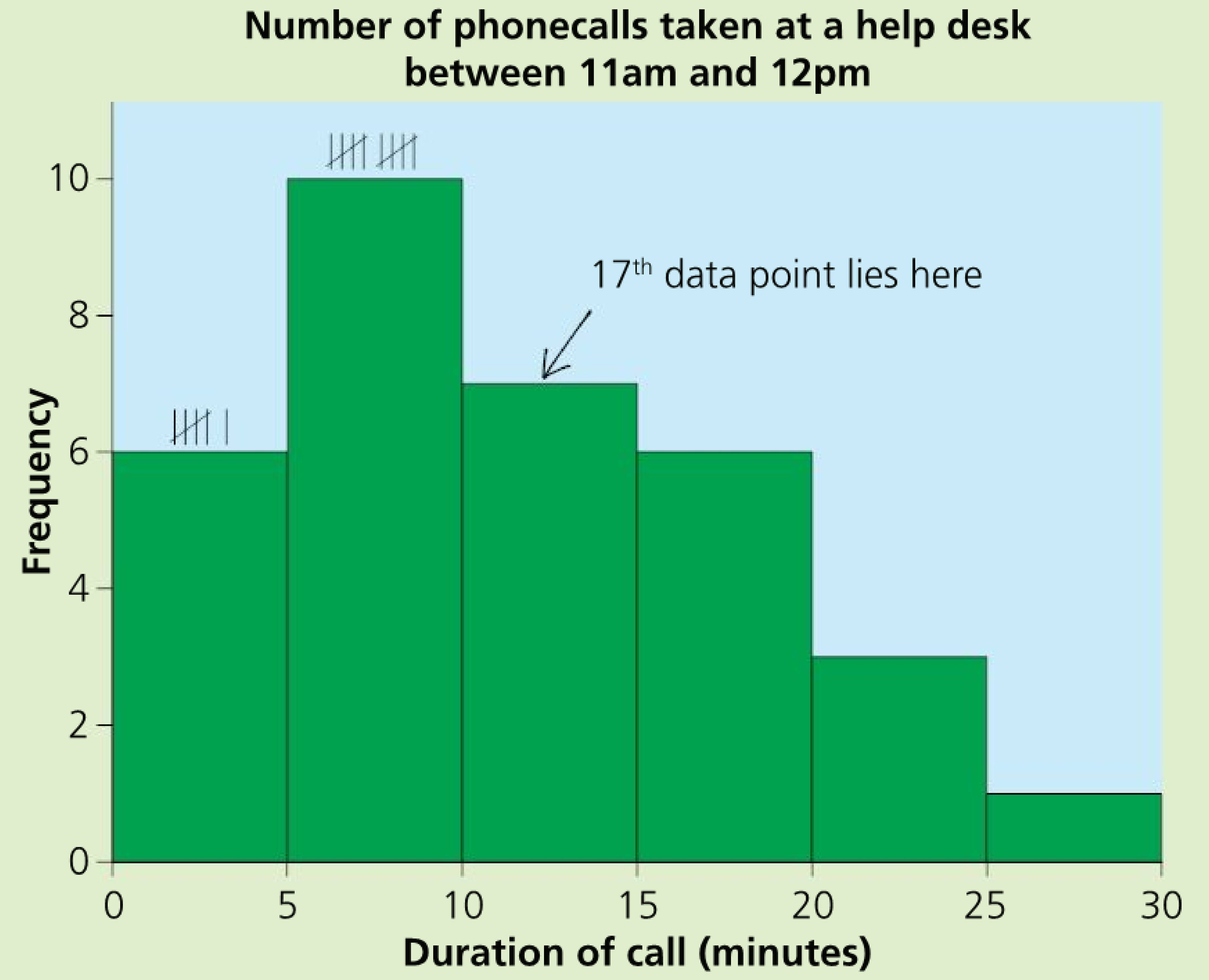
Distance from school (km)	Frequency
up to but not including 5 km	IIII
between 5 km and up to but not including 10 km	卌 卌 II
between 10 km and up to but not including 15 km	卌 卌
between 15 km and up to but not including 20 km	III

15th data point

This means that half the students surveyed travel as much as 5–10 km to get to school, and half travel more than that. When the median is not a single number but an entire interval, we refer to this as the **median interval**.

3 This is a very similar problem to the last question, but instead of reading from a table, we are reading off a graph. If we add up all the frequencies we have 33 data points: (6 + 10 + 7 + 6 + 3 + 1). The next number up, divided by two, is 17 (34 ÷ 2). Searching for the seventeenth data point on the graph, we arrive at the third interval, 10–15.

This means that the median interval is 10–15 and half of the people working at the help desk take up to 10–15 calls in an hour.



4 We can count 13 data points on the stem-and-leaf diagram. The middle is the seventh datum.

Waiting times at a doctor's clinic

Stem	Leaf
0	8
1	0 4 7 7 8
2	1 5 5 9
3	2 3
4	5

Remember, this is a stem-and-leaf diagram. We circled the number '1', but this actually represents 21. Thus, half the patients in the waiting room are there less than 21 minutes, and half wait longer than that to see the doctor.

PRACTICE EXERCISE

Determine the median in each data set.

1 2 3 7 12 20

2 **a** 7 9 13 1 6

b

Stem	Leaf
0	5
1	1 2 4
2	0 6 8 9
3	2 2 7

3

Interval	Frequency
0–2	卌
2–4	I
4–6	卌 I
6–8	III

4 Calculate the median in each case. Which is a good representation of the 'centre' of the numbers, and which is misleading? Why?

a 10 20 30 40 50 60 70

b 10 10 10 10 40 700 700 700 800

Why does the average person use average?

This all seems very convenient – the number of data points is always odd so there is always a middle number. Real life is not always so convenient. When there is an even number of data points, we have two numbers competing for the 'median' title – consider the cartoon of the 'Median Cup'.

To determine how the median is concluded in this situation, we first need to review our third and final measure of central tendency, the mean.

Most of the time, when trying to generalize a set of uni-variate data into one number, people look to the mean. Although this is a *type* of average, it is not the *only* average. Any measure of central tendency can, technically, be used as an average, but rarely will anyone refer to anything but 'mean' when they use the term 'average'.

SO WHAT DOES 'MEAN' ACTUALLY *MEAN*?

As we saw on page 85, there are occasions when the middle number is a poor representation of the data. Let's look again at those numbers:

10 10 10 10 40 700 700 700 800

The mode is 10 and the median is 40. If we calculate the mean by adding the numbers together and dividing by the number of data points (remembering that each number is a data point), we get:

$$\frac{10+10+10+10+40+700+700+700+800}{9}$$

$$= \frac{2980}{9}$$

$$= 331.11$$

ACTIVITY: Football's rising stars

ATL

- Critical thinking skills: Consider ideas from multiple perspectives

Rank	Name	Age
20	Mason Holgate	20
19	Álex Grimaldo	21
18	Manuel Locatelli	19
17	Frederico Chiesa	19
16	Aaron Martin	20
15	Allan Saint-Maximin	20
14	Harry Winks	21
13	Patrick Roberts	20
12	Jesús Vallejo	20
11	Lucas Tousart	20
10	Tom Davies	19
9	André Onana	21
8	Kai Havertz	18
7	Matthijs De Ligt	17
6	Malang Sarr	17
5	Christian Pulisic	18
4	Kasper Dolberg	19
3	Moussa Dembélé	21
2	Theo Hernández	19
1	Kylian Mbappé	18

PRACTICE EXERCISE

Calculate the mean in each data set.

1	2 6 9 13 5 7 6
2	13.4 61 278 13.6 171.6

3 What is the mean age of students participating in the Taekwondo class recorded in the table below?

Age of student in Taekwondo class	Frequency
11	3
12	5
13	4
14	2

Isabella and Liam determined the mean age of 'Rookies of the Year', as ranked on the sports blog 'Bleacher Report' in 2017:

Isabella's approach:

$$\frac{20+21+19+19+20+20+21+20+20+20+19+21+18+17+17+18+19+21+19+18}{20}$$

$$= \frac{387}{20}$$

$$= 19.35$$

Liam's approach:

$$\frac{2(17) + 3(18) + 5(19) + 6(20) + 4(21)}{20}$$

$$= \frac{387}{20}$$

$$= 19.35$$

How did Liam arrive at his numbers? Which approach is most efficient? Is that method always most efficient when calculating the mean? If not, what might efficiency depend on?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

Thinking back to the competition for the Median Cup on the facing page, when two numbers compete for the median place (this happens when there is an even number of data), we take the mean of those two numbers. So in the case of the Median Cup:

$$\text{median} = \frac{10 + 14}{2}$$

$$= 12$$

PRACTICE EXERCISE

Calculate the median, mode and mean for this set of data points:

4 8 9 13 17 21

Which is the best measure of central tendency?

Now that we have thoroughly reviewed what we know about mean, median and mode, let's summarize it all so that we have all the information in one place and need never confuse them with one another.

Copy and complete the table below somewhere where you may easily refer to it at any time. This could be in your agenda, on the back of your notebook or even on a sticky note placed on your bathroom mirror. Be creative! If you are not a risk-taker yet and choose to do it in your notes, use coloured pencils or highlighters – whatever you need to make it really stand out.

You will be using this information in mathematics and for sciences, and possibly for future courses (like psychology, medicine or economics, for example). You will also be using it in your day-to-day life all the time. Promise.

	Mean	Median	Mode
How to calculate			
When to use			

Making a Pro/Con list is a strategy many people use when taking very serious decisions. Essentially, you list the good points – the pros – and the bad points – the cons – for each of your choices.

Write a Pro/Con list for each measure of central tendency.

Sometimes it is the context and sometimes it is the numbers that determine the best choice of method.

Examples

Calculate using the most appropriate measure of central tendency in each case.

1 **Contextual:** A store needs to renew its supply of a brand of shoes and looks over the sizes from its past orders:

4 5 6 6 7 7 7 7.5 8 8
8 8.5 8.5 8.5 8.5 9 9 10

Solution

In this case, we want to order a larger quantity of the size that was most commonly sold, and smaller quantities of the rest, so we are most interested in the mode. The mean of 7.5278 is not even a shoe size so is hardly helpful! We note that 8.5 is listed four times, so this is the mode.

2 **Numerical:** A list of numbers is given for the height in metres of six different trees. What is the central tendency of these trees?

120 cm 160 cm 800 cm 860 cm
890 cm 1016cm

Solution

Using the median value here would give us a very inaccurate representation of these numbers.

$$\text{median} = \frac{800 + 860}{2}$$
$$= 830\,\text{cm}$$

Two of the trees are extremely small and, if we call 830 cm the 'centre', it implies that most trees are around the 700–900 cm mark.

Calculating the mean we get:

$$\frac{120 + 160 + 800 + 860 + 890 + 1016}{6} = 641$$

This gives us a much more accurate picture of what the trees are like!

EXTENSION

Convert the tree height measurements from centimetres to metres. Does this change your choice for which measure of central tendency you would use?

THINK-PAIR-SHARE

Beside each scenario below, draw the appropriate emoji to express whether it is best to use mean, median, or mode:


Mean


Median


Mode

Scenarios

- **Average income of customers staying in a particular hotel**
- **Favourite pizza topping**
- **The numbers 4 56 57 58 59**
- **Average time spent exercising per day, in minutes**
- **The numbers 4 4 1200 1250 1500**
- **Average points scored per game**
- **Class average of mathematics grade for the year**
- **Item numbers purchased in a grocery store**

ACTIVITY: Tampering with data

ATL

- Creativity and innovation: Create novel solutions to complex problems; Make guesses and generate testable hypotheses

1. Find a set of five **different** numbers that give a mean of 17.25.
2. Think of a set of ten **different** numbers that give a median of 21.
3. Think of three consecutive numbers that give a mean **and** median of 35.
4. Think of a set of numbers in which the mode would best represent the group of numbers, then repeat for mean and then median.

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

MEET A MATHEMATICIAN: JOHN FORBES NASH JR (1928–2015)

Learner Profile: Thinker

> 'I wouldn't have had good scientific ideas if I had thought more normally.'
>
> *J.F. Nash*

When Sylvia Nasar wrote her biography about John Nash, she very appropriately titled it *A Beautiful Mind*. Not only did Nash have a brilliant mathematical mind, which made significant contributions to game theory, geometry and calculus, but he was able to provide these contributions while fighting a lifelong battle with mental illness.

In his early years at Princeton, Nash wrote a paper about non-cooperative games that led to the 'Nash equilibrium' and later won him a Nobel Memorial prize. His theory opened up a branch of mathematics which helps deal with 'competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants'. This has been applied in war, business and biology. According to Wikipedia, Nash's work has provided 'insight into the factors that govern chance and decision-making inside complex systems found in everyday life'.

Shortly after Nash's first accomplishments, he began treatment for paranoid schizophrenia, a mental illness for which he received hospital treatment over ten years. At that point, Nash still had delusions but his behaviour was relatively moderate, so he began to use his own strength of mind and will to cast away delusional thoughts.

A Beautiful Mind has since become the basis for the Academy Award winning film with the same name.

How do we keep track of how far we've come?

CUMULATIVE FREQUENCY

DISCUSS

Why do cash registers keep a running total as items are scanned, prior to giving the final total?

In Chapter 2 you learned about frequency tables, which we use to organize data as we collect it. A helpful extension of the frequency table is the **cumulative frequency** table, which keeps a running total of frequencies for each category. This cumulative frequency will be especially helpful later when calculating percentiles or grouped medians … but it can also be used quite simply if you'd like to know how many data there are up to a particular value. For instance, in the example below, how many students achieved a level below 6? How many of these students are there in the class?

Criterion B level	Frequency	Cumulative frequency
1	0	0
2	1	1
3	2	3
4	7	10
5	4	14
6	2	16
7	1	17
8	0	17

We want to know the number of students who achieved a level below 6 … so we look at the cumulative frequency at Level 5.

Criterion B level	Frequency	Cumulative frequency
1	0	0
2	1	1
3	2	3
4	7	10
5	4	**14**
6	2	16
7	1	17
8	0	17

So there are 14 students who achieved a level below 6. If the question were worded 'How many students achieved a level of 6 or lower?', how would that change your answer?

Because the final number in the cumulative frequency table is 17, that means the number of students in the class is 17.

THINK-PAIR-SHARE

Think about how you could answer the questions in this example with a basic frequency table, like the one you learned about in Chapter 2.

Discuss your ideas with a partner and then share with the class.

How did we arrive at the numbers in the cumulative frequency column of the table? Before reading further, look closely at the frequency and cumulative frequency columns – how are the numbers related? There are two approaches you can take to calculate cumulative frequency.

Approach 1

Add all the numbers in the frequency column up to and including the row in question. For example, to determine the cumulative frequency for Criterion B Level 4, we would add 0 + 1 + 2 + 7 = 10.

Criterion B level	Frequency	Cumulative frequency
1	**0**	0
2	**1**	1 (0 + 1)
3	**2**	3 (0 + 1 + 2)
4	**7**	**10 (0 + 1 + 2 + 7)**
5	4	14 (0 + 1 + 2 + 7 + 4)
6	2	16 (0 + 1 + 2 + 7 + 4 + 2)
7	1	17 (0 + 1 + 2 + 7 + 4 + 2 + 1)
8	0	17

Approach 2

Add the frequency in the row of interest to the cumulative frequency from the previous row. To determine the cumulative frequency for a grade of 4, we would add 3 + 7 = 10.

Criterion B level	Frequency	Cumulative frequency
1	0	0
2	1	1 (0 + 1)
3	2	**3 (1 + 2)**
4	**7**	**10 (3 + 7)**
5	4	14 (10 + 4)
6	2	16 (14 + 2)
7	1	17 (16 + 1)
8	0	17

Example

Using the table from our example, what percentage of students achieved a Level 4 or lower? Round your answer to the nearest percent.

Solution

From the table we see that 10 students received a level of 4 or lower, and that there are 17 students in total.

$\frac{10}{17} \times 100\% = 58.82\% = 59\%$

Thus, 59% earned a Criterion B Level 4 or below.

PRACTICE EXERCISE

1 Copy and complete the table below by filling in the cumulative frequency column.

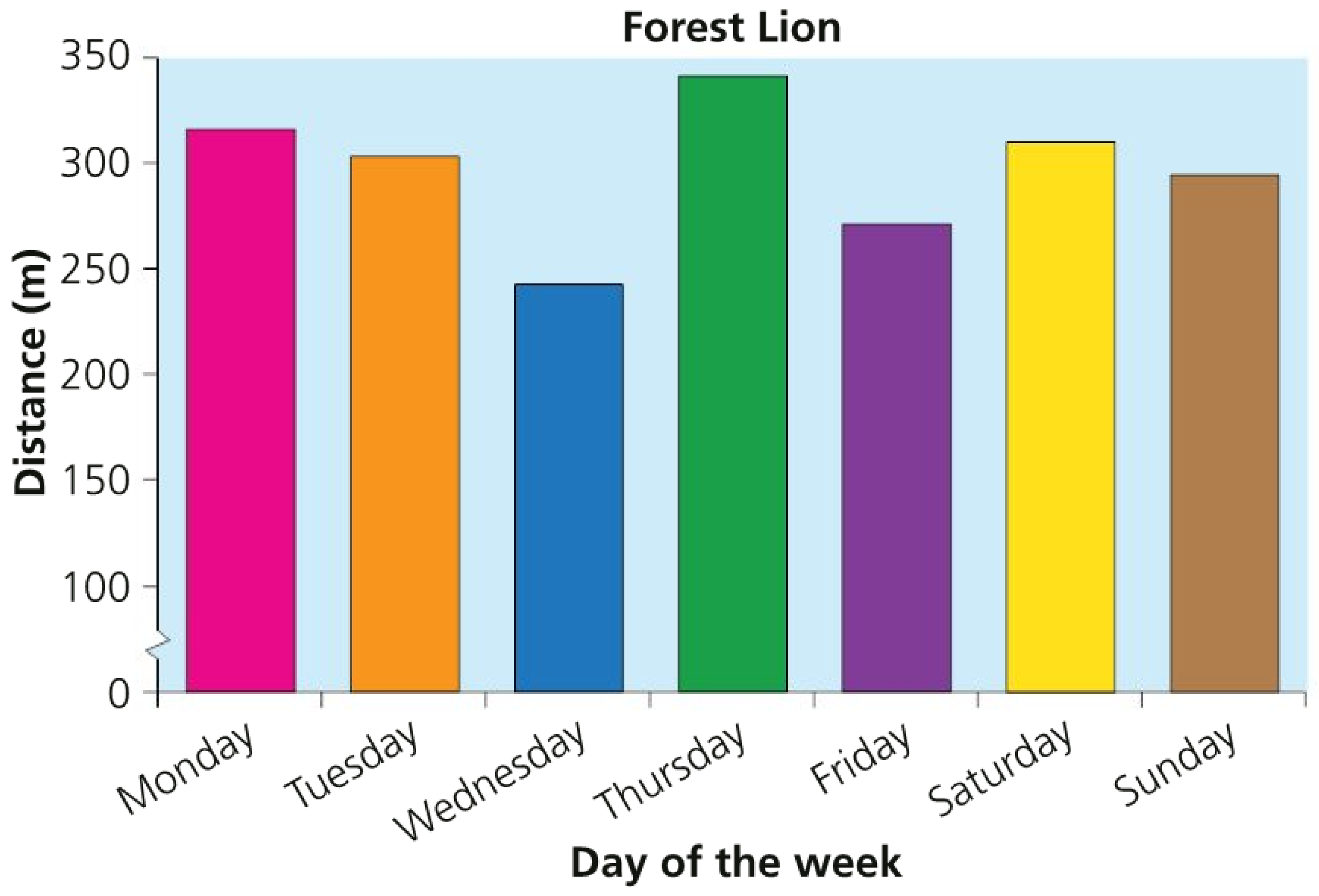
Number of days people exercised in week	Frequency	Cumulative frequency
0	1	
1	3	
2	4	
3	10	
4	8	
5	7	
6	4	
7	2	

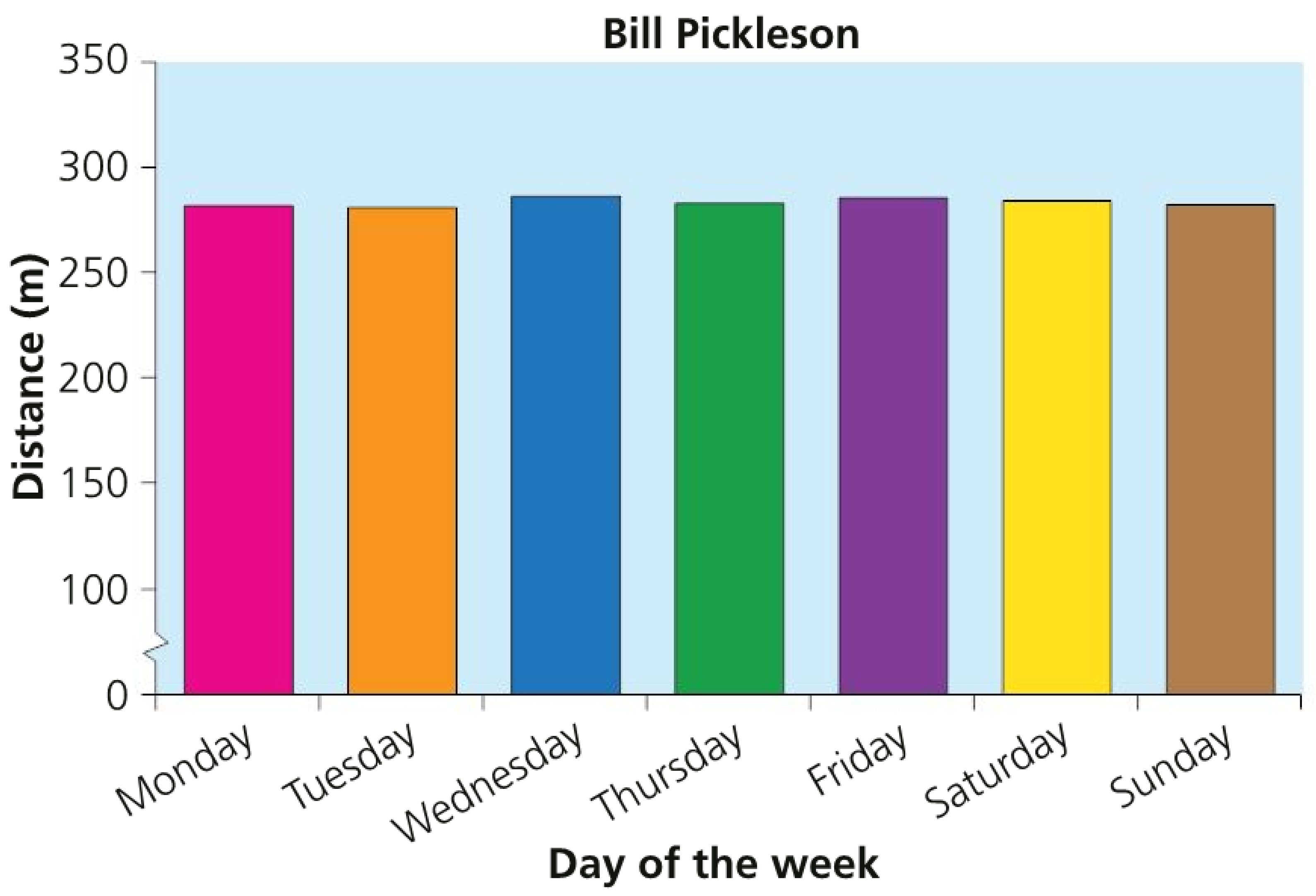
2 How can you use the cumulative frequency to calculate the percentage of people who exercised on three days or fewer?

How can outliers affect range?

Last year you learned to use range to determine the spread of your data. Range is important because it tells us how *consistent* our data are. You also learned about outliers.

Recall this example from *Mathematics for the IB MYP 1*, page 73, which shows how range can vary in data.





Note that Forest Lion's maximum driving distance was 340 m and his minimum was 240 m. This gives his range for the week as:

maximum – minimum

= 340 m – 240 m

= 100 m

DISCUSS

How did Pickleson's Saturday outlier affect his range?

Bill Pickleson's drives were between 260 m and 280 m, making his range for the week:

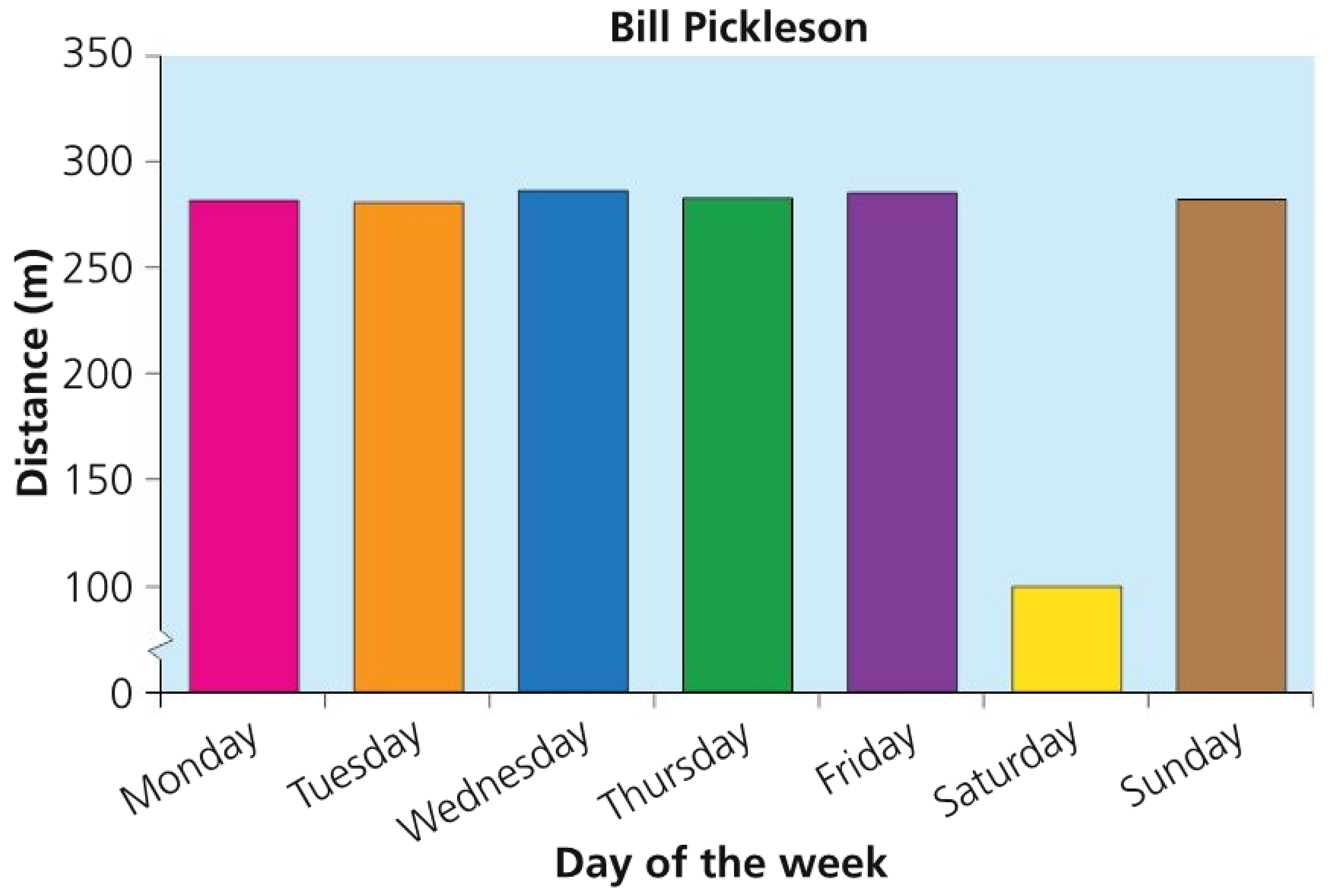
maximum – minimum

= 280 m – 260 m

= 20 m

So, while Forest Lion had longer maximum drives, he was far less consistent than Bill Pickleson.

Now, if Pickleson's graph looked like the one below, it would mean Saturday was a bit of an outlier – it stands out completely from the rest of the week. Perhaps he had injured himself, or only took one bad shot before suddenly leaving the golf course – people love to speculate about outliers!



Example

Calculate the range of the ages below. What do you notice?

4 45 45 45 45 46 46 101

Solution

range = 101 – 4 = 97

This is a very large range, and suggests the numbers are scattered widely, when in fact most of them hover close to 45. The 101 and even the 4 are outliers that have greatly changed the perception of this data's spread.

The range in the previous example misleadingly suggests a wide variety of ages when more than half the numbers are within the 45–46 range (which is, in fact, quite a small range). To avoid problems of this kind, statisticians shift their focus onto the middle 50% of the data when considering central tendencies, eliminating any misleading outliers. They ask 'what is the range of the middle 50%?' and proceed to calculate the **Interquartile range** (**IQR**). In the case of the ages that we were examining, we start by finding the middle number – the median.

$$\left.\begin{matrix}4\\45\\45\\45\end{matrix}\right\}50\%\qquad \text{median} = Q_2 = 45 \qquad \left.\begin{matrix}45\\46\\46\\101\end{matrix}\right\}50\%$$

Note this median is also known as the **second quartile** or Q_2.

Next we look at the top half of the data and find its median. We call this Q_1.

4

45

45 } $Q_1 = 45$

45

Repeat with the bottom half of the data to find Q3.

45

46

46 } $Q_3 = 46$

101

ACTIVITY: Ski Bowl qualifiers

ATL

- Critical-thinking skills: Interpret data

Devon can qualify for the Ski Bowl if her combined ski times are **below** the first quartile among all of the racers. The table here shows the times of all competitors in the Ski Bowl.

If Devon's race time is 2:13, will she make it to the Ski Bowl?

Race times (minutes)
1:98
2:03
2:09
2:13
2:14
2:20
2:20
2:21
2:23
2:25
2:29
2:34

Assessment opportunities

In this activity you have practised skills that can be assessed using Criterion D: Applying mathematics in real-life contexts.

Now we have successfully divided the data into four.

25% { 4, 45

$Q_1 = 45$

25% { 45, 45

$Q_2 = 45$

25% { 45, 46

$Q_3 = 46$

25% { 46, 101

DISCUSS

Why are there three quartiles if we are dividing the data into four sections?

With the data divided this way, we can easily calculate the range of the middle 50% of the data, that is, the IQR. This is sometimes called a 'trimmed range', as we are trimming off the top 25% and the bottom 25% of the data:

$$\mathbf{IQR = Q_3 - Q_1}$$

In this case, $Q_3 = 46$ and $Q_1 = 45$ so:

$$IQR = 46 - 45$$

$$IQR = 1$$

How can we visually represent spread?

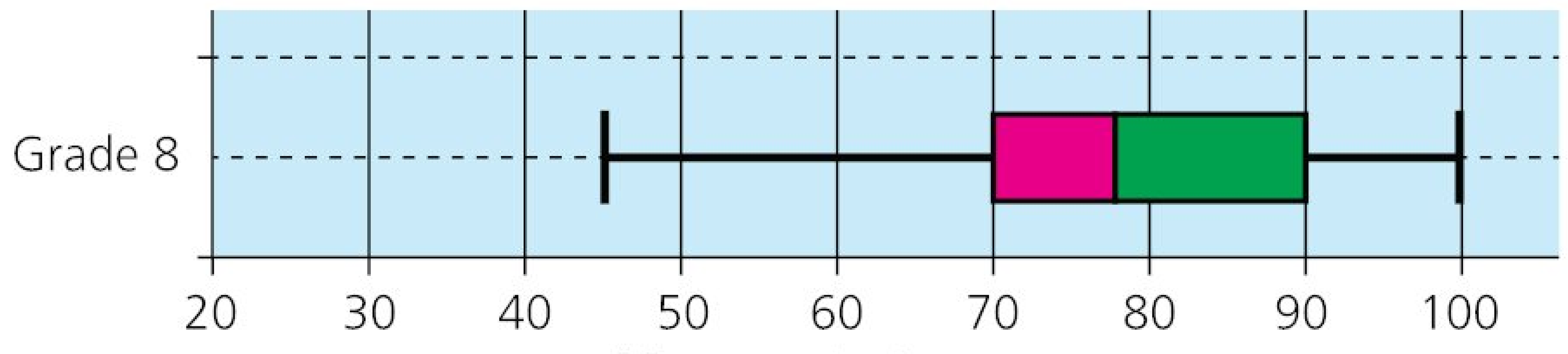
Hint

*Remember that in a box-and-whisker plot, the **size** of each section has nothing to do with the number of data points in that section but everything to do with the spread.*

Just as data are difficult to comprehend without visual aid, spread has little meaning when only described in words, such as: ‘IQR of 1 and range of 97’. In order to better understand how data are spread out, we use box-and-whisker plots.

BOX-AND-WHISKER PLOTS

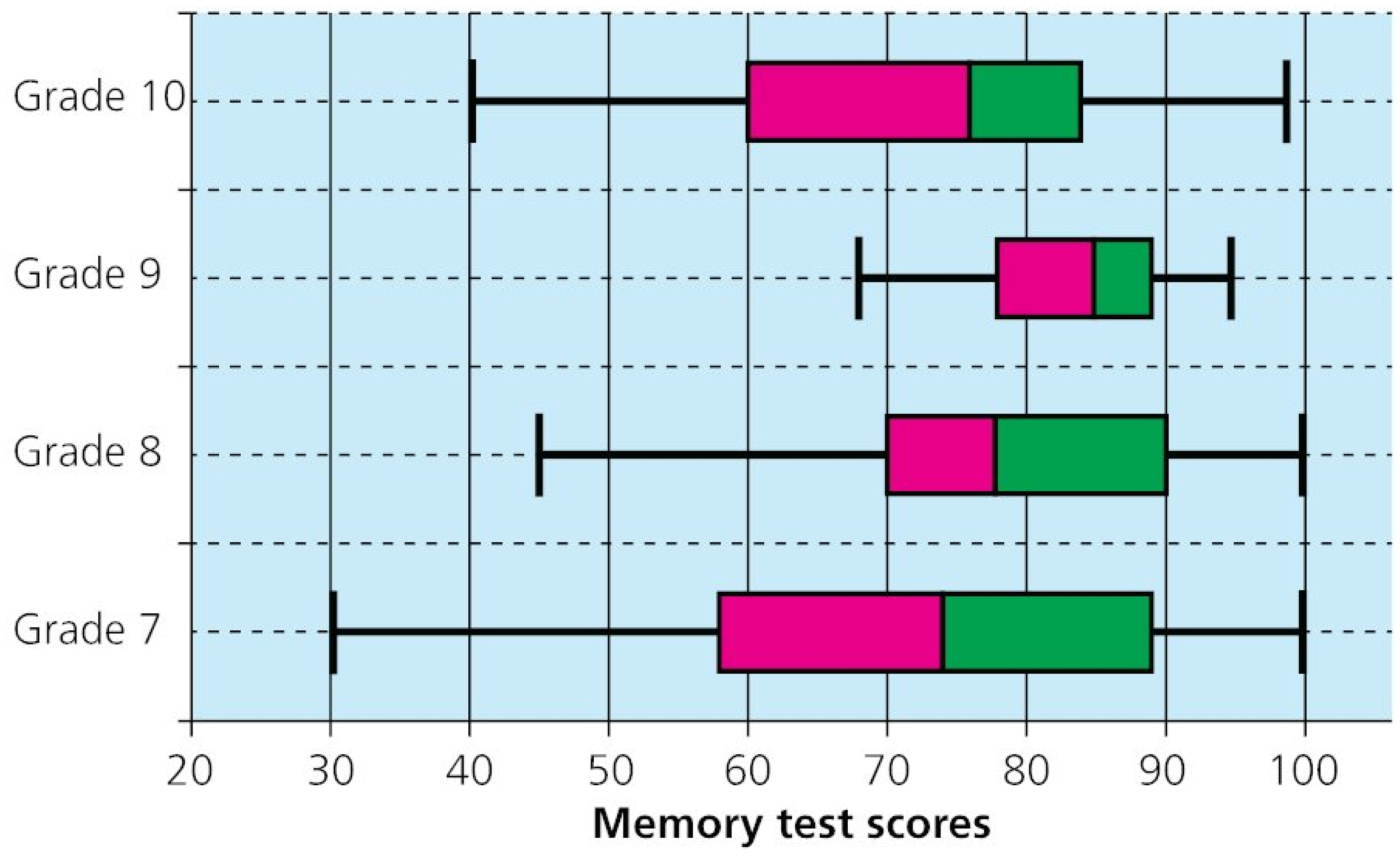
The graph below displays the distribution of memory test scores for 100 Grade 8 students. The data are divided into four, and the ‘box’ in the middle represents the middle 50%. This means that half the students scored between 70 and 90 out of 100. The box is divided in the middle by the median. **The number of students in the pink zone is equal to the number of students in the green zone.** The fact that the green box is larger than the pink box does not mean there are more students in this range. It is **not** a bar graph. Each area describes the performance of 25 students. The green zone is larger because the second quartile is more spread out. Students in this zone scored between 78 and 90. Students in the pink zone scored in a smaller range, 70–78, but **the number of students** is the same.



Because we have learned mostly about graphs that show exact data points, it takes some getting used to graphs that show how data are **spread out**. From the graph above we can determine that:

- one or more students scored the best possible result (maximum score = 100)
- no students submitted blank entries as the minimum score = 45
- 25% of the class scored under 70 – but we cannot determine if the majority of that group performed closer to 45 or scored in the high 60s
- 25% of the students scored between 70–78, and another 25% scored between 78–90
- the top 25% scored above 90! Wow! That means an entire quarter of the students did exceptionally well!
- three-quarters of students scored above 70, so in general this Grade 8 group did very well.

Box-and-whisker graphs rarely stand alone. They are often used to compare the spreads of different sets of data, as shown below.



EXTENSION

Suppose the Grade 8 class comprised of 32 students. Estimate the number of students who achieved a memory test score of 90 or above. Give reasons for your answer.

Now we can begin to compare the spread of different data sets. We can answer the following questions by looking at the second box-and-whisker plot:

As a whole, which grade level performed the best?

The Grade 9 class had the least fluctuation. Although nobody reached 100, as a whole, they performed consistently well, with the highest median, third and fourth quartiles. 75% scored above 78!

Which class had the least consistency?

Without even looking at the numbers, we can see the Grade 7 class has the widest intervals in all but the top quartile.

Which class had the lowest scorer?

The Grade 7 class had at least one student who scored 30.

Which class had the highest number of passing students?

We cannot determine this, as the graph does not show us individual data points, only how they are spread out as a whole.

ACTIVITY: Raising awareness about the Great Barrier Reef

ATL

- Communication skills: Make inferences and draw conclusions, Use and interpret a range of discipline-specific terms and symbols
- Critical-thinking skills: Interpret data; Evaluate evidence and arguments; Draw reasonable conclusions and generalizations

The Great Barrier Reef is the largest coral reef system in the world. Its 2900 individual reefs stretch across more than 2300 km, and can be seen from outer space. It is composed of billions of organisms (coral polyps) and is home to over 6750 different species.

Study the box-and-whisker plots in Figure 2 at this link:

www.ck12.org/statistics/Applications-of-Box-and-Whisker-Plots/rwa/Counting-Coral/

- **What major problem do you notice from the series of box-and-whisker plots?**
- **How can the plots help to raise awareness about this problem?**
- **Compare specific elements of the graphs as you justify your response.**

Assessment opportunities

In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns and Criterion D: Applying mathematics in real-life contexts.

What steps are needed to draw a box plot?

Example

Now that you are an expert on interpreting box-and-whisker diagrams, it's time to create your own.

M. Vincent has found a home to rent in Paris for 8500 euro. Below are the rent prices in euros of 11 homes in the eleventh arrondissement in Paris:

8250 8750 8490 8000 8520 8610 8500
10 000 8570 8620 8595

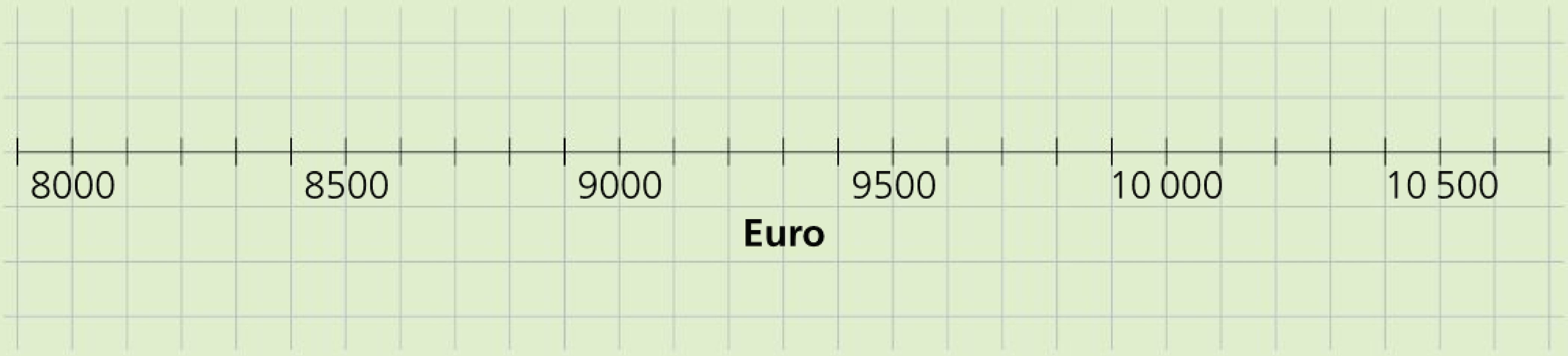
Does this neighbourhood have a large variety of prices? Are there outliers? Draw a box-and-whisker plot to determine if the home M. Vincent has selected falls in line with other homes in the location.

Solution

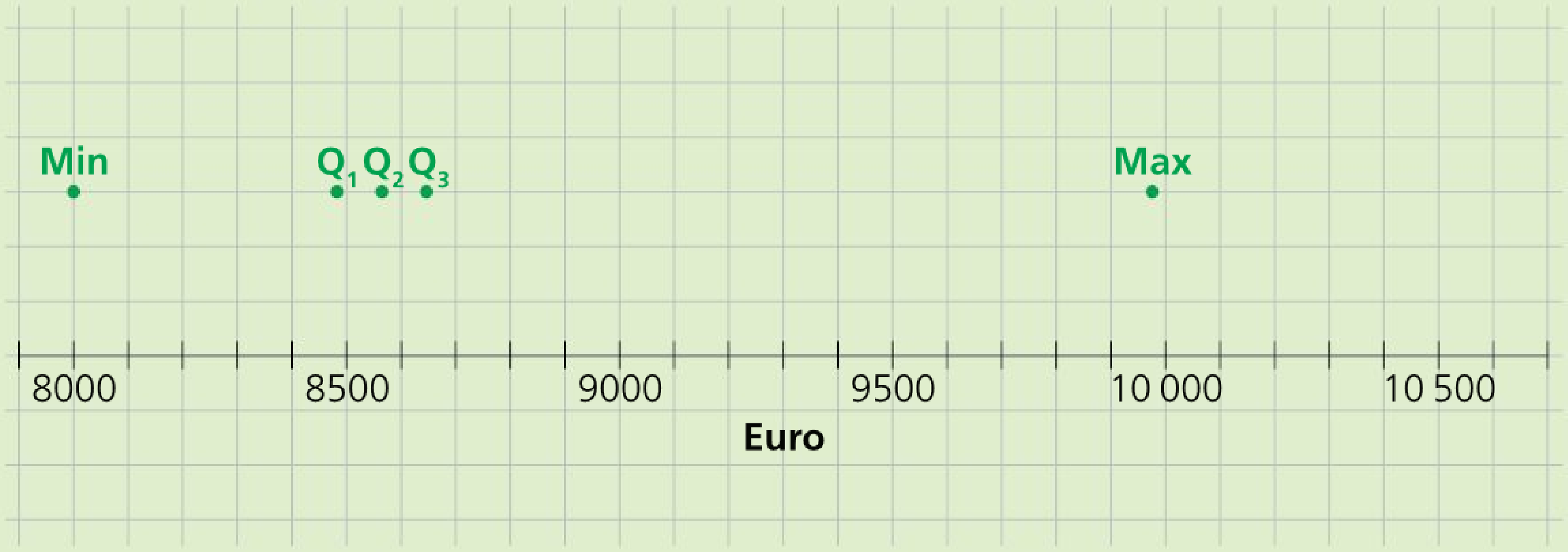
First we need to organize the information. List it in numerical order and find the quartiles:

8000
8250
8490 Q_1
8500
8520
8570 Q_2
8595
8610
8620 Q_3
8750
10 000

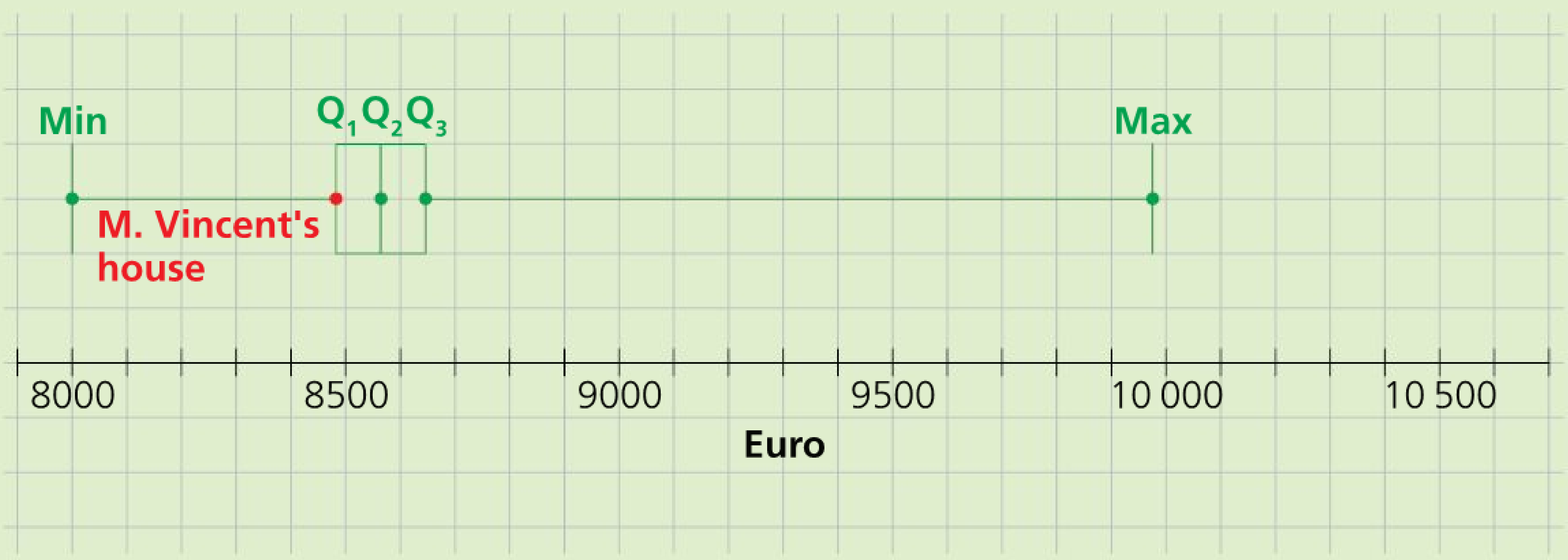
Now we can start graphing. Begin by drawing a scale on the x-axis …



… and now plot points above the maximum, minimum and each quartile:



Finally, we draw vertical lines at each quartile, create a box around them, and extend lines out to the maximum and minimum points (the whiskers).



As you can see from the graph, the home M. Vincent is considering is below the median, and only just hovering above the first quartile. This makes it better priced than most homes in the area … although for the most part they are all consistently between 8490–8620 euro. The occasional home is quite a lot higher in price, and those that are priced lower are still very close to the price of M. Vincent's home.

ACTIVITY: Tech desk

ATL

- Critical-thinking skills: Interpret data; Evaluate evidence and arguments
- Communication skills: Understand and use mathematical notation

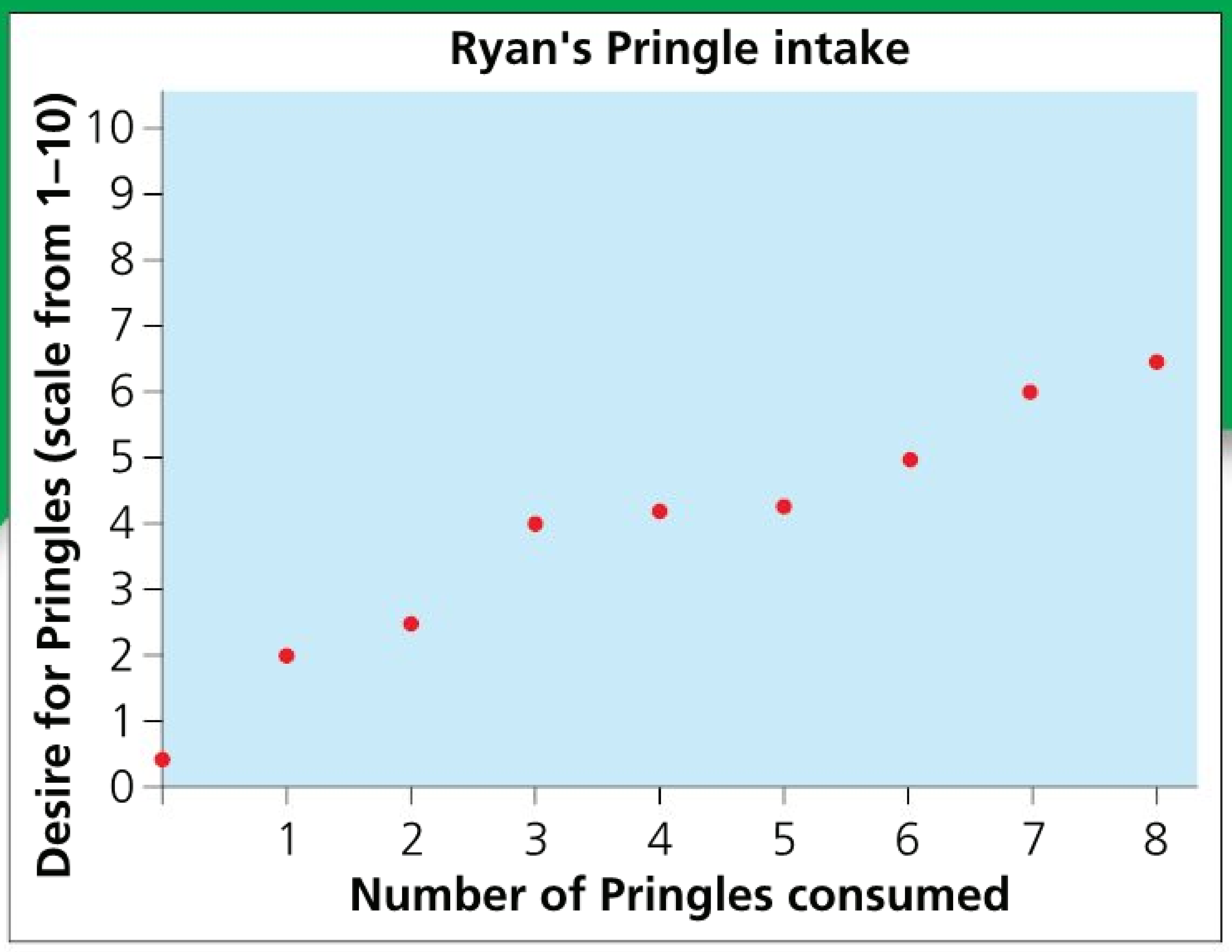
Shi-Vaughn and Lucas both work at a technical support desk. Following each phone conversation, callers are asked to complete a satisfaction survey about the service they have received. The survey results score from 1 to 10. Draw a box-and-whisker plot for each call handler to determine the consistency of their performance ratings. Who is more reliable according to the survey? Could there be more going on than the box plots show?

Shi-Vaughn	Lucas
2	7
5	6
7	7
10	8
6	7
7	9
8	8
3	8
4	7
5	5
2	8
6	6

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion D: Applying mathematics in real-life contexts.

What are positive and negative correlations?



Until now we have been examining uni-variate data – data with only one variable. When we start looking at **bi-variate** data, we can really start digging into relationships between variables. How does one affect the other, if at all?

WARM-UP

For each research title listed below, state whether the research involves uni-variate or bi-variate data:

- How many auto parts does the average assembly worker make in a day?
- How many times does a healthy one-year-old typically visit the doctor in a year?
- Is there a relationship between driving speed and gas consumed?
- What is the most popular song in an all-school survey?
- What is the relationship between rainfall and number of ripe tomatoes?

You may have noticed a cheat tactic for answering the warm-up question: anytime we are looking for a relationship, we are looking at bi-variate data.

Recall that the most common way to represent bi-variate data is through a scatter plot. These help us to identify patterns and trends, and draw conclusions about how two variables are related.

The slogan for Pringles chips is 'Once you pop, you can't stop!' This implies that the more Pringles chips you eat, the stronger your desire to eat more.

Recalling what we learned about scatter plots last year, can you determine a trend in Ryan's Pringle intake and desire from the graph above?

To determine if two variables really do have a relationship – if they fluctuate together – we study their **correlation**.

You may have noticed from the graph that the more Ryan eats, the more he wants to eat. This is an example of a **positive correlation**. As one variable increases, so does the other. Here are some other examples of positive correlations:

- temperature and ice cream sales
- smoking and incidences of lung cancer
- height and weight
- reading frequency and spelling ability.

Correlations are not always positive. Correlation simply describes how two variables fluctuate together. As one variable increases does the other increase? Decrease? Change randomly and sporadically?

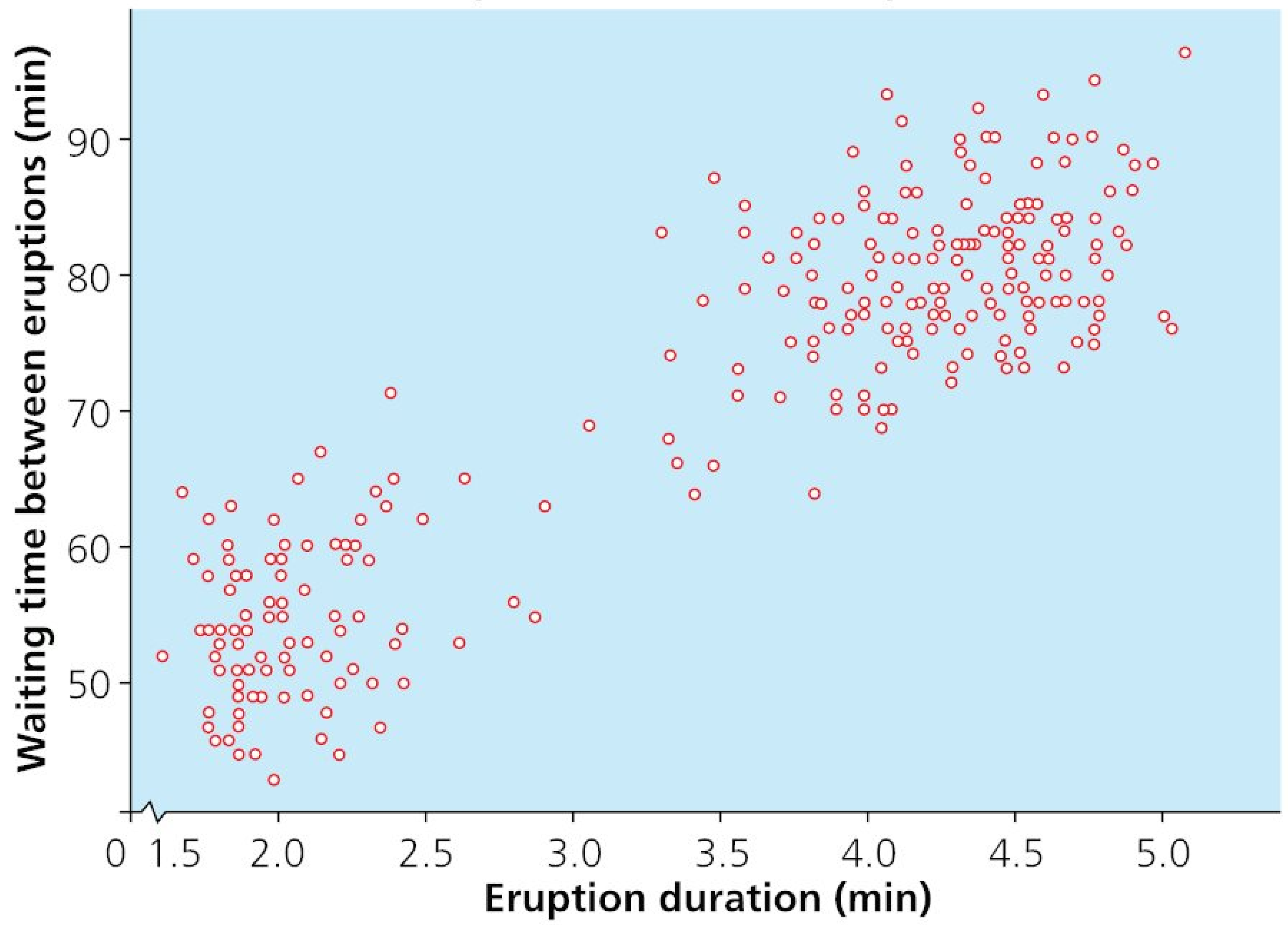
SEE-THINK-WONDER

Look at the graphs on this page and on the facing page. What do you think about any correlation shown?

What do you wonder about the reasons for the relationships (or lack of relationships)? Discuss your thoughts with a partner.

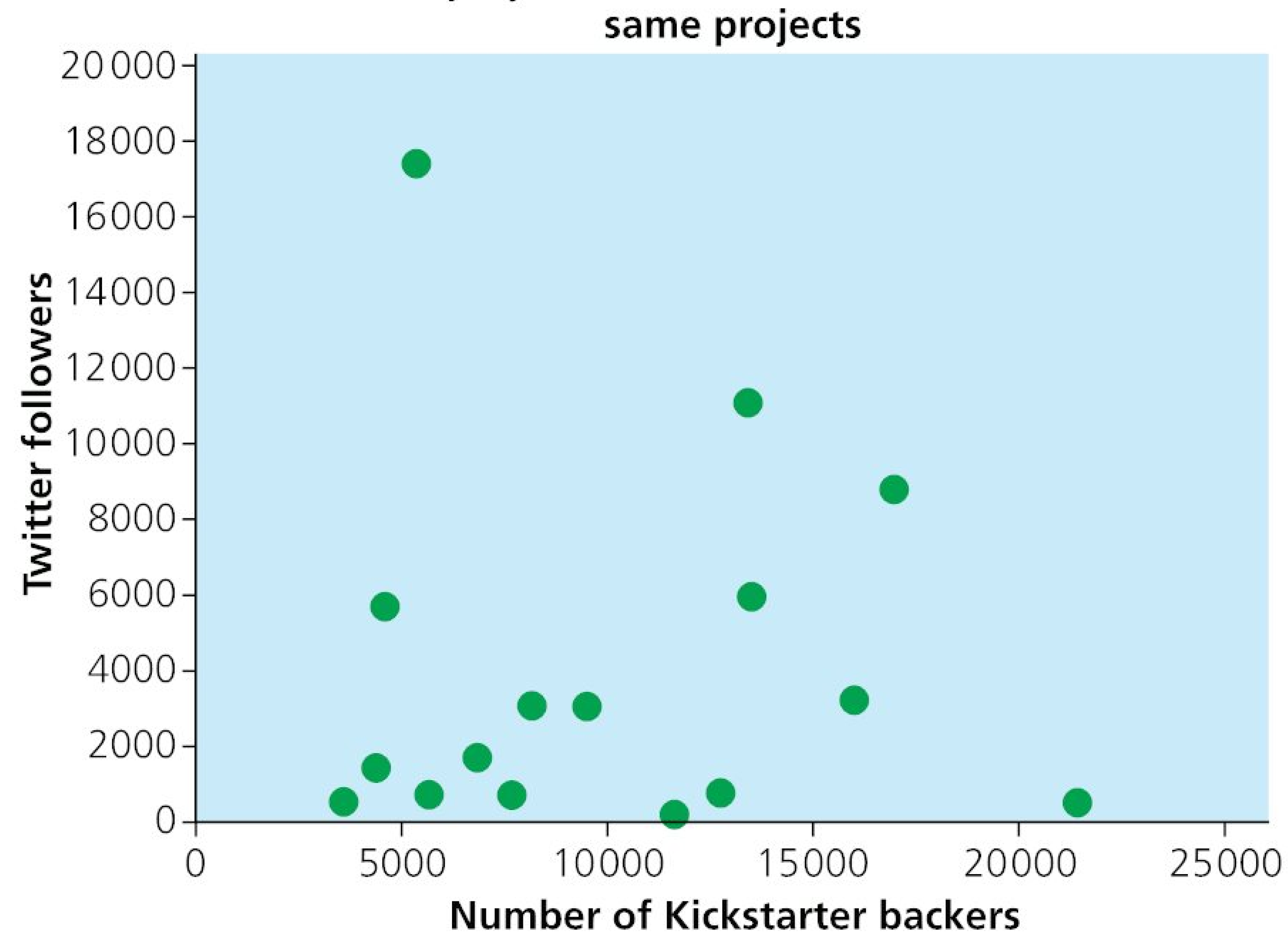
In your discussion, you may choose to use a phrase like this: 'As (one variable) increases, the (other variable) decreases/increases/does nothing.'

Graph A: Old Faithful Eruptions



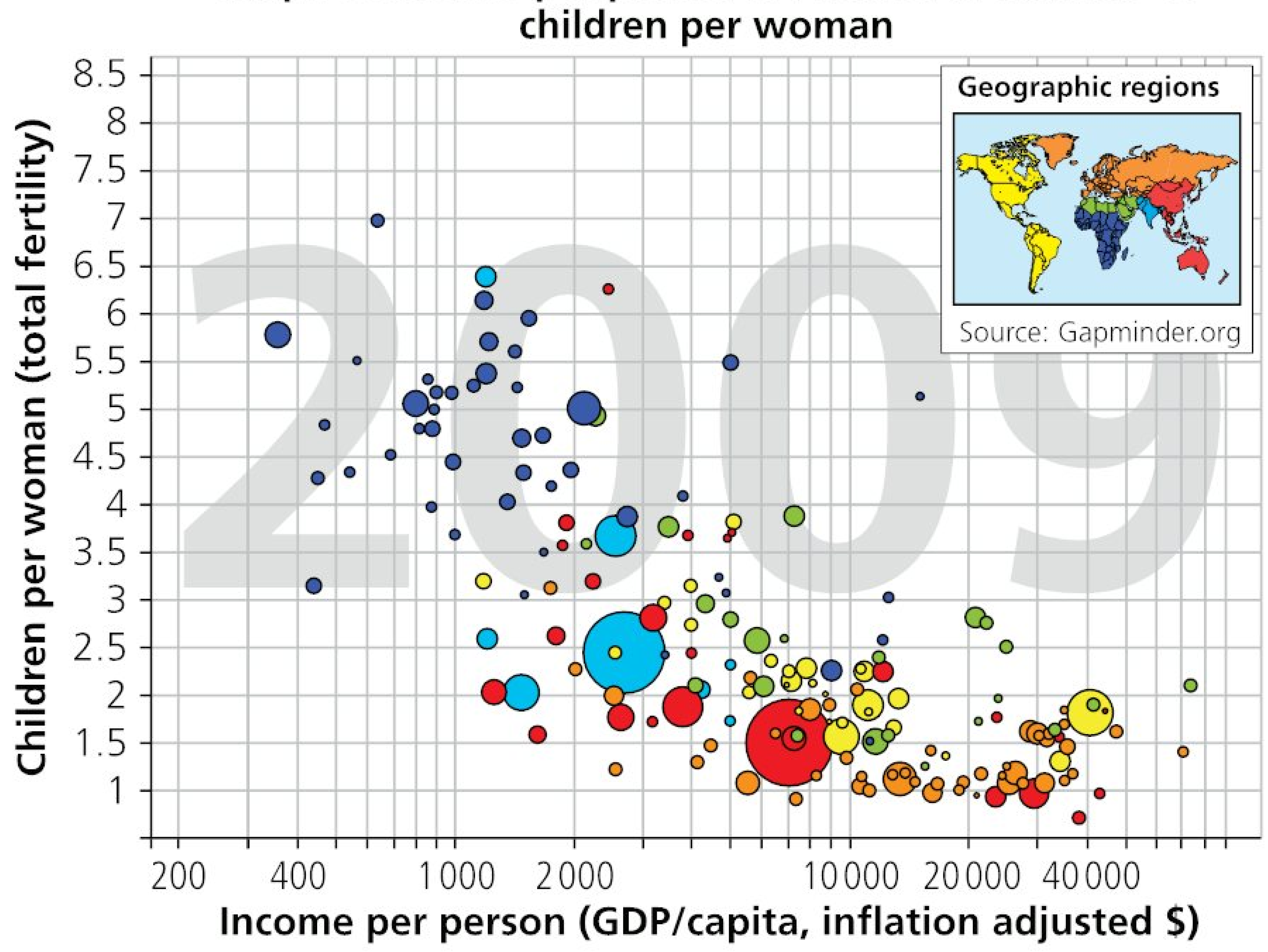
Source: Wikipedia: **en.wikipedia.org**

Graph C: Correlation between Twitter followers for Kickstarter projects and number of investors in those same projects

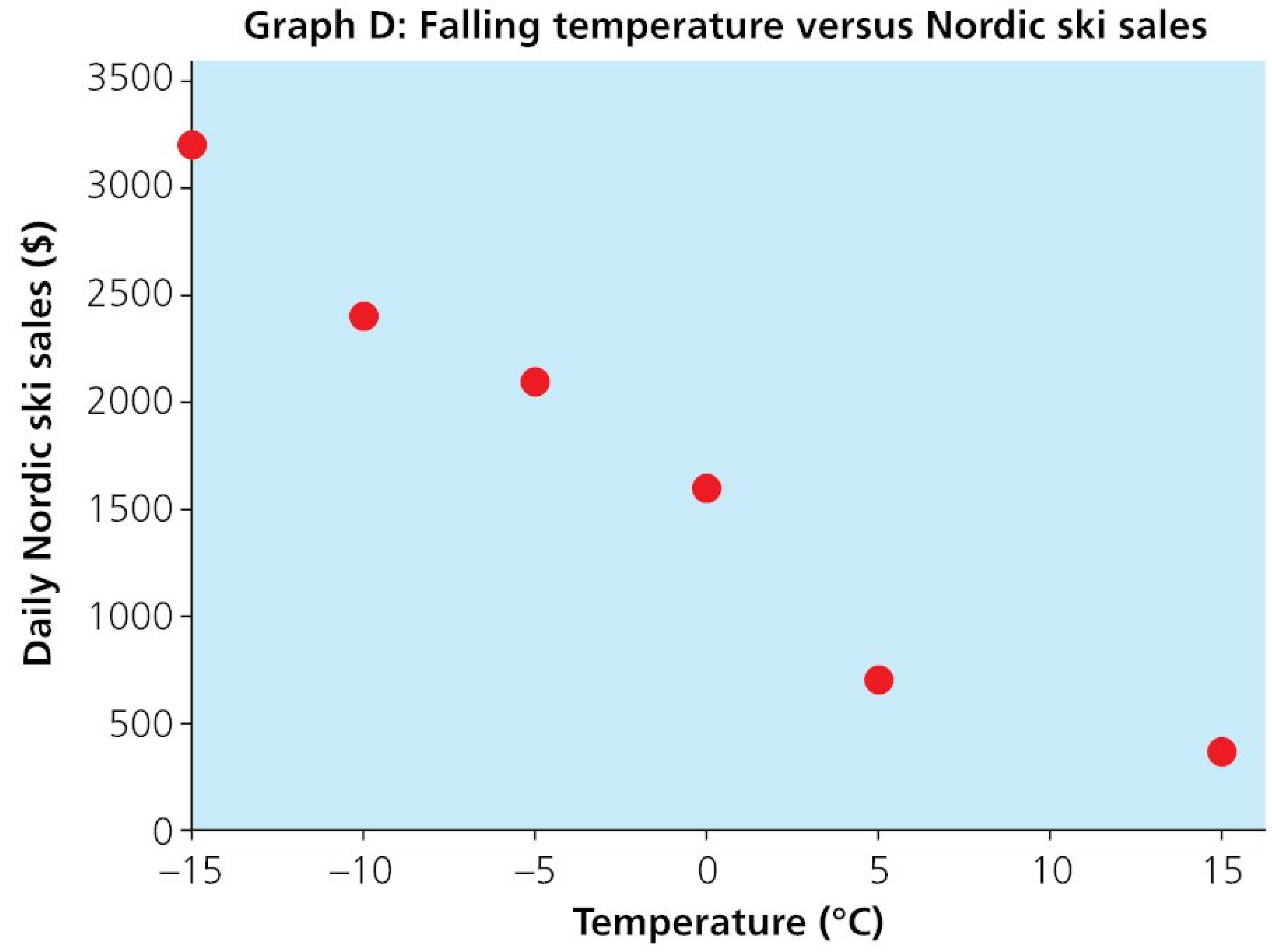


Source: **www.ockhamrazorcompany.com**

Graph B: Income per person in relation to number of children per woman



Graph D: Falling temperature versus Nordic ski sales



Source: **http://princeofslides.blogspot.com**

You may have noticed in your discussions about Graph A that the variables move in tandem – that is, as one variable increases the other increases as well. The reverse is also true: as one variable decreases the other variable also decreases. This is known as a **positive correlation**.

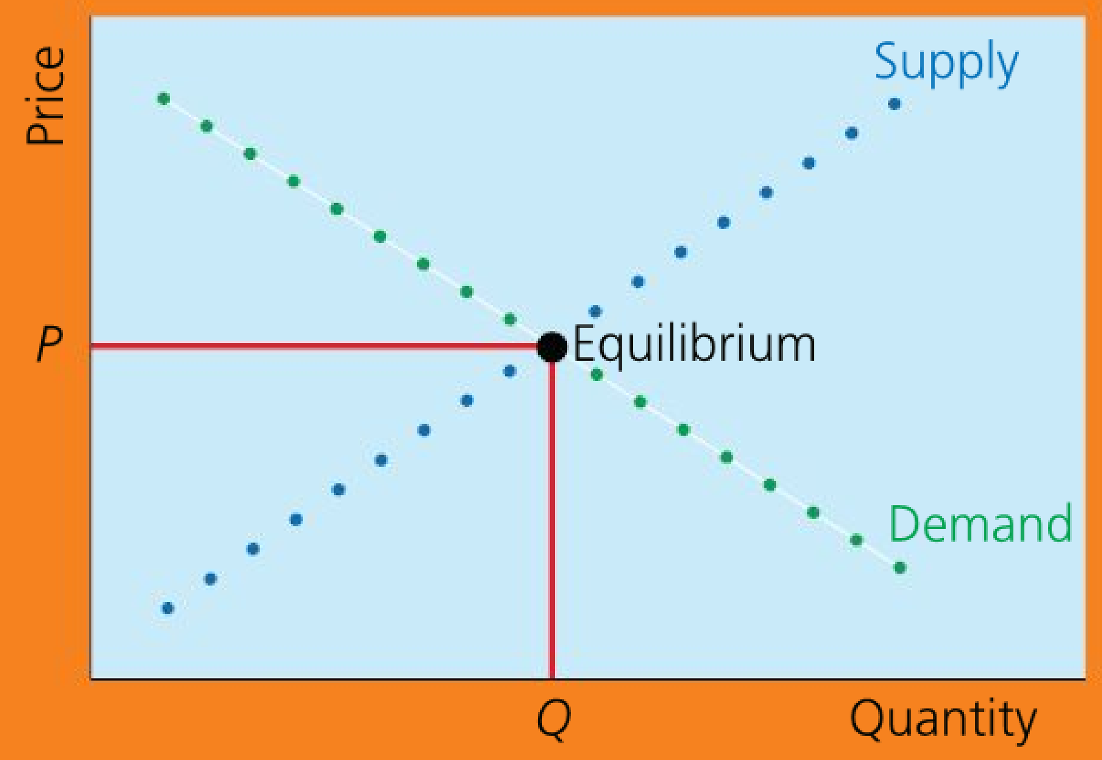
In Graphs B and D, one variable increases as the other decreases. They are moving in opposite directions – they have a **negative correlation**.

In Graph C, the data are completely scattered. One Kickstarter project had close to zero Twitter followers but was able to raise over $20 000 while other projects with similar Twitter numbers barely raised $5. We can conclude there is no relationship between the number of Twitter followers and the success of your start-up business!

Links to: Individuals and societies

One of the first things most students learn about in economics is the law of supply and demand. It suggests

- there is a negative correlation between the quantity demanded of a good or service and its price (that is, the higher the price, the fewer the people who are willing to pay)
- there is a positive correlation between quantity supplied and price (the higher the price, the more of the good/service merchants are willing to sell)
- the ideal price falls where these two correlations meet.



Correlation prediction

Predict whether each pair of variables in the list will have a positive, negative or zero correlation:

- distance driven and gas (petrol) level in a car
- US economy and Canadian economy
- hours of weekly exercise and physical health
- square footage (area) in home and electricity bill
- shoe size and IQ performance.

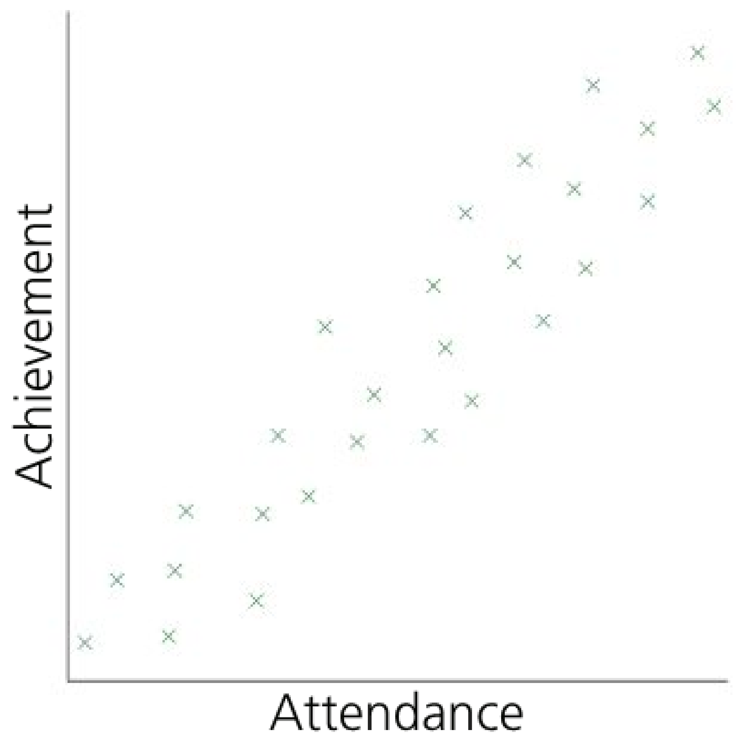
Explain why you made your predictions. Can you think of any more examples of positive and negative correlation?

EXTENSION

If a study concludes 'zero correlation', was it a waste of time? Research some examples to justify your conclusion.

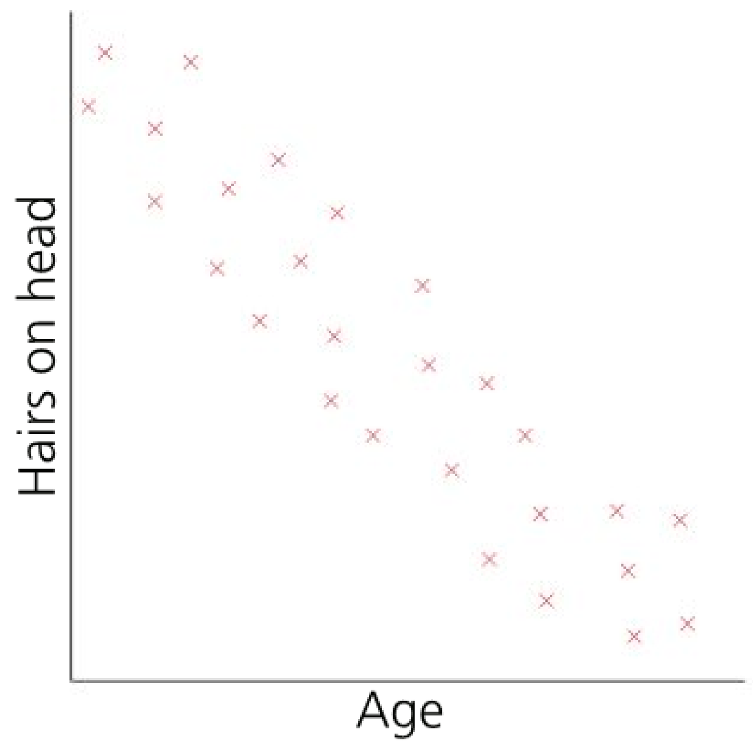
SUMMARY GRAPHS

Positive correlation



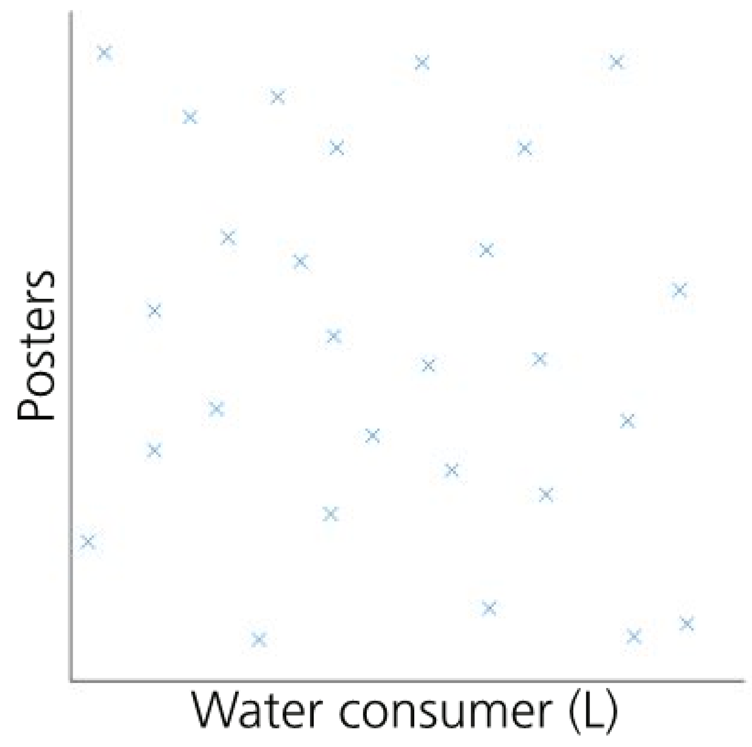
- School attendance versus school achievement.

Negative correlation



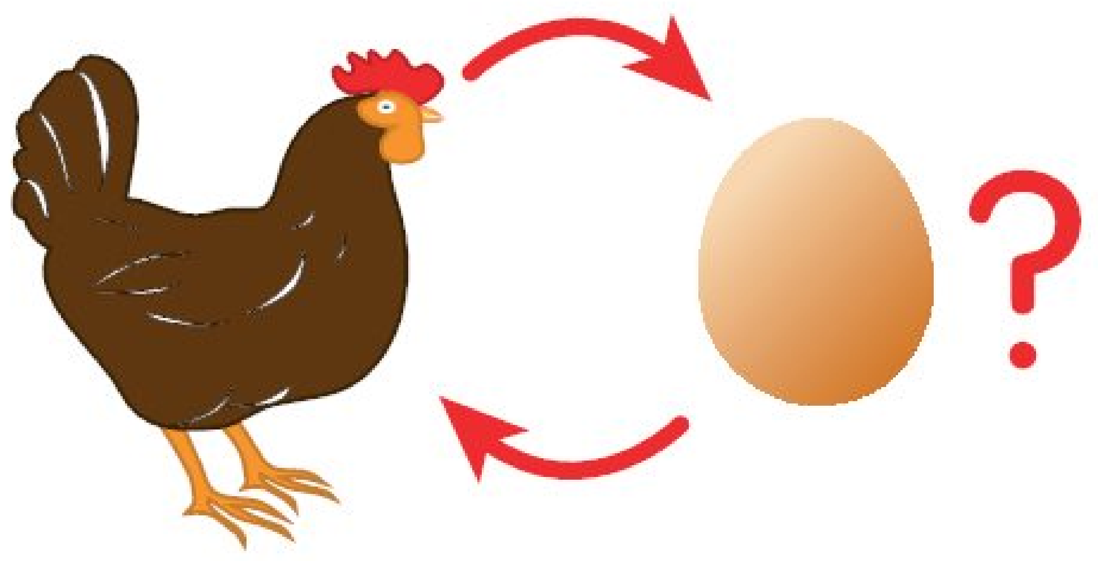
- Age versus number of hairs on head.

Zero correlation



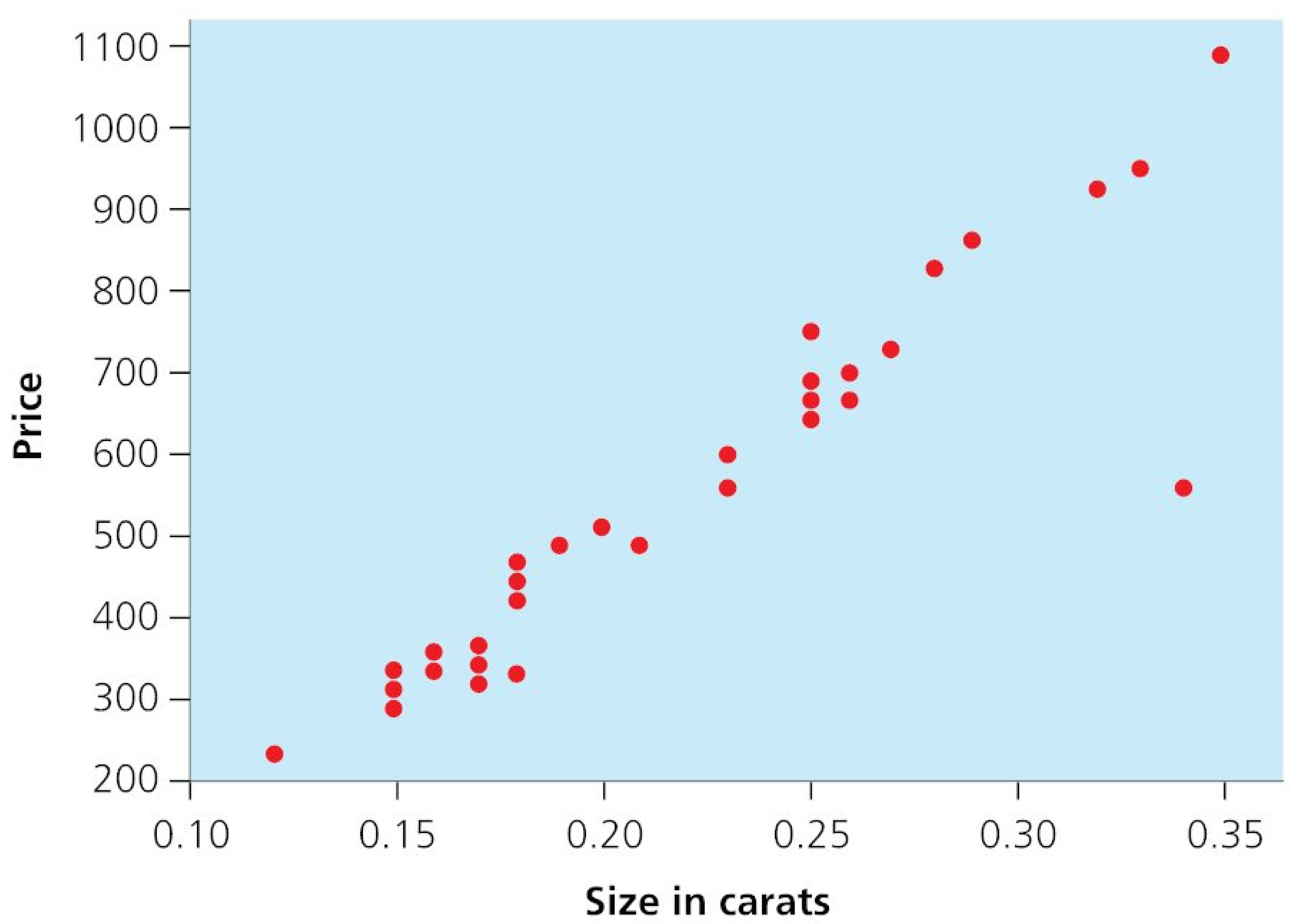
- Litres of water consumed versus number of posters on bedroom walls.

It is interesting to note that in scatter plots it is difficult to determine which, if either, variable is the dependent one. It is something like the chicken-and-egg paradox. Did the low school attendance lead to the low achievement, or did students who were not performing well stop going to school due to low morale?

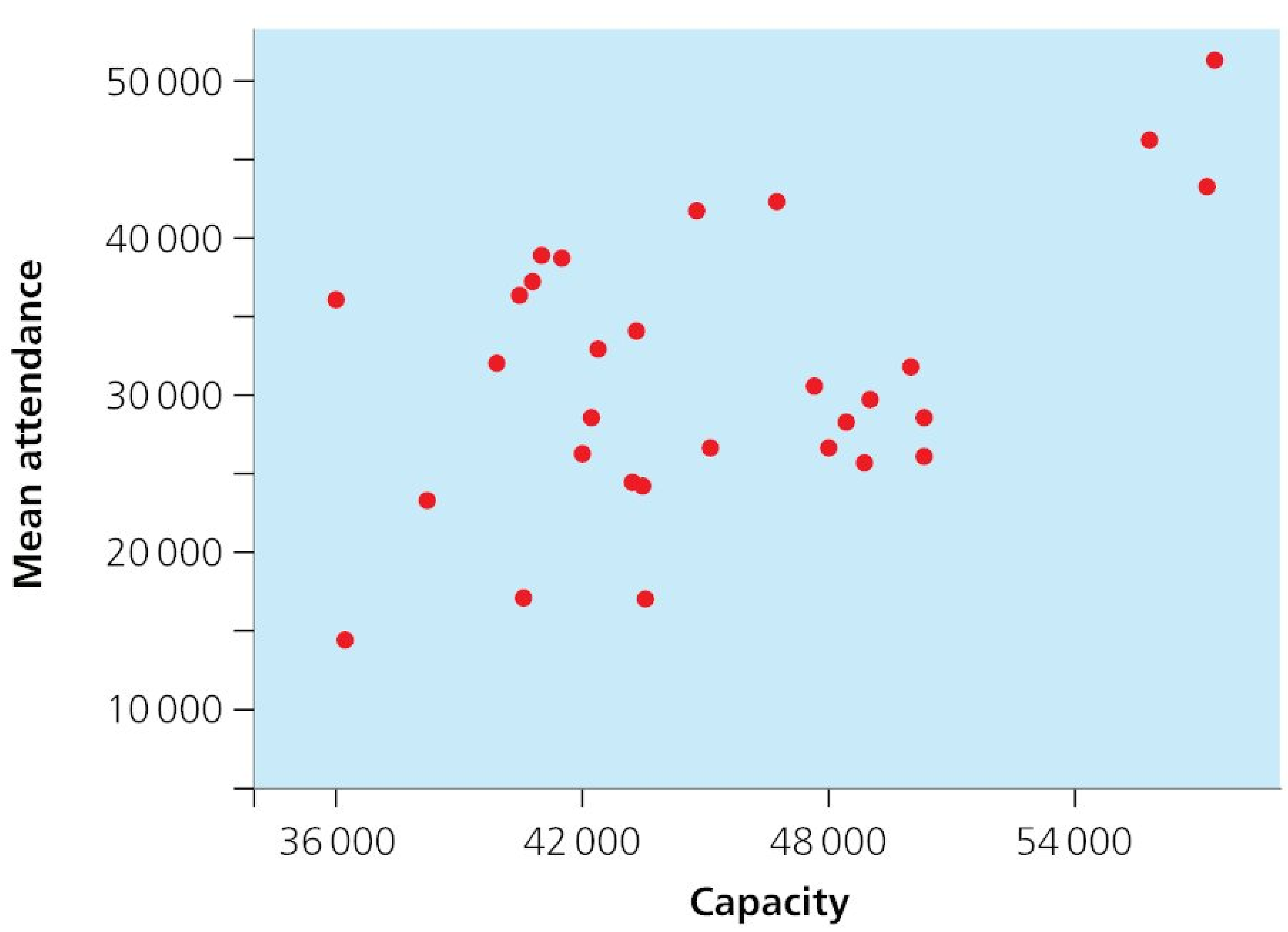


Can one positive correlation be 'more positive' than another?

A graph such as the retail price of a diamond and its size in carats shows a very obvious positive correlation. These variables are said to have a **strong correlation**. On the other hand, we often see correlations that look more like the second graph below. The data do not follow a perfectly straight line and outliers hover above and below. Perhaps other variables affect the trend, in addition to those being studied. But looking closely, there is still a **tendency** for one variable to increase as the other increases. Vague trends like these have **weak correlations**.

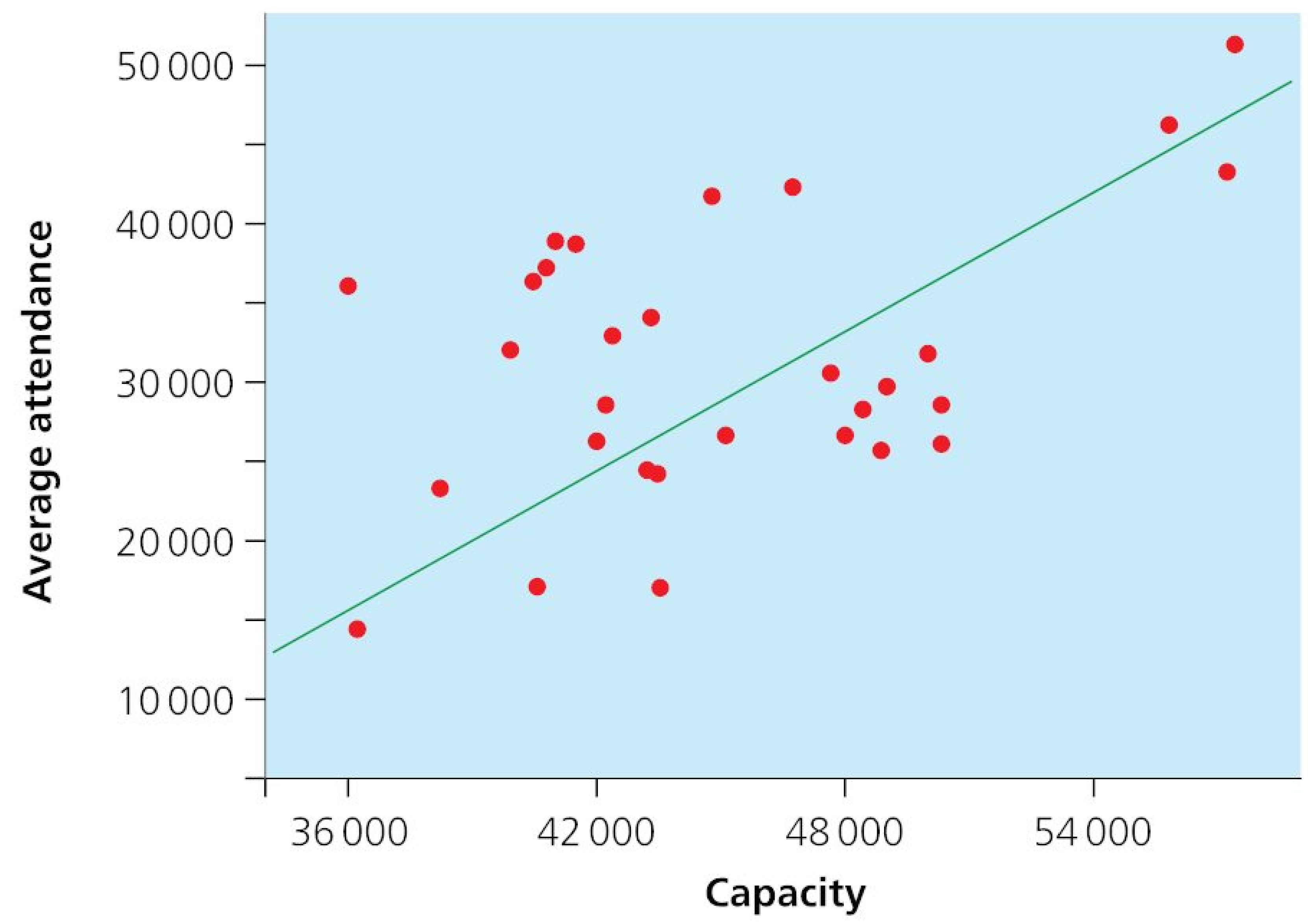


■ Strong positive correlation.



■ Weak positive correlation.

To show a vague trend, statisticians use a **line of best fit** (LOBF).

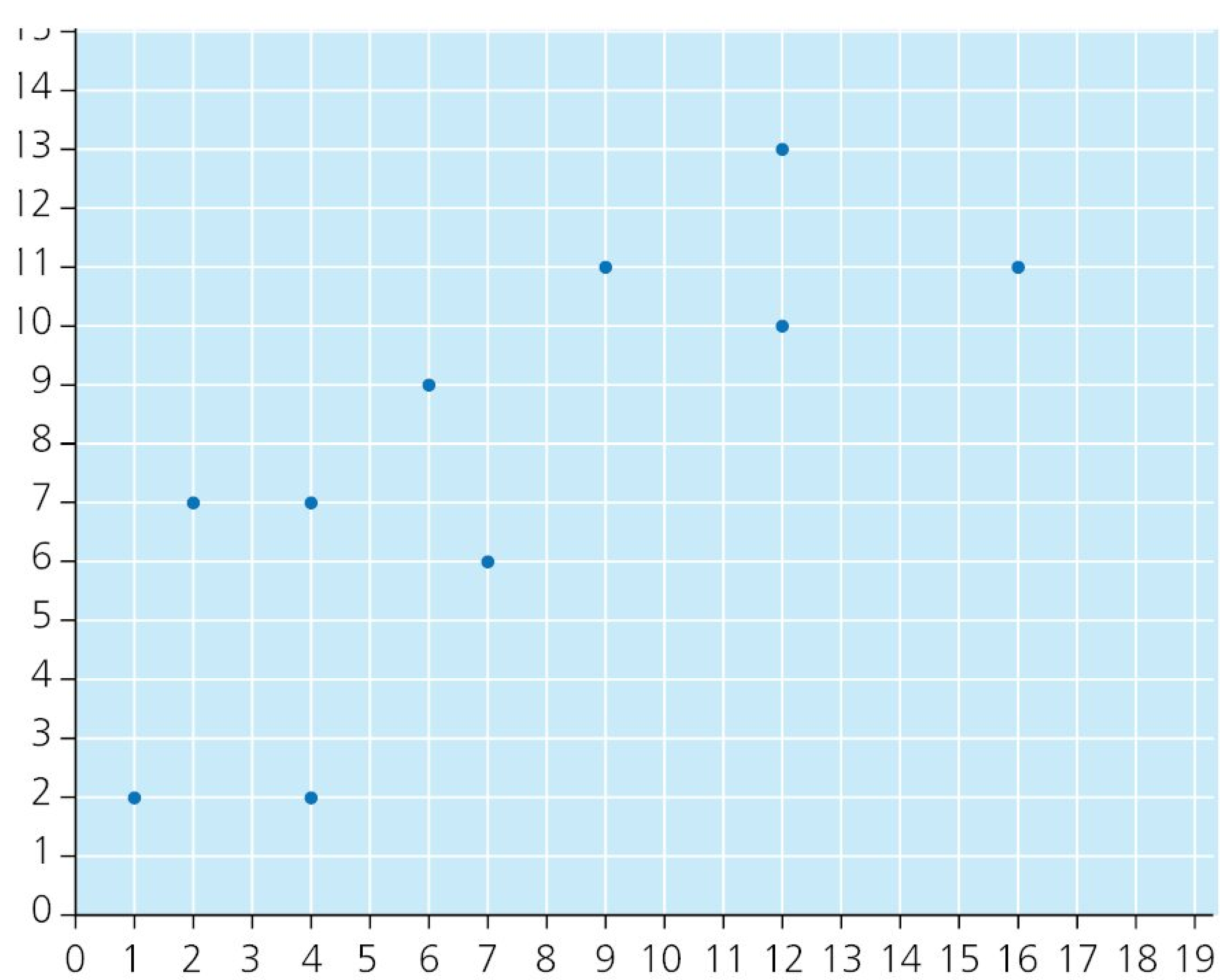


■ A line of best fit makes the trend more obvious.

A LOBF can be used to make predictions in the future based on current data.

LOBF are very helpful when interpreting data, finding upward and downward trends, and making predictions. Very often, we must draw our own LOBF to make sense of the data. There are several approaches to coming up with a good LOBF.

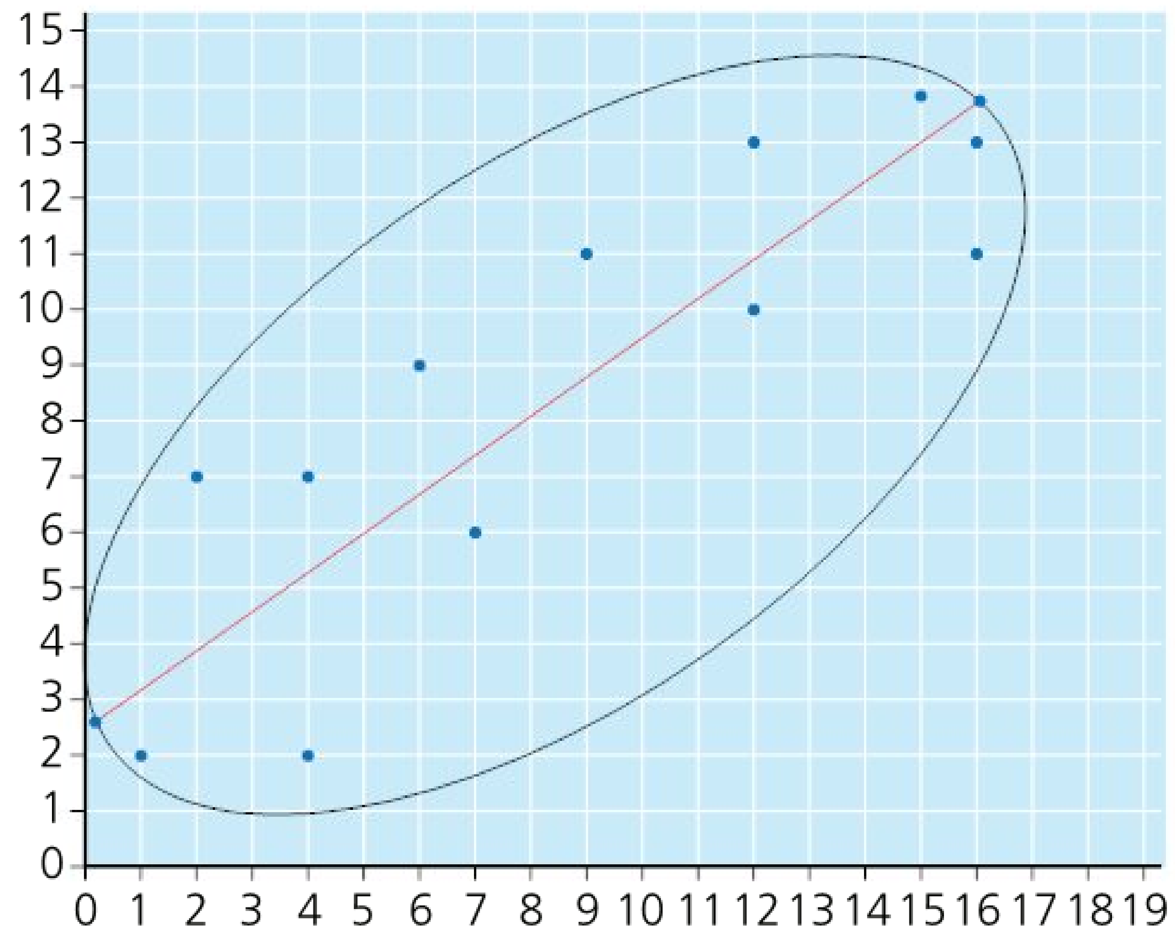
Suppose we want to draw a LOBF for the data below.



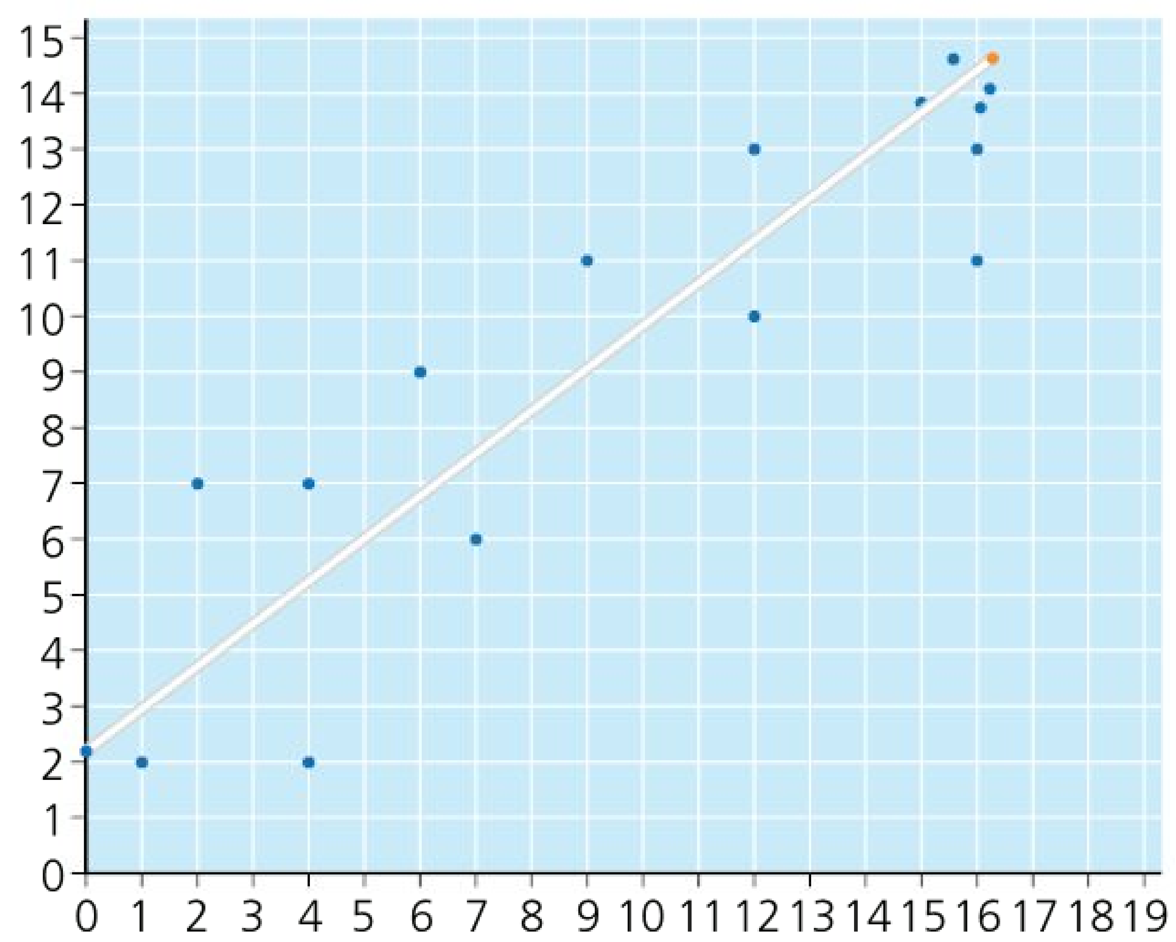
DISCUSS

Is the placement of the LOBF completely subjective (unbiased)? Can the placement ever be wrong? Are some LOBF better than others? Which of the three methods below do you prefer and intend to use? What types of scatter plots make it easier to draw a LOBF and what types make it challenging?

Betty likes to circle the area containing the data and cut this in half.



Veronica prefers to hold her ruler on its side and move it around until she is as close to as many points as possible.



Midge applies the same ruler technique as Veronica, but takes care that she has as many points above her ruler as she does below, with similar distances on either side.

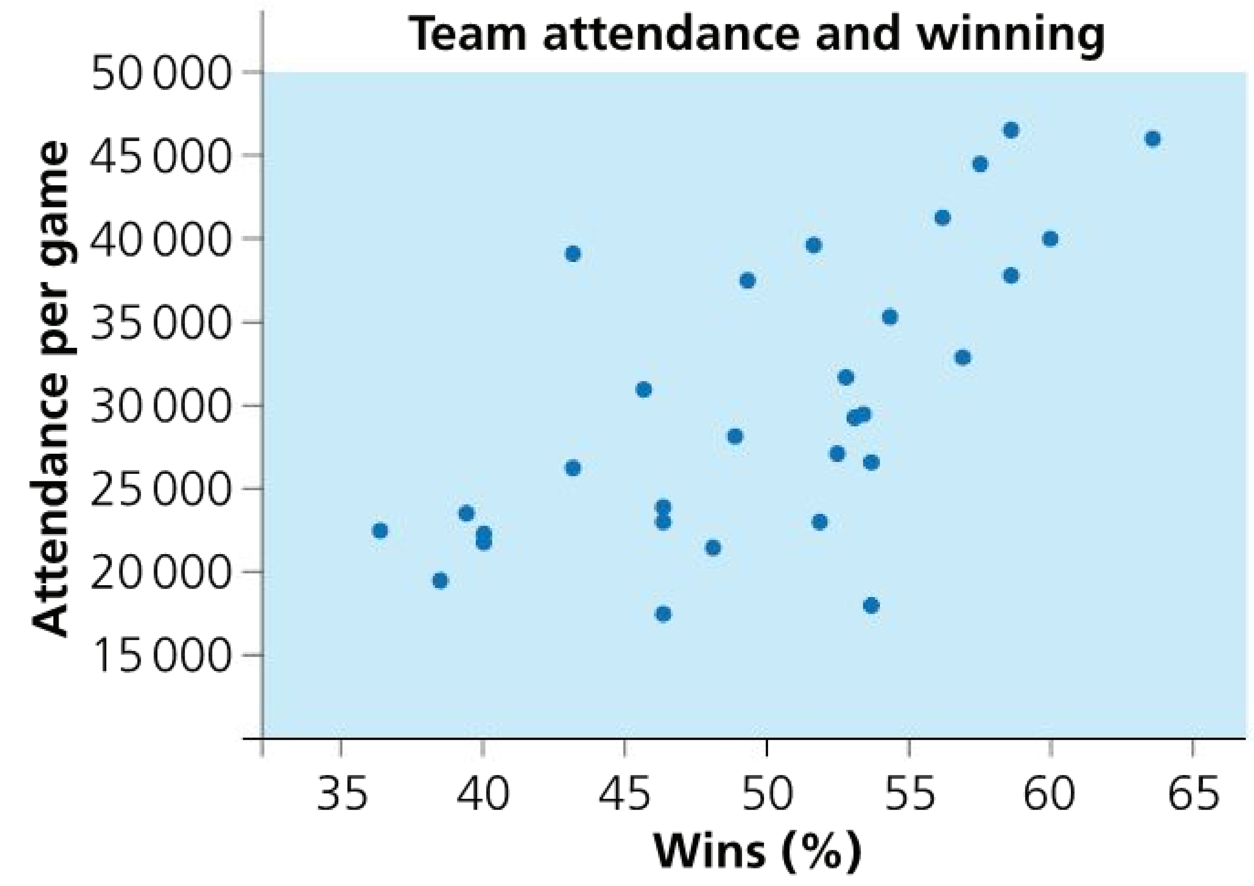
There is a 'best' LOBF that can be determined using complex mathematics, but a visual estimate is still extremely helpful and often very close to a computer-generated LOBF.

PRACTICE EXERCISE

Describe each graph using appropriate terms:

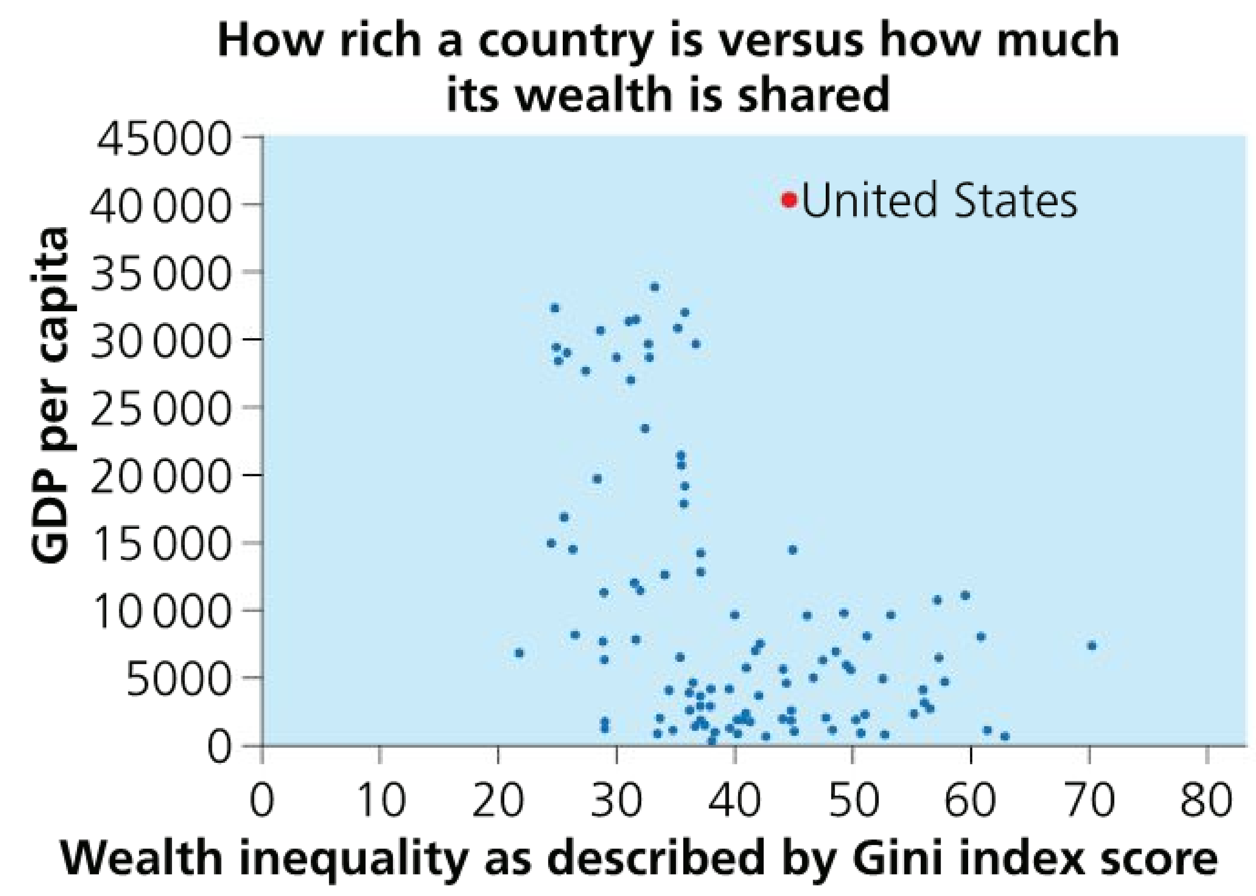
- **positive, negative or no correlation**
- **strong or weak correlation.**

1



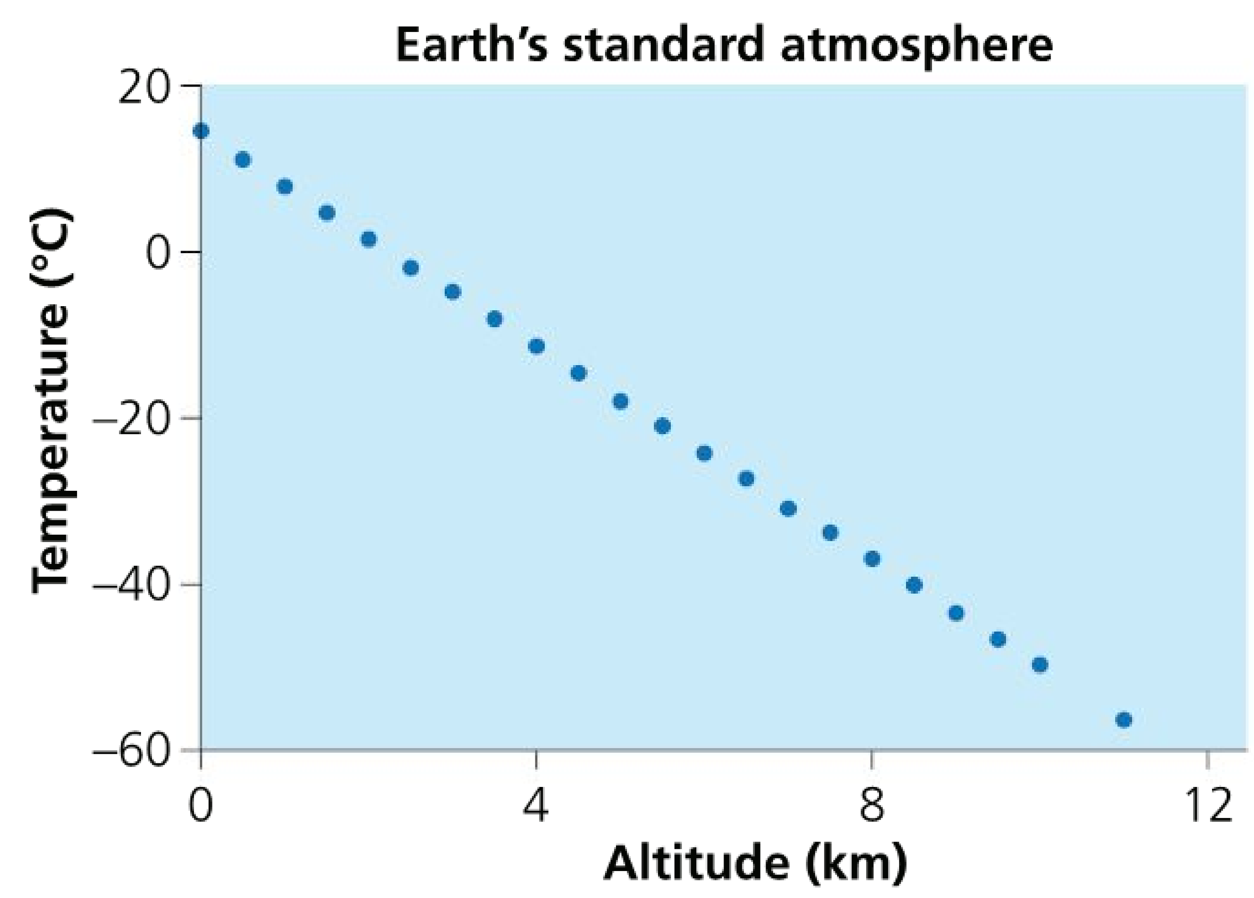
Source: **http://princeofslides.blogspot.com**

2



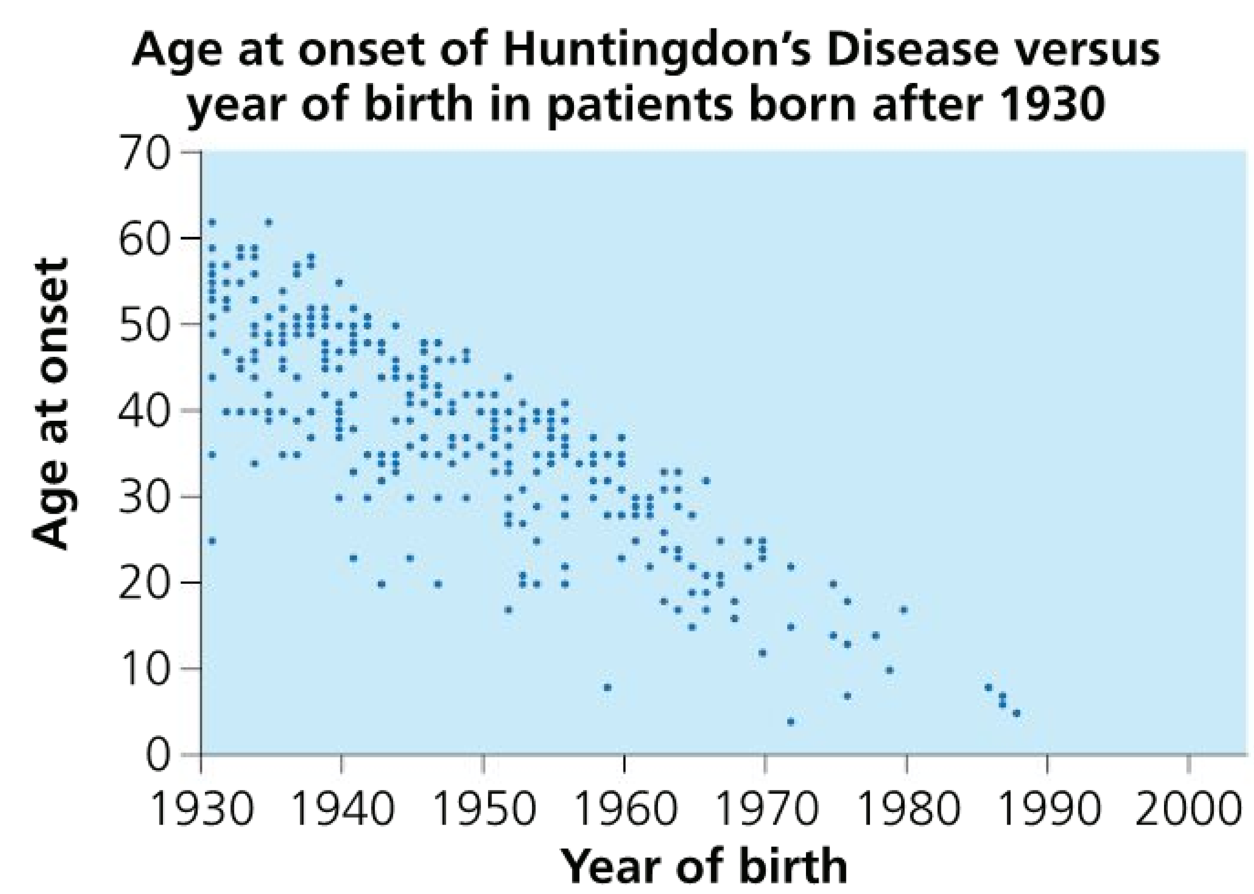
Source: **http://visualizingeconomics.com**

3



Source: **www.seattlecentral.edu**

4



Source: **http://onlinelibrary.wiley.com**

Can statisticians be replaced by computers?

Imagine your friend's father is driving you to soccer practice and a car from the opposite side of the road runs through a stop sign and nearly hits you! You see the driver briefly – a shaken up middle-aged lady. 'Of course, another crazy woman driver', you hear your friend's dad say.

What does this comment imply? Not only that your friend's father believes all bad drivers are women, but that every time he witnesses dangerous driving, he *notices* the driver is a woman. He believes that every time this has happened to him, there was a woman at the wheel of the other car. What is actually happening is that when the dangerous driver is a man, he does not take note!

Here are some statistical facts from the US about car accidents and gender:

- For every year from 1975 to 2015, the number of male crash deaths was more than twice the number of female crash deaths, but the gap has narrowed recently.
- From 1975 to 2015, male crash deaths declined by 24% and female crash deaths declined by 14%.

Source: **www.iihs.org**

- Men cause 6.1 million accidents per year and women cause 4.4 million per year.

Source: National Highway Safety Administration

- 105.7 million women and 104.3 million men have a driver's license.

Source: University of Michigan's Transportation Research Institute

To be fair, men tend to drive far more than women, so when the numbers of accidents are considered per hours driven, the statistics for the genders equal out. But this still does not support your friend's father's incorrect observations.

EXTENSION

Find a scatter plot on the internet which shows a critical health-related positive or negative correlation that affects people in your age group. This could involve nutrition, physical activity or mental health for example. Draw a LOBF directly over the data in the scatter plot and extend it past the data. Write about the relationship between the two variables in the scatter plot and, if possible, publish this on your school website, blog or newspaper.

So what is the discrepancy between your friend's father's observations and reality? The father is only noticing what he expects. He expects the bad driver to be a woman – if it is, he notices, comments and remembers. If it is not, it slips his mind before he has a chance to store it in his memory bank. Is he doing this because he wants to falsify data? Not at all.

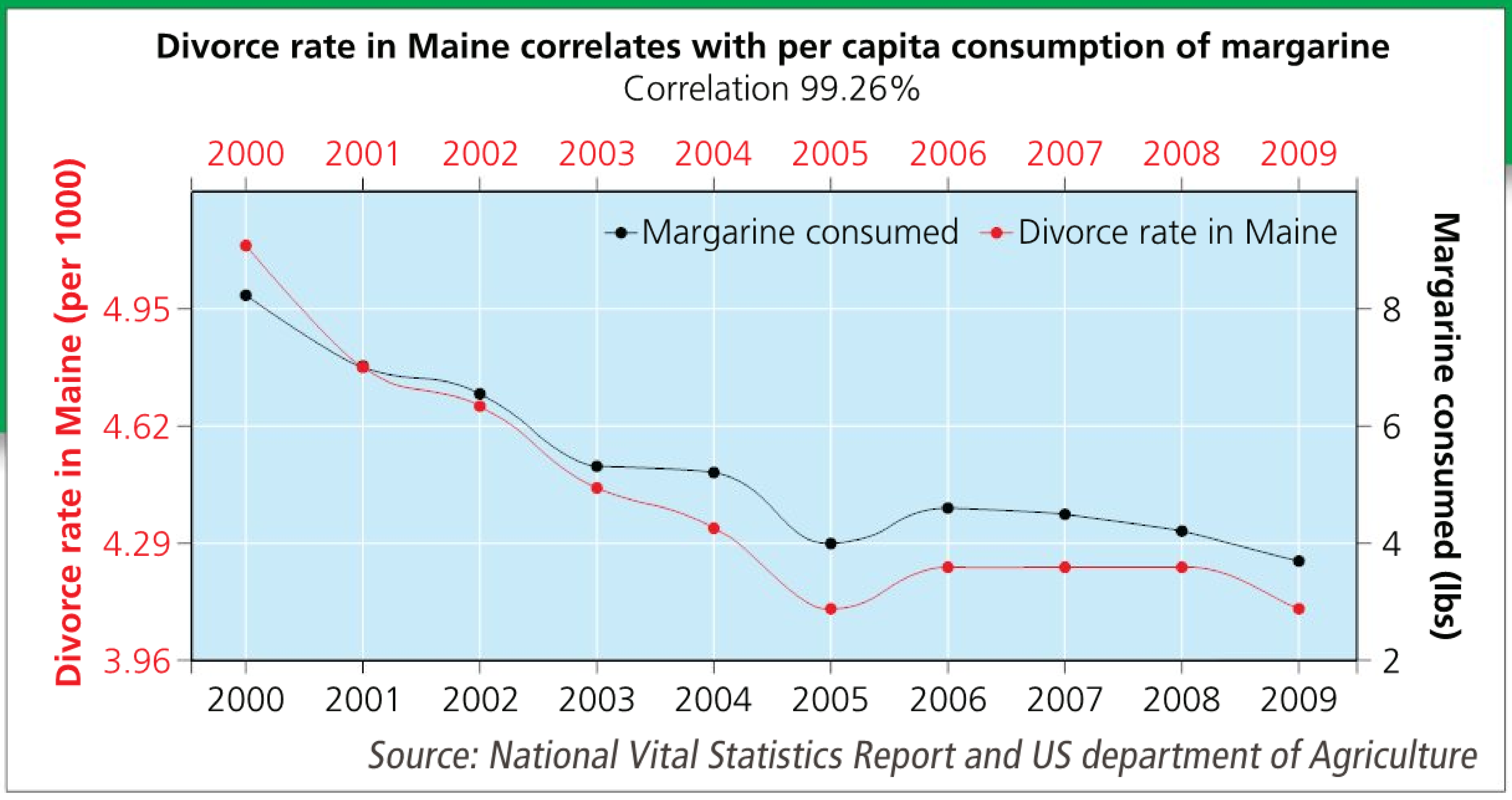
> *'Humans are evolutionarily predisposed to see patterns and gather information that supports pre-existing views, a trait known as confirmation bias.'*

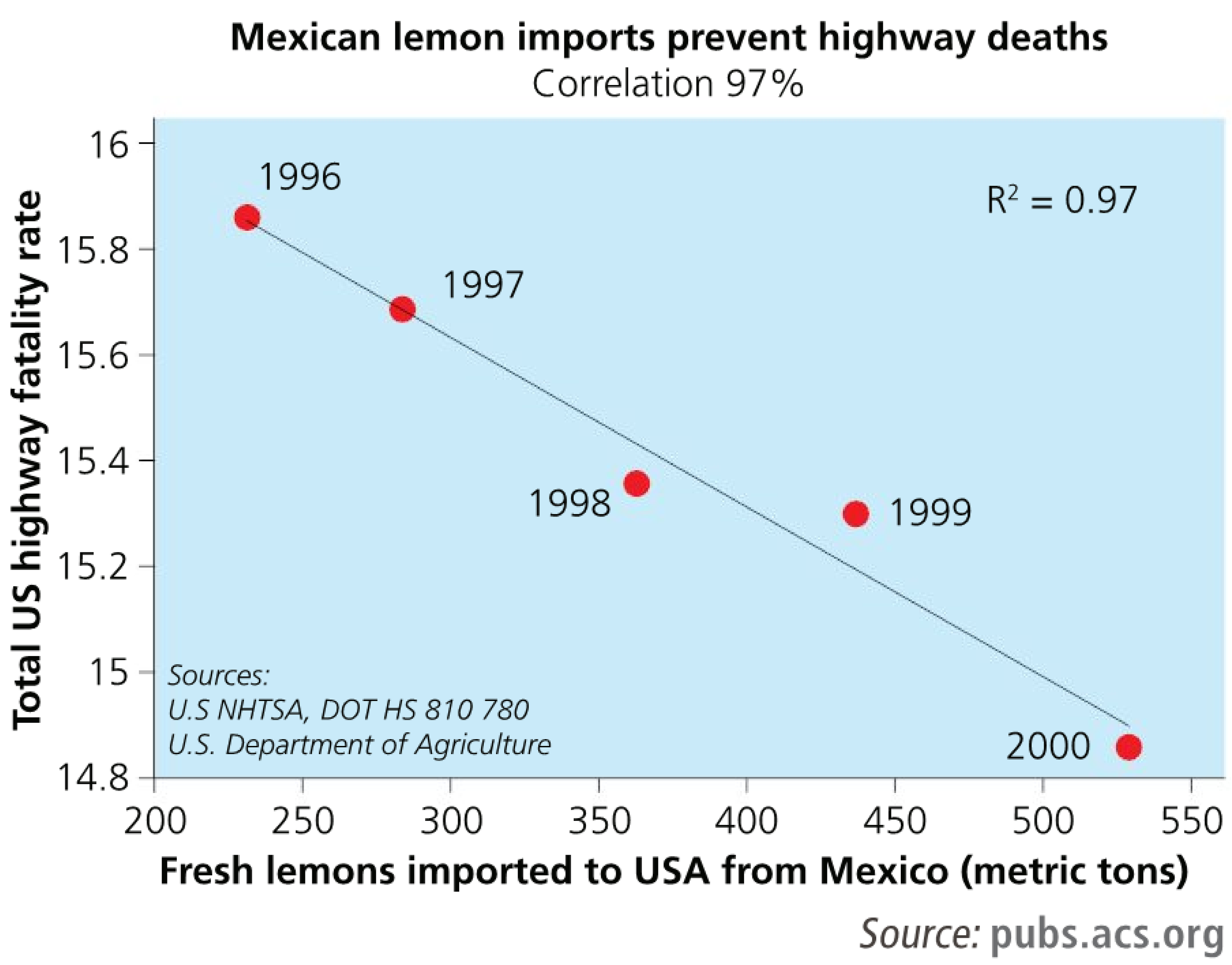
Source: **http://science.howstuffworks.com**

In other words, if we have a strong opinion about something, we look for patterns that will confirm our opinion, and we turn a blind eye to anything that will disprove it.

DRAWING CONCLUSIONS

We have determined that correlations can be used to show whether or not two variables fluctuate together. This usually implies that one variable is dependent on the other and **causes** it to fluctuate in the way it does. On the next page are some examples of strong correlations – but can we say that one variable *causes* changes in the other? What sorts of conclusions can we draw from these graphs?





DISCUSS

What is meant by the phrase 'correlation is not causation'?

If correlation does not always mean causation, why do so many researchers in psychology, medicine, advertising, business, politics and in so many other areas still spend so much time studying it?

Information literacy

What is the **Hawthorne effect** and how might it affect conclusions drawn about correlations?

Look at these links to find out about the Hawthorne effect:

http://science.howstuffworks.com/innovation/science-questions/10-correlations-that-are-not-causations.htm

www.economist.com/node/12510632

Statisticians can never be replaced by computers. While computers can generate and process large amounts of data, graph them and even tell us their exact correlation, it takes a human to *interpret* the data.

THE STEEL-DRIVING MAN

American folklore tells of John Henry who manually hammered a steel drill into rock to make holes for explosives during the construction of a railroad tunnel. Other labourers used a steam-powered hammer. John Henry raced against the new technology and won – only to die from the stress he'd placed on his heart.

This story tells us about a correlation bias – the John Henry effect. When one group sees another group receive additional supplies or tools, they want to prove their worth and start working harder. This makes it very difficult to determine whether the new tools have actually resulted in increased productivity.

SUMMATIVE ASSESSMENT

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

Does your height determine your best sport?

To excel in sport takes commitment to training and lots of practice. However, some people also have physical genetic (inheritable) advantages over others. Let's examine the heights of athletes, to determine whether or not there is a relationship between height and chosen sport. Do ice skaters tend to be tall? Are weightlifters less tall than most other sports people? Do some sports have more height outliers than others?

Although we are studying a relationship, there is only one set of quantitative data, so let's explore each sport individually using all the uni-variate data analysis methods you have learned about. See if you can think of reasons behind any interesting patterns, relationships and outliers you observe, and justify them by referring to measures of central tendency, range and IQR as they apply.

Hockey	Volleyball	Soccer	Horse racing	Basketball
1.78	1.96	1.79	1.46	2.03
1.82	1.91	1.65	1.48	2.10
1.85	1.97	1.83	1.50	1.98
1.61	2.00	1.91	1.43	1.97
1.96	2.00	1.80	1.52	2.45
1.89	1.97	1.95	1.50	2.20
1.87	1.93	1.73	1.49	1.95
1.91	1.89	1.79	1.48	2.00
1.88	1.97	1.68	1.51	2.31
1.85	1.96	1.92	1.49	2.00

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

Reflection

Use this table to reflect on your own learning in this chapter.

Questions we asked	Answers we found	Any further questions now?			
Factual: How do we keep track of how far we've come? What are positive and negative correlations? Can one positive relationship be 'more positive' than another? What steps are needed to draw a box plot?					
Conceptual: What is a mathematical echo? Why does the average person use average? How can outliers affect range? How can we visually represent spread?					
Debatable: Which is the best measure of central tendency? Is it a coincidence or a correlation? Will statisticians become obsolete?					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Critical-thinking skills					
Communication skills					
Information literacy skills					
Learner Profile attribute(s)	Reflect on the importance of being a thinker for your learning in this chapter.				
Thinker					

5 Can mathematics be beautiful?

In many **cultures**, arguments about what is perceived as beautiful can be **justified** by a mathematical **relationship** between **equivalent** images.

CONSIDER THESE QUESTIONS:

Factual: How do I turn a table into a graph? How do two-dimensional figures 'move'?

Conceptual: What is mathematical about mirrors? How many ways can you rotate a figure? What qualifies as 'similar'?

Debatable: Where do I stand?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

IN THIS CHAPTER, WE WILL…

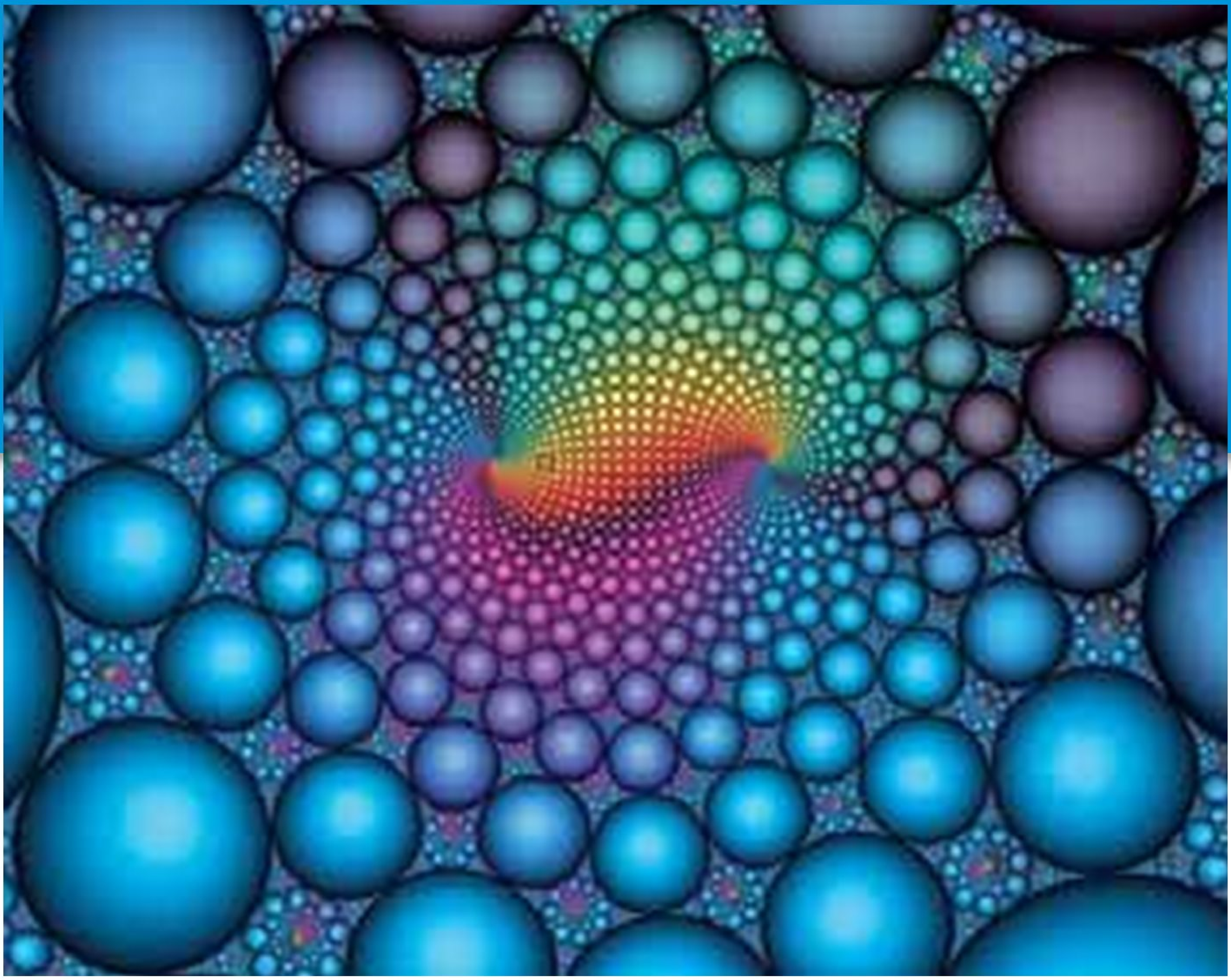
- **Find out** how to plot points on a Cartesian plane given coordinates or a table of values.
- **Explore** the different patterns that result from performing transformations on a single figure.
- **Take action** by creating a hands-on physical model to demonstrate transformations.

These Approaches to Learning (ATL) skills will be useful …

- Communication skills
- Critical-thinking skills
- Transfer skills

Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion B:** Investigating patterns
- **Criterion C:** Communicating
- **Criterion D:** Applying mathematics in real-world contexts



We will reflect on this Learner Profile attribute …

- **Open-minded** – We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view and we are willing to grow from the experience.

SEE-THINK-WONDER

Take a close look at the image above. It is computer generated.

What mathematics do you see in this image?

What do you think about it?

What does it make you wonder?

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how to plot points on a number line
- how to create and interpret a scatter plot
- how to substitute numbers into expressions/ equations and solve them.

KEY WORDS

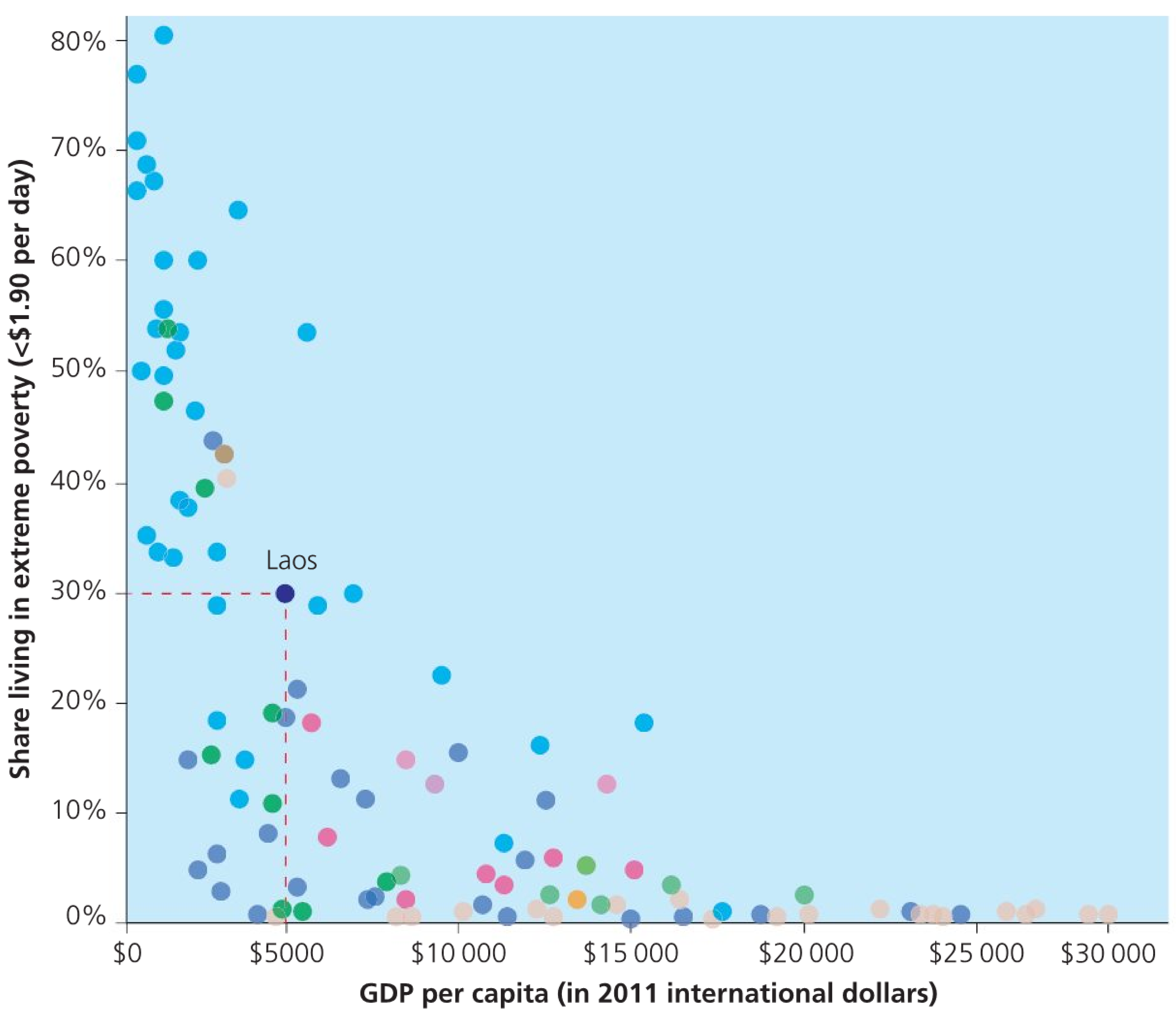
coefficient
GDP
retail chain

From scatter plot to Cartesian plane

Recall from the last chapter that to plot points on a scatter plot, we needed two variables. We looked at the variable on the horizontal axis and the variable on the vertical access; the point where they met was the point that represented both those values at the same time.

The share of people living in extreme poverty vs GDP per capita, 2008 to 2014

Both measures are adjusted for inflation over time and for price differences between countries and are expressed in 'international dollars'. Extreme poverty is defined as living with less than $1.90 per day.

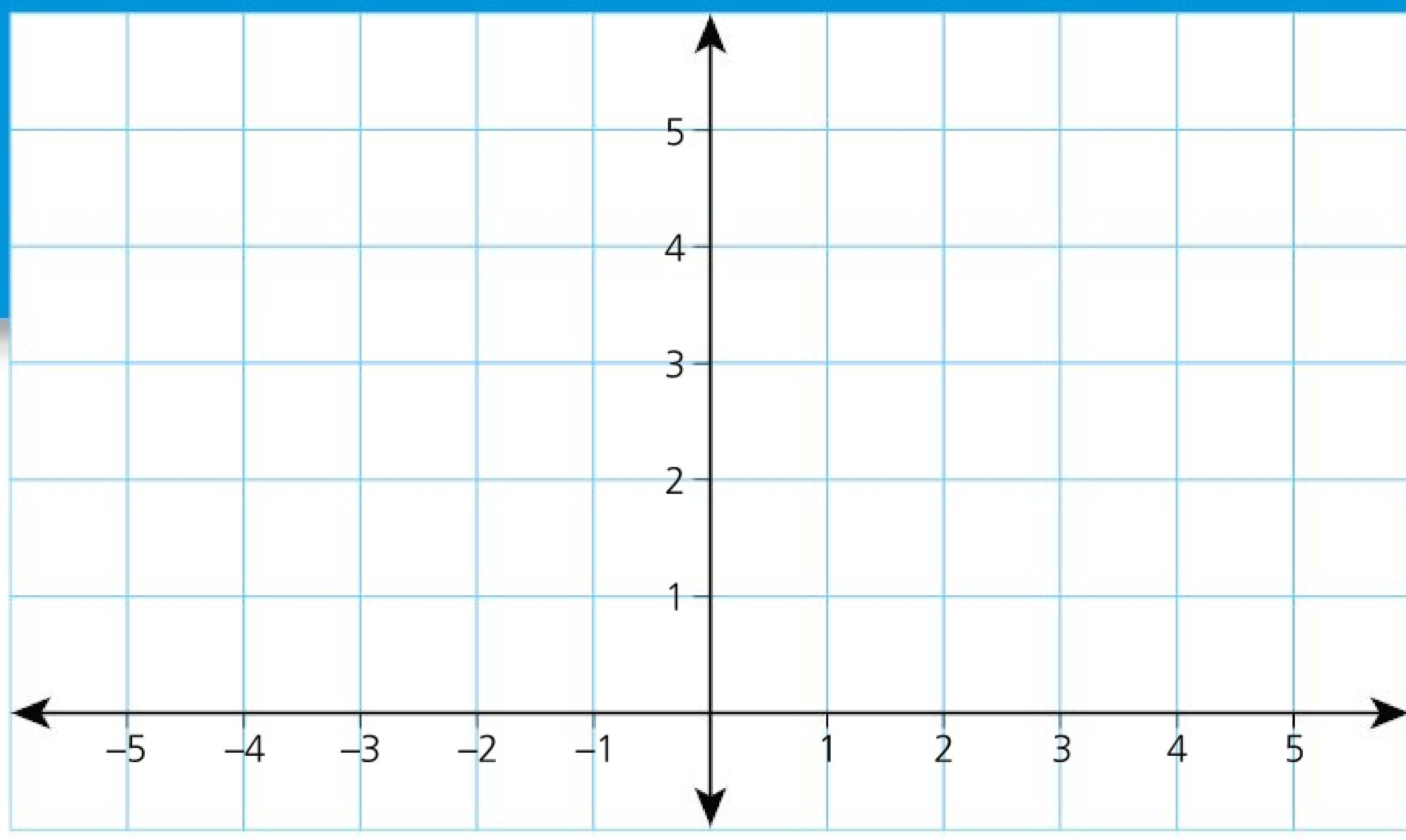


Source: **ourworldindata.org**

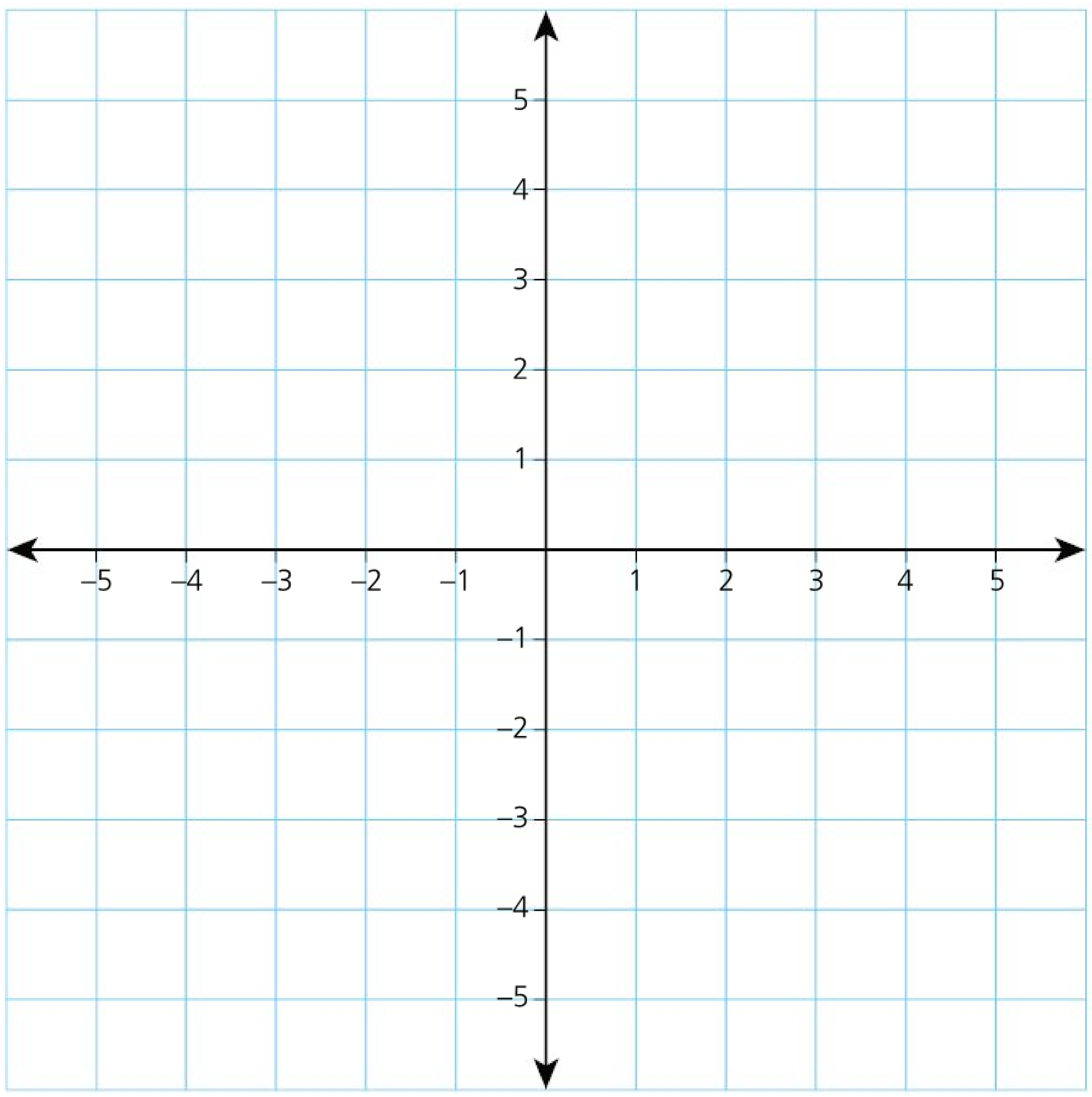
This is a very convenient graph as gross domestic product (GDP) values and percentages of populations can only have positive values. Suppose, however, we are examining a variable on the horizontal axis that has negative values. We would need to extend the x-axis to the left and label it the way we would a number line.

Links to: Sciences

Cartesian planes are also used in Sciences. See Chapter 1 of *Sciences for the IB MYP 2: by Concept* to find out more about how they are used in this subject.



Likewise, we need to extend the vertical axis downwards to include negative numbers for y.

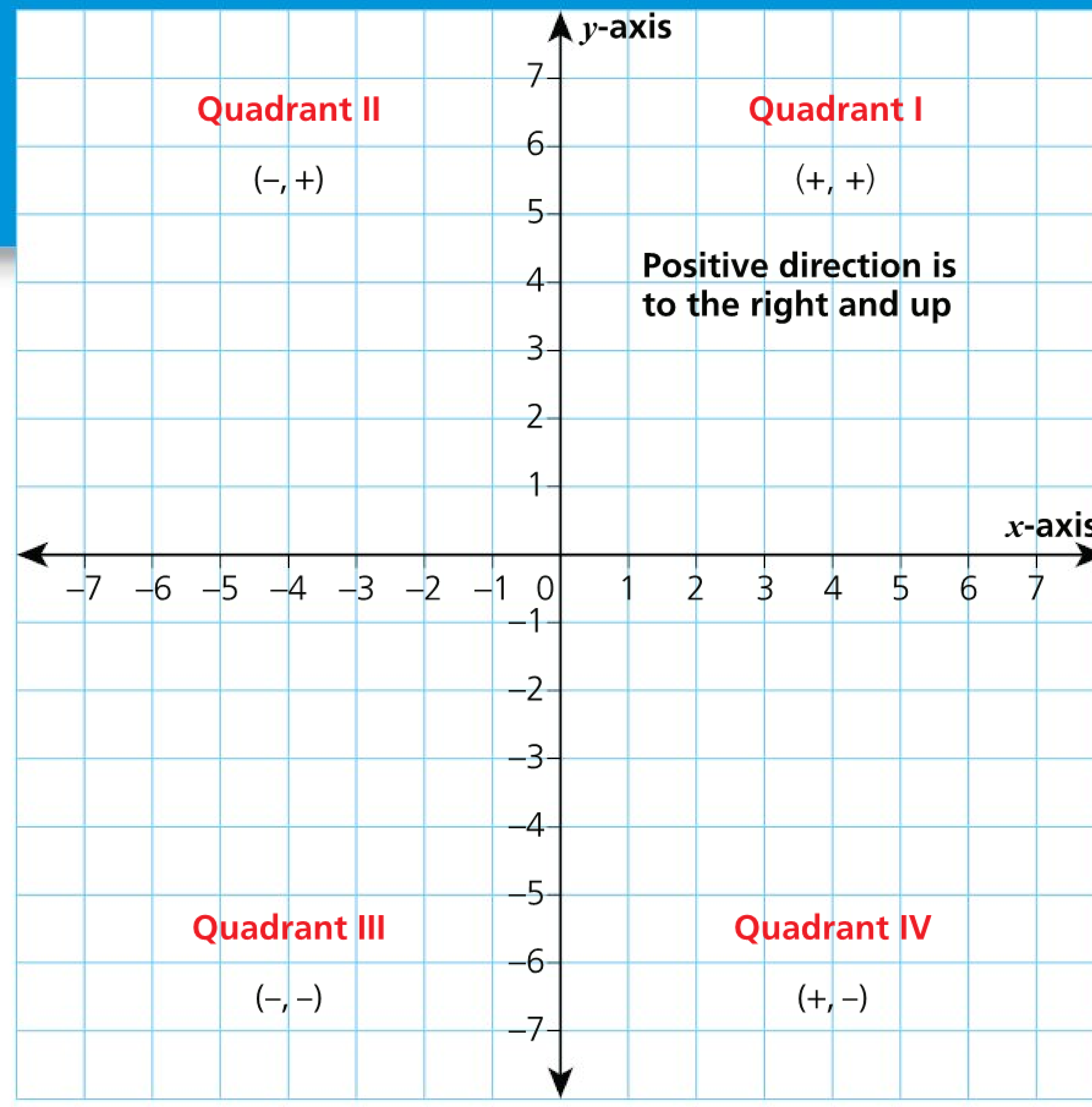


We can now plot relationships between any two variables on what is known as the **Cartesian plane**. This is a two-dimensional graph divided into four sections, known as **quadrants**. The horizontal axis is referred to as the x-axis, while the vertical axis is the y-axis. The point where the x and y-axes meet is known as the origin (0, 0).

Note that **axes** is actually the plural of **axis** – it wasn't a typo!

DISCUSS

Why do you think there are arrows on the ends of each axis in both directions?



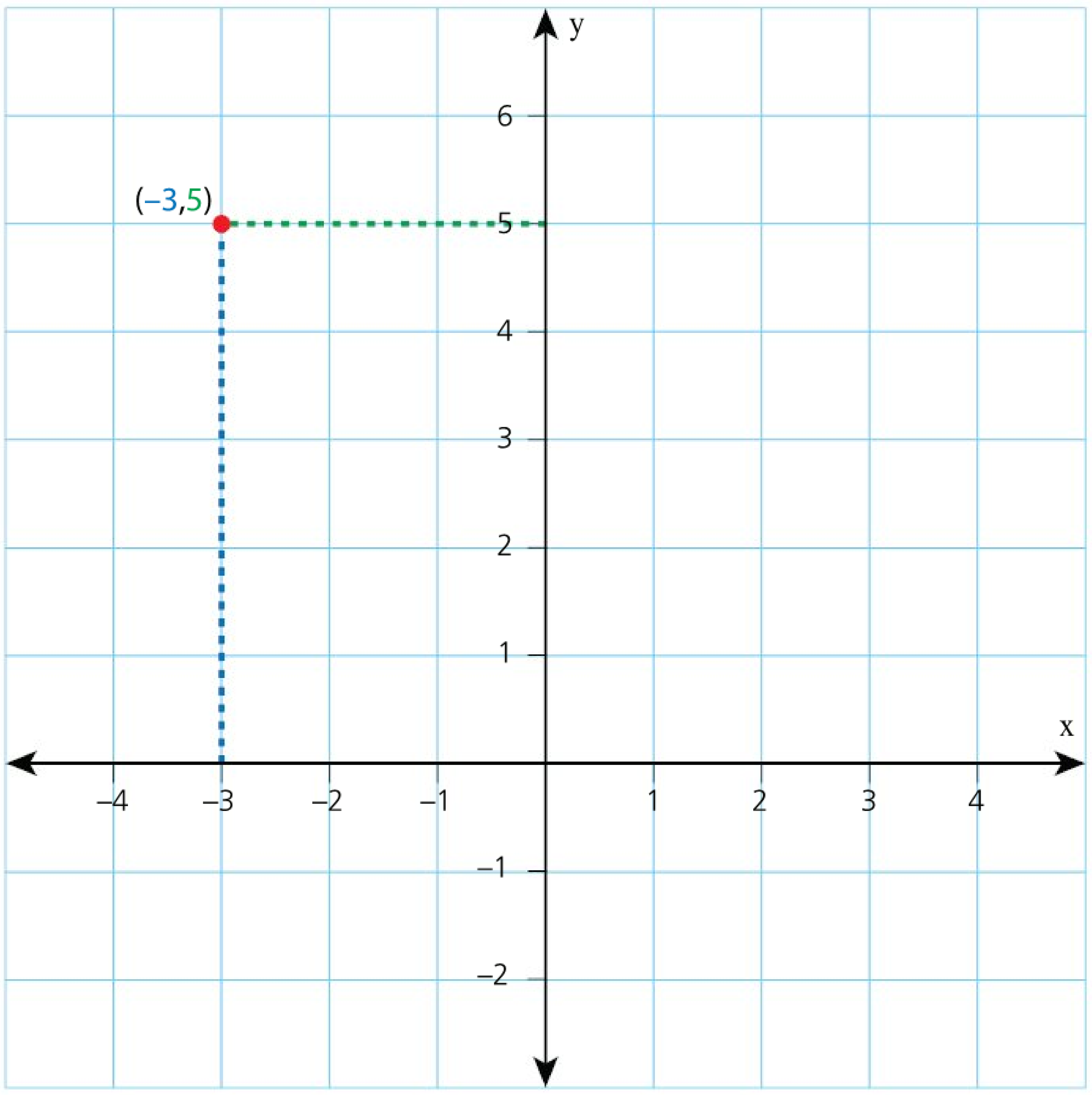
Why 'Cartesian'?

There is a legend that tells of a morning when French mathematician René Descartes was observing a fly on the ceiling and asked himself how he could recount to someone else the fly's exact position. His conclusion was that he could describe the position by relating it to its distance from the walls. He transferred this knowledge to points, and eventually developed the Cartesian coordinate system, the word Cartesian coming from the name 'Descartes'. The story is not confirmed, but the name of the inventor of the Cartesian plane is a matter of record. For a longer account of this story, read *The Fly on the Ceiling* by Dr Julie Glass.

Where do I stand?

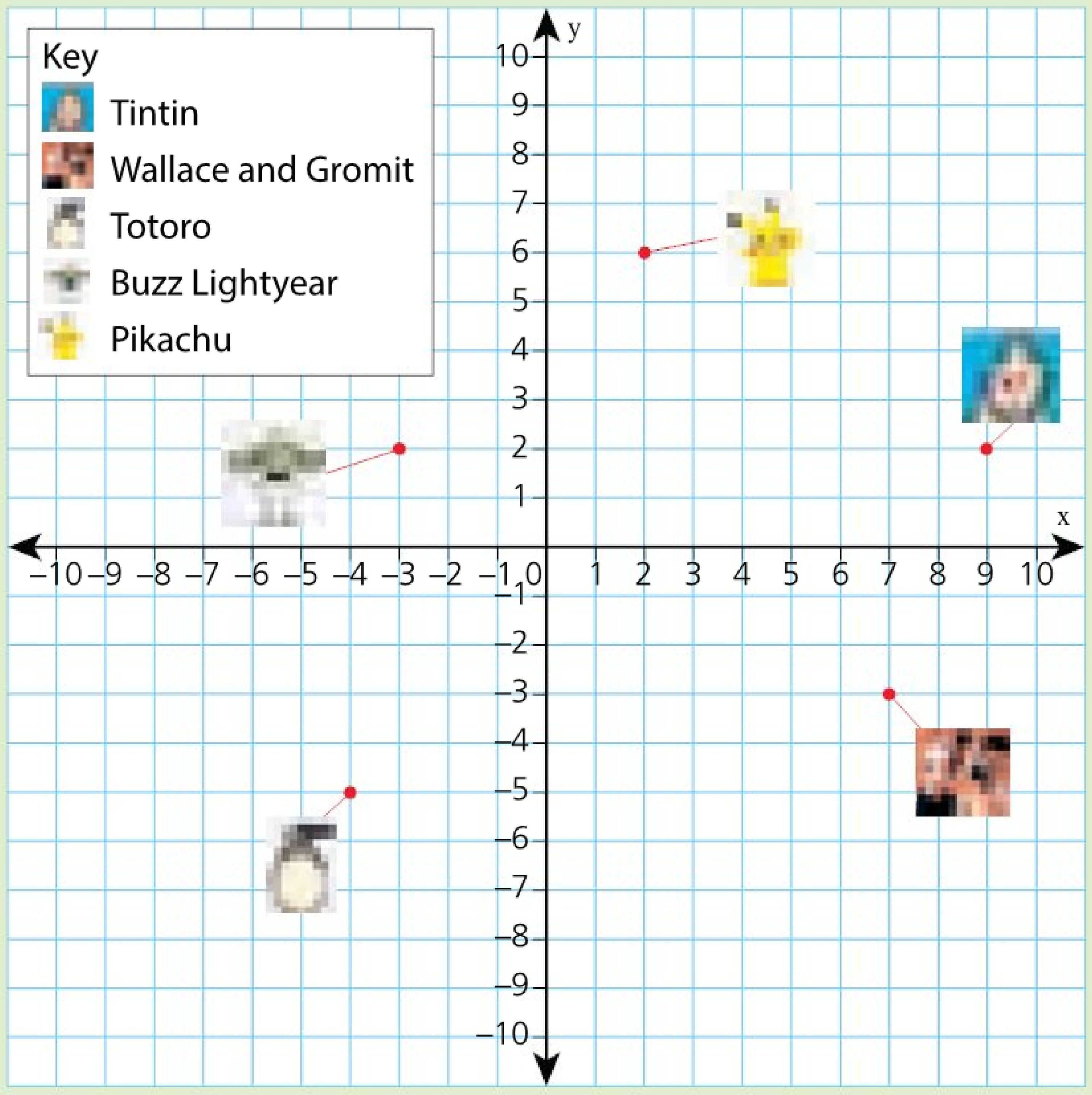
Just as a point on a scatter plot is located using two values, each point on a Cartesian plane also has two values associated with it, the x-variable and the y-variable. We sometimes refer to points as 'ordered pairs', because to find them we need a pair of numbers given in the correct order: first the x-value, then the y-value.

Suppose we want to plot the point (–3, 5) on the Cartesian plane. The first number is always referred to as the x-coordinate (as it is where you plot the x-value), and the second is the y-coordinate. They are always given in alphabetical order. So, we find the –3 on the x-axis (blue line) and the 5 on the y-axis (green line), and draw the point where the two coordinates meet.



PRACTICE EXERCISE

1 Write down the ordered pairs of the spot where each character is standing.



ACTIVITY: Battleships

ATL

- Communication skills: Give and receive meaningful feedback; Understand and use mathematical notation

Now with numerous versions of the app available, *Battleships* was a pencil-and-paper game during the First World War and in 1967 became a popular Milton Bradley board game. It involves placing a fleet of ships on a grid system that is hidden from the opponent. Players then take turns 'firing canons' at one another by giving coordinates. (You see where this is going, don't you?)

The original game had letters across the x-axis and numbers across the y-axis, so that even young children could play with ease. However, we are mathematically skilled enough to do it on a proper Cartesian plane!

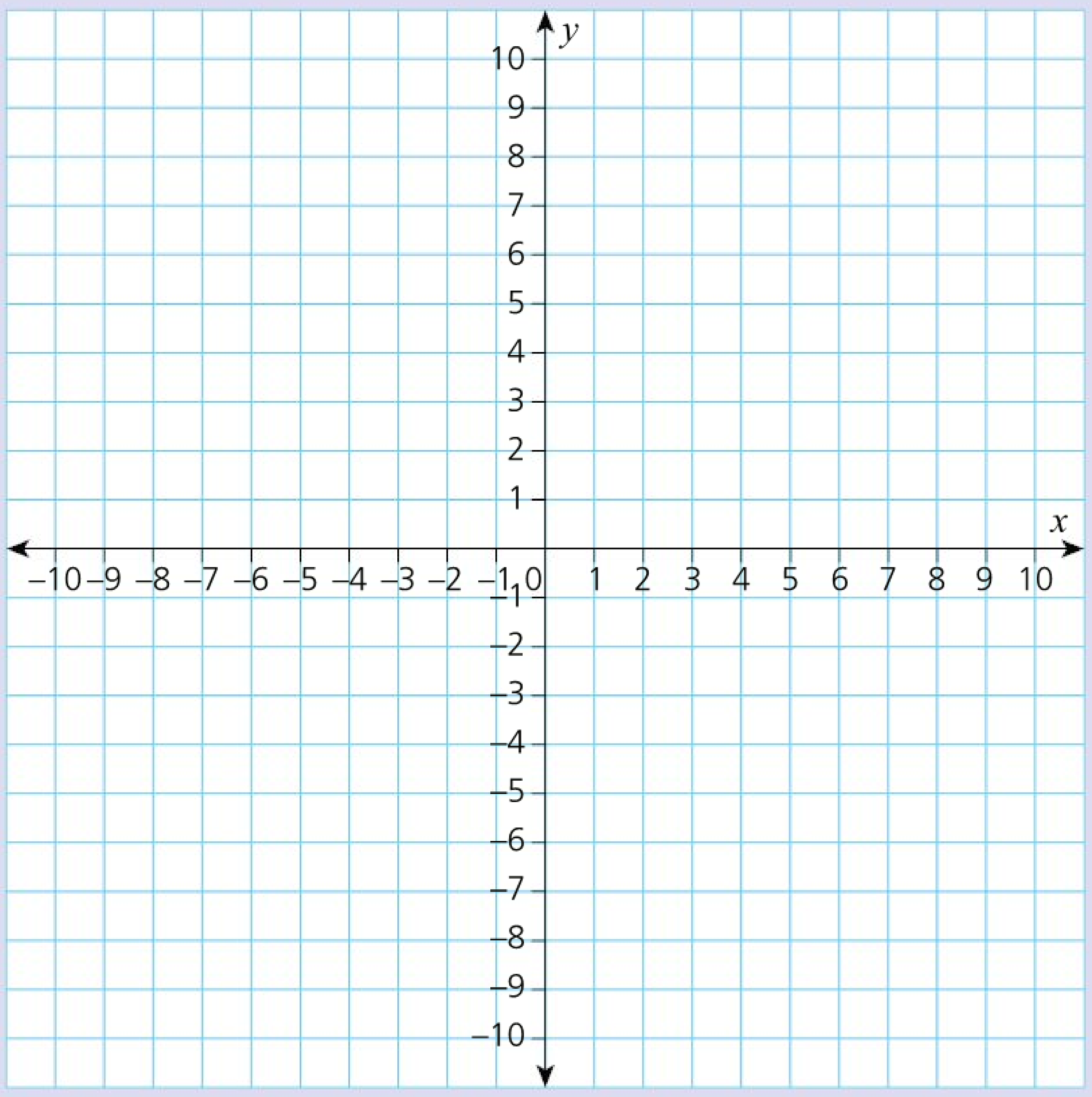
2 Copy the Cartesian plane provided and plot each point on it. Label the point with its corresponding letter:

A (2, 5)

B (–9, –2)

C (0, 7)

D (9, 0)

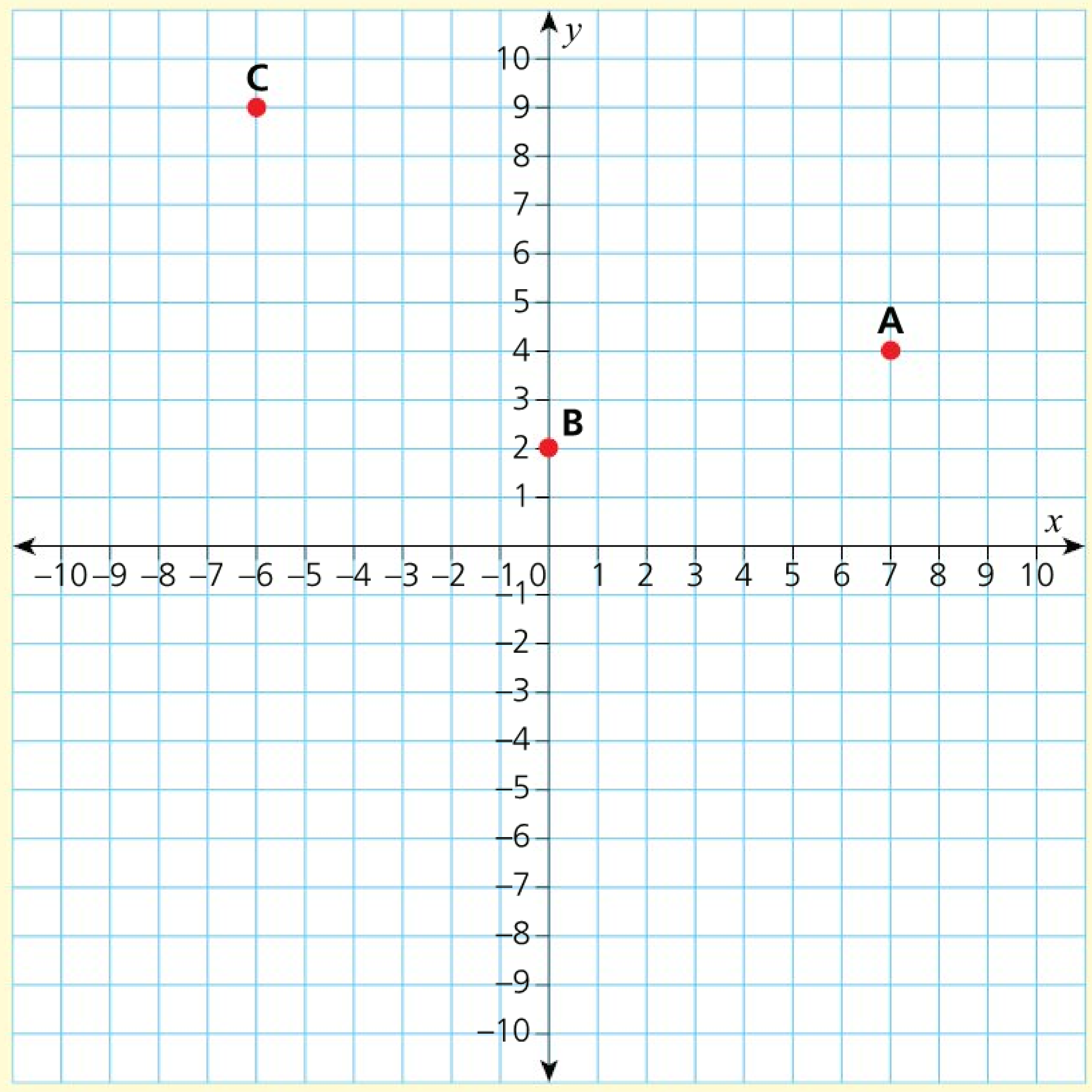


3 Katarina was asked to plot each point on the graph. What did she do wrong?

A (4, 7)

B (2, 0)

C (9, –6)



1 **Play in pairs. Each of you should draw and label a Cartesian plane like the one on page 109. Call this plane 'My ships'.**

2 **Now both station your fleets. Draw side-by-side points in a horizontal or vertical line for each of the five ships in the table. The table tells you how many points to draw for each ship:**

	Class of ship	Size (points)
1	Carrier	5
2	Battleship	4
3	Cruiser	3
4	Submarine	3
5	Destroyer	2

3 **Now both draw a second plane so that you can keep track of where you fired at your opponent, and whether it was a hit or a miss. Call this plane 'Their ships'.**

4 **PLAY! Have your opponent call out an ordered pair. Find the ordered pair on your 'My ships' plane … if they hit you, draw a red cross over the point; if they missed, draw a blue cross over the point. Let your opponent know if they hit or missed you (and if they hit every point in a ship, let them know they've sunk it!).**

5 **Take your turn to fire at your opponent, keeping track of where your hits and misses are on your 'Their ships' plane.**

6 **Continue to play until one player has sunk all of the other's ships. Compare your planes when finished to see if your markings were the same. Note any differences and suggest reasons for these.**

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion A: Knowing and understanding.

How do I turn a table into a graph?

Last year you learned about **substitution** – plugging numbers into expressions in place of variables to find the result. This is going to come in handy in just a minute, so let's quickly review the process.

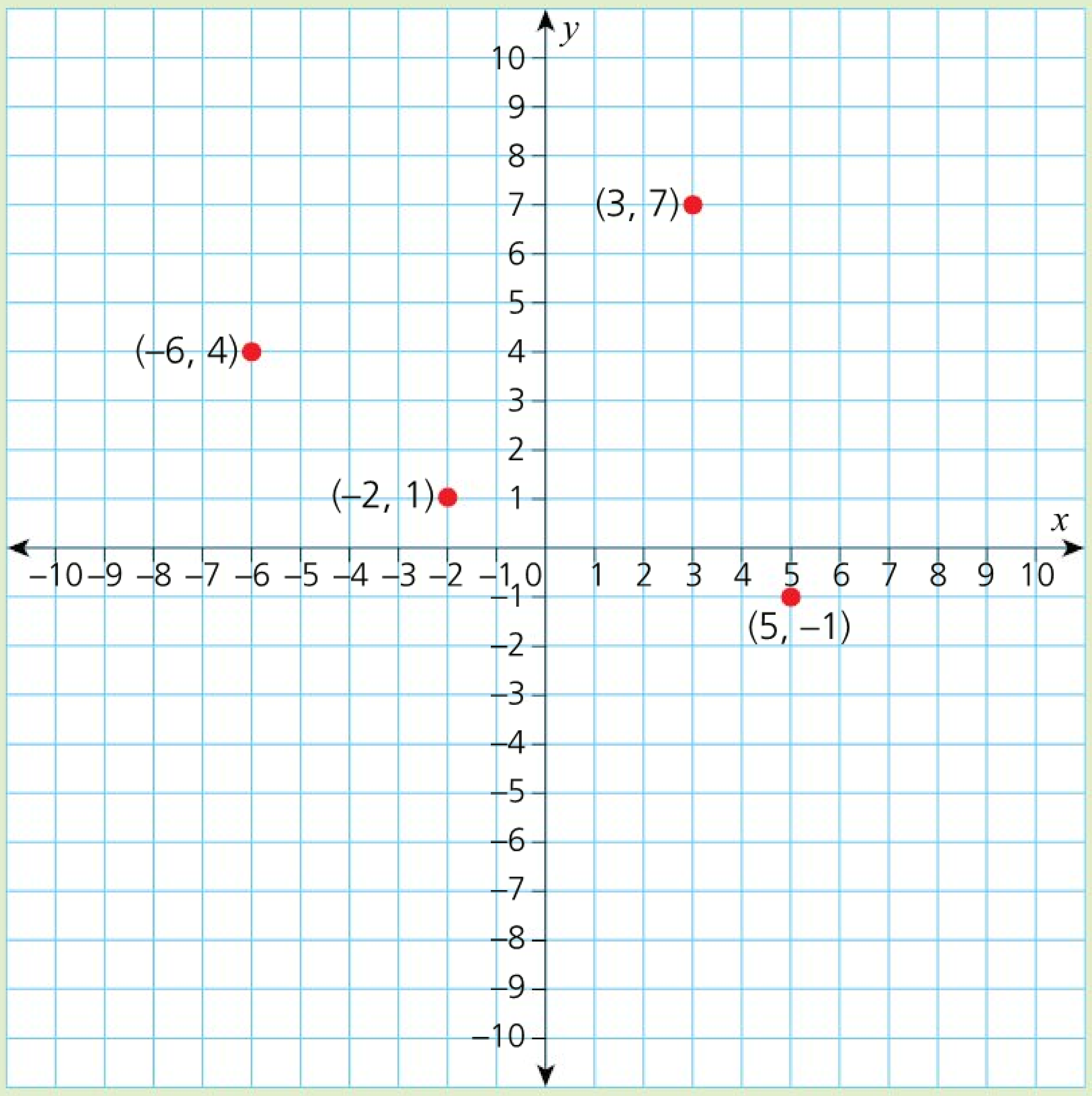
Example

Plot and label the points (–2, 1), (3, 7), (5, –1) and (–6, 4).

Solution

A list of points is disorganized; plotting them is so much easier if we put the coordinates in a table. This is known as a **table of values**, and is simply a different way to express ordered pairs. It is especially useful when there are lots of them, or when trying to find patterns among them.

x-coordinate	y-coordinate
–2	1
3	7
5	–1
–6	4



Example

Substitute $x = 3$ into $y = 2x + 4$.

Solution

Wherever we see an 'x' in the equation, we will replace it with '(3)'. The brackets are very important!

$y = 2x + 4$

$y = 2(3) + 4$

$y = 6 + 4$

$y = 10$

Try some more just to refresh your memory:

1 Solve $y = x^2$ if $x = 2$.

2 Solve $y = 4$ if $= -3$.

3 Solve $y = 6\ ^2 + 1$ if $= -2$.

All set? Let's resume …

Sometimes a table of values doesn't quite spell things out for us. Often, there is a pattern in the y-coordinates, and the pattern is expressed as an equation that relates to x. For example, suppose $y = x + 3$ and the x-values are given to us. What would this look like without a table of values?

Example

1 Substitute $x = -2$ into $y = x + 3$. State your x and y-values.

2 Substitute $x = -1$ into $y = x + 3$. State your x and y-values.

3 Substitute $x = 0$ into $y = x + 3$. State your x and y-values.

4 Substitute $x = 1$ into $y = x + 3$. State your x and y-values.

5 Substitute $x = 2$ into $y = x + 3$. State your x and y-values.

Solution

Instead of completing each question separately, we can write out a table like this.

x-coordinate	$y = x + 3$
–2	
–1	
0	
1	
2	

And then we treat each row as a separate substitution problem.

x-coordinate	$y = x + 3$	y-coordinate
–2	$y = -2 + 3$ $y = 1$	1
–1	$y = -1 + 3$ $y = 2$	2
0	$y = 0 + 3$ $y = 3$	3
1	$y = 1 + 3$ $y = 4$	4
2	$y = 2 + 3$ $y = 5$	5

Example

Complete a table of values that displays various side lengths of squares and their respective areas.

Solution

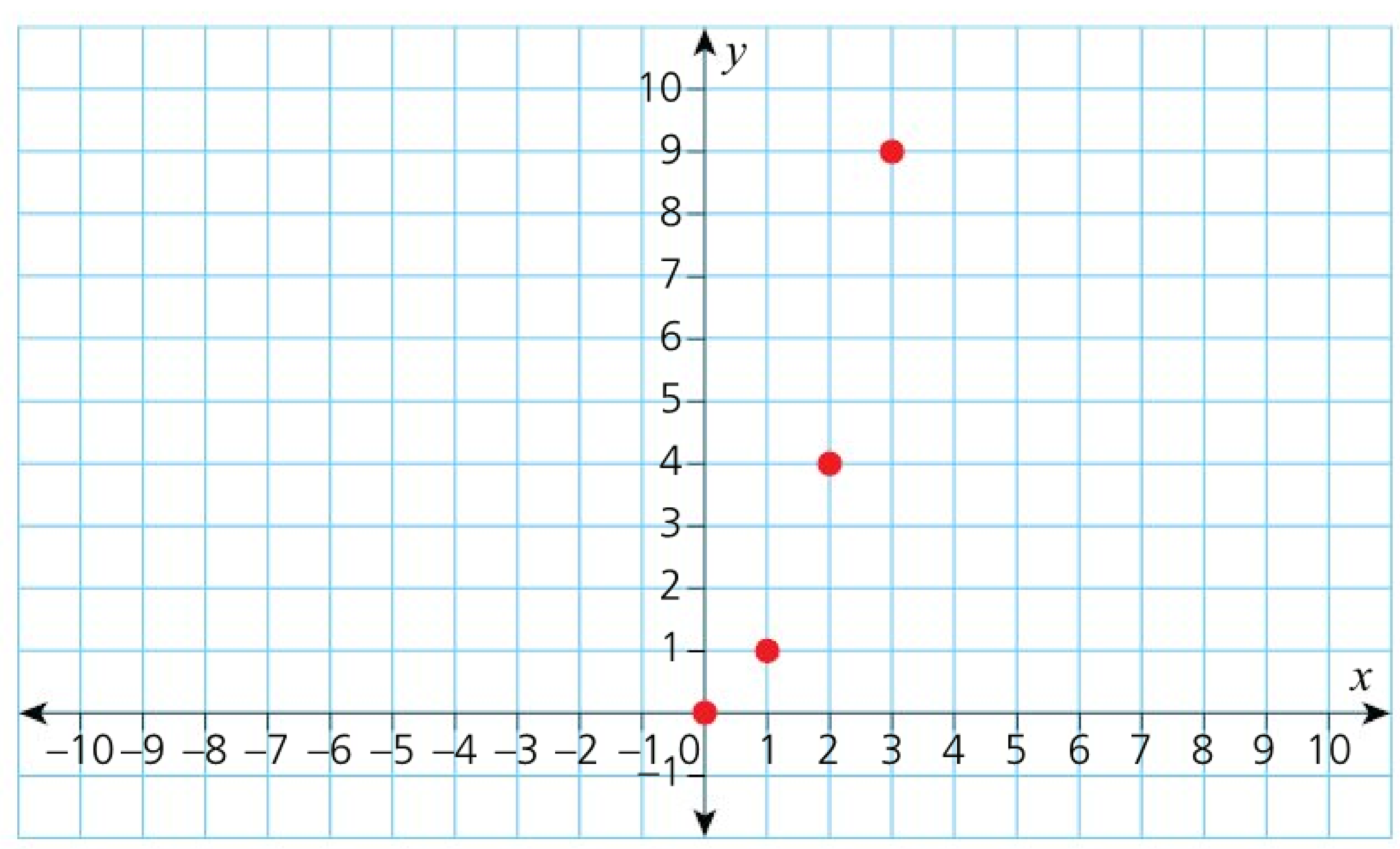
If x represents the length of a square's side, and y represents the area of that square, and we know the area of a square is just $l \times l$, or l^2, to determine y (the area) we simply square whichever x-value we are looking at.

x-coordinate (length)	$y = x^2$ (area)
0	$y = 0^2 = \mathbf{0}$
1	$y = 1^2 = \mathbf{1}$
2	$y = 2^2 = \mathbf{4}$
3	$y = 3^2 = \mathbf{9}$

DISCUSS

Why are there no negative values for the x-coordinate?

Now that we have found our y-values, we can graph. Does this look like a straight line?



Can you read off the area of a square with side length 3 units by looking at the points on the graph? Does the value match the table?

ACTIVITY: Plot it

■ ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

1 Plot these points on a Cartesian plane.

x	y
–2	5
–1	4
0	3
1	2
2	1

2 Plot these points on a Cartesian plane.

x	$y = x - 3$
–2	
–1	
0	
1	
2	

3 A taxi driver records the number of minutes a journey takes in their cab but forgets to turn on the meter until 10 minutes into the drive. Copy and complete the table of values, and draw a graph of actual time in the car versus time on the meter.

x (time on meter)	$y = x + 10$ (actual time)
10	
20	
30	
40	
50	

4 A retail chain offers to double customer donations to a charity. Write an equation that would give the total amount in donations depending on customer donations, create a table of values and plot your points on a Cartesian plane. Do you notice any patterns? Why do you think this is?

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

ACTIVITY: Graphs and equations

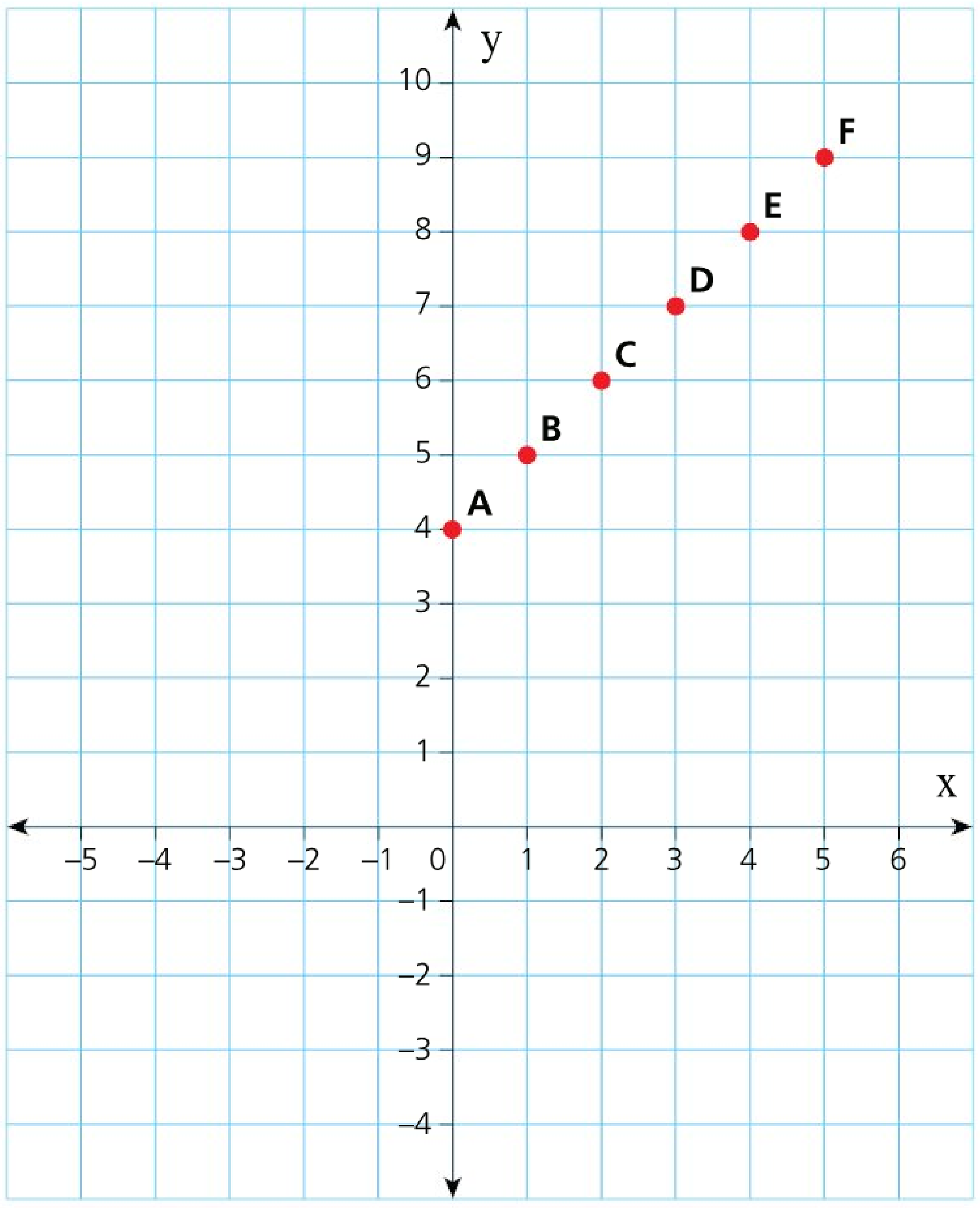
■ ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions

Create a table of values from the graph of the plotted points. Do you notice a relationship between the x- and y-values? Can you generalize it and come up with an equation for y? Justify your results with examples of your own.

◆ Assessment opportunities

- In this activity you have practised skills that can be assessed using: Criterion B: Investigating patterns.



ACTIVITY: Gradient/slope investigation

ATL

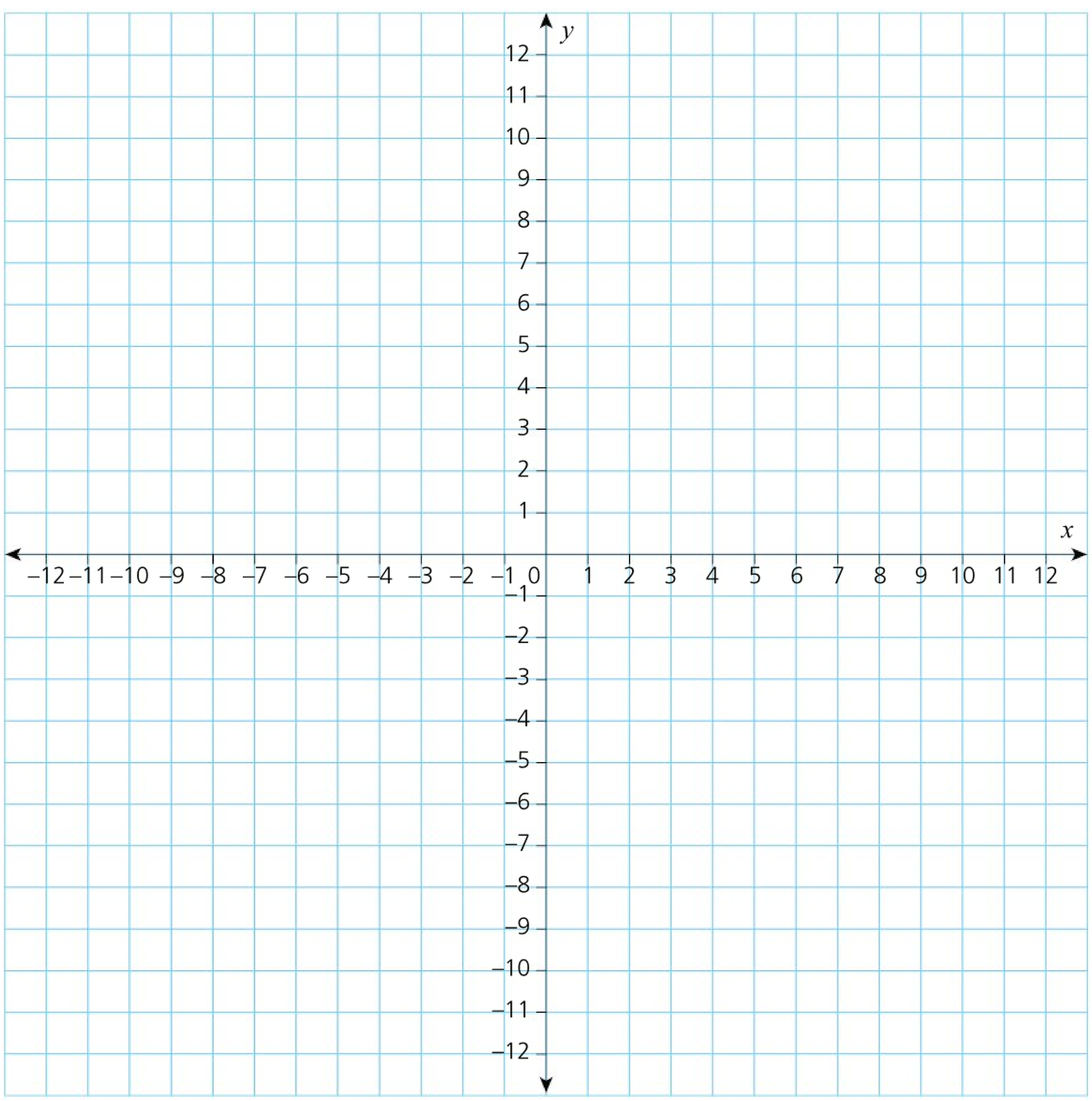
- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Communication skills: Make inferences and draw conclusions

Copy and complete the tables of values below, and graph their points on the same plane, using a different colour for each set of points. Using the same colours as before, connect the points for each table together.

x	$y = 2x$
–2	
–1	
0	
1	
2	

x	$y = 6x$

x	$y = \frac{1}{2}x$



What do these equations all have in common? How are they different?

Look at the connected points on each graph. What do these lines all have in common? How are they different?

Could you **predict** what the graph of $y = 8x$ would look like? Graph it to test your hypothesis.

Describe the relationship between equations with coefficients of x and their graphs.

EXTENSION

Investigate the relationship between equations with negative coefficients and their graphs. Describe and justify any patterns you notice.

Assessment opportunities

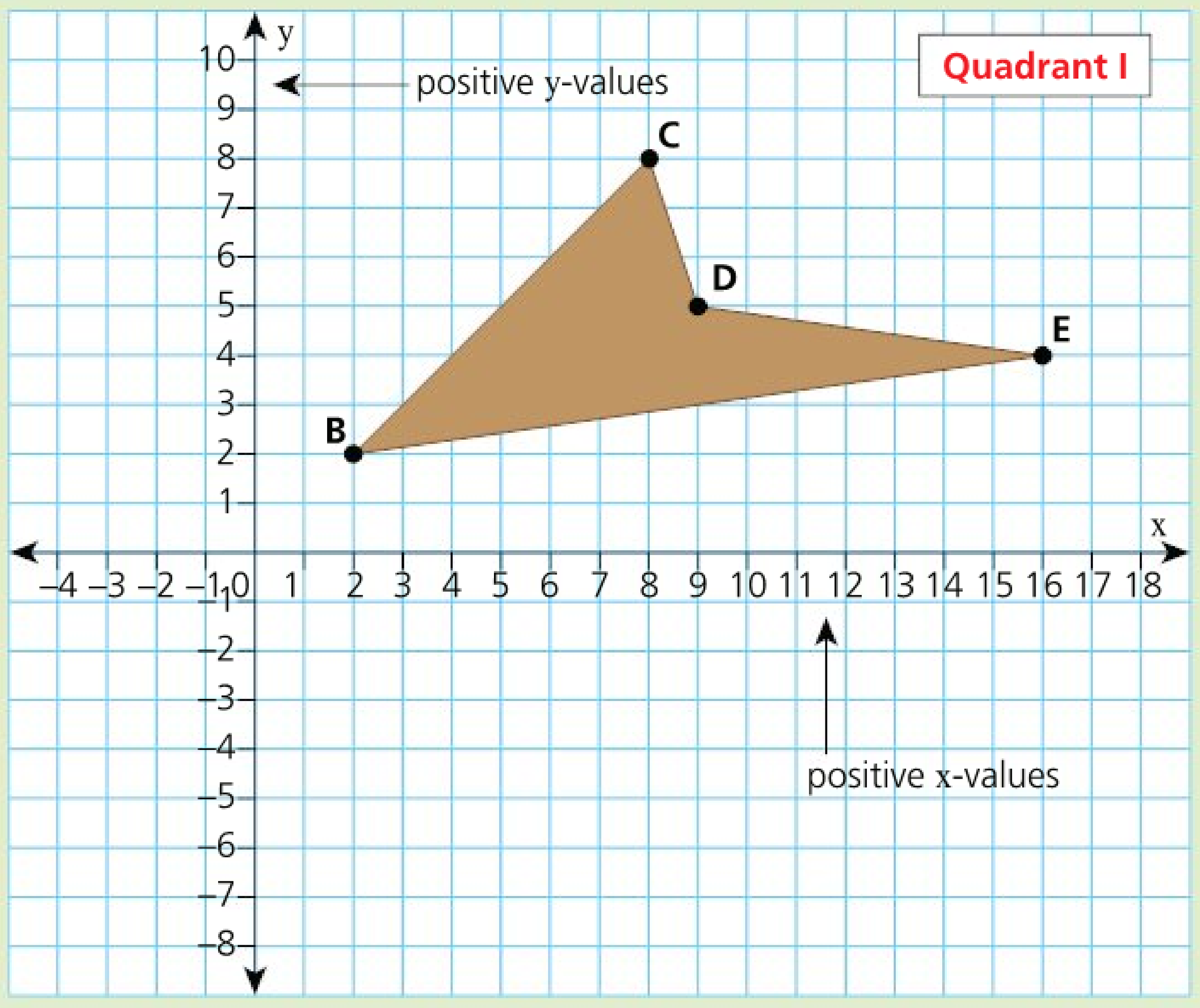
- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

What is mathematical about mirrors?

When a shape is flipped over a line, creating a **mirror image**, this is called a **reflection**. The line acting as the mirror (which is equally distant from both images) is referred to as the **reflection axis** or **line of reflection**. To start with, we will use the x- and y-axes as the lines of reflection, but as you learn more about linear equations you will come across many other lines of reflection.

Example

Reflect the figure in the x-axis.



ACTIVITY: Reflection investigation

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Communication skills: Understand and use mathematical notation

Draw a Cartesian plane. **Plot** the points shown in the table of values (right) and join them together to form a triangle. We will call this table (and image) A.

x	*y*
3	4
6	1
7	7

Plot the points in this second table of values on the same plane as the first, and connect them to one another. Call this table (and image) B.

x	*y*
–3	4
–6	1
–7	7

How are the numbers in the two tables related? How are the objects you drew related?

Now recreate table A, this time making all the y-values negative, and plot the shape. What do you notice? Remember to use as much mathematical language as possible in your response.

Can you draw any conclusions about the effects of changing the signs of the x-coordinates? Of the y-coordinates? Justify your response using examples.

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

Anastasia's algebraic solution

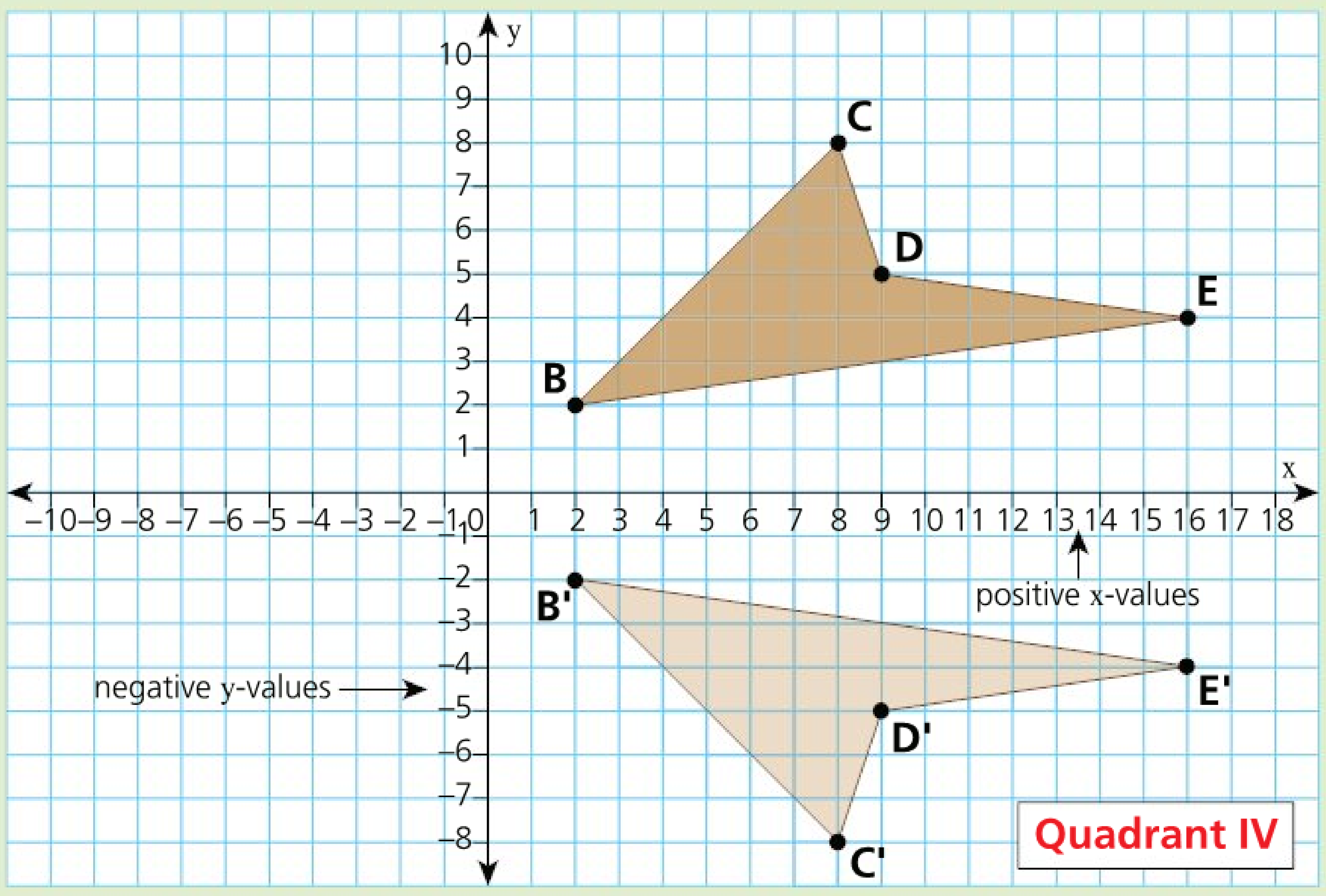
Anastasia makes a table of values from the points in the original image.

x	y
2	2
8	8
9	5
16	4

Since she is reflecting the shape in the x-axis, her image will be in quadrant VI where the y-values are negative (look back at page 109 to remind yourself about the different quadrants in the plane). So, Anastasia's new table of values looks like this.

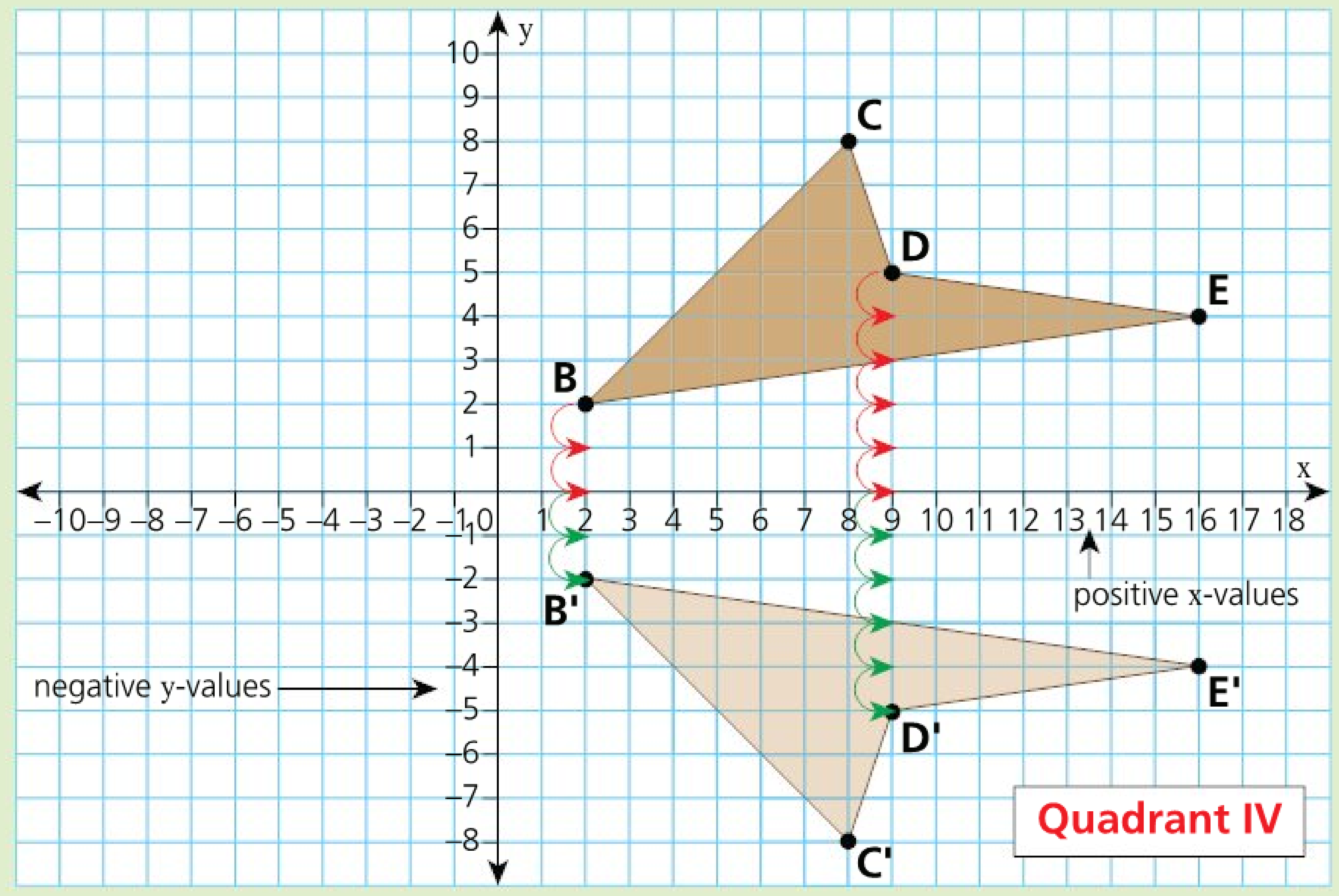
x	y
2	–2
8	–8
9	–5
16	–4

And she plots these values.



Gideon's graphical solution

Sometimes, if there aren't too many points to reflect, we can find the new image by counting the distance from each point to the line of reflection, and then counting away the same distance on the other side.



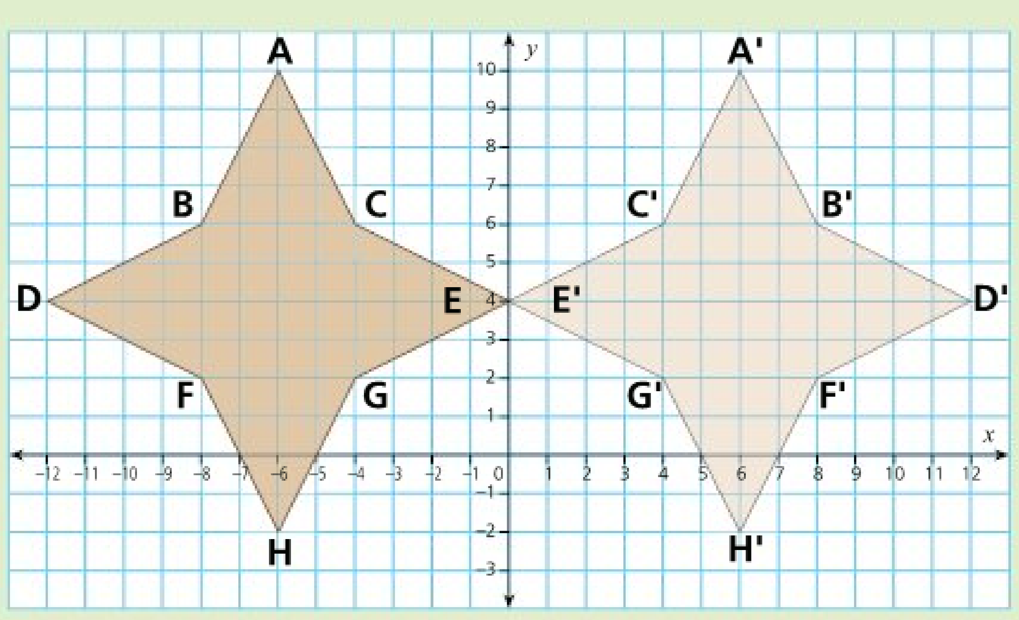
Notice that the image points are labelled using the same letters as the original figure, but with an added apostrophe (in mathematics this is called the **complement** sign). Why do you suppose mathematicians prefer this convention?

Note that in reflections, the figure and its image have the same shape, size, area and angles. This means the two figures are **congruent**.

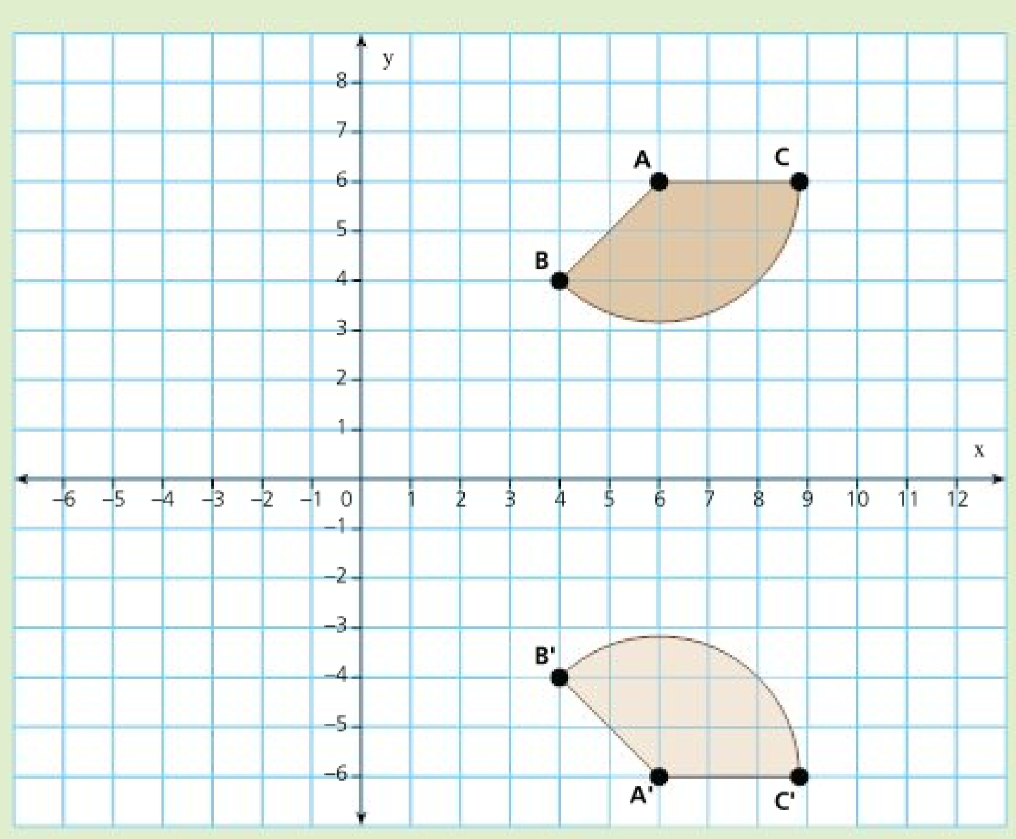
PRACTICE EXERCISE

1 State whether each transformation is a reflection in the x-axis or in the y-axis.

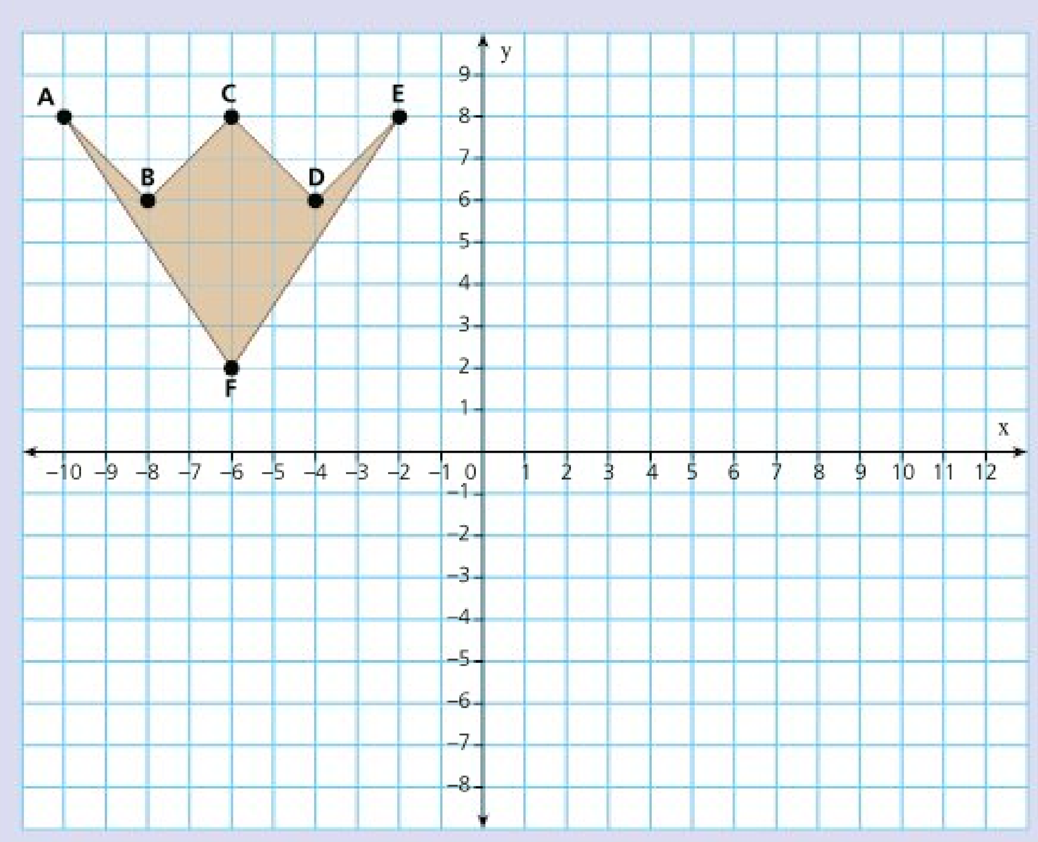
a



b



2 Reflect this image in the y-axis.

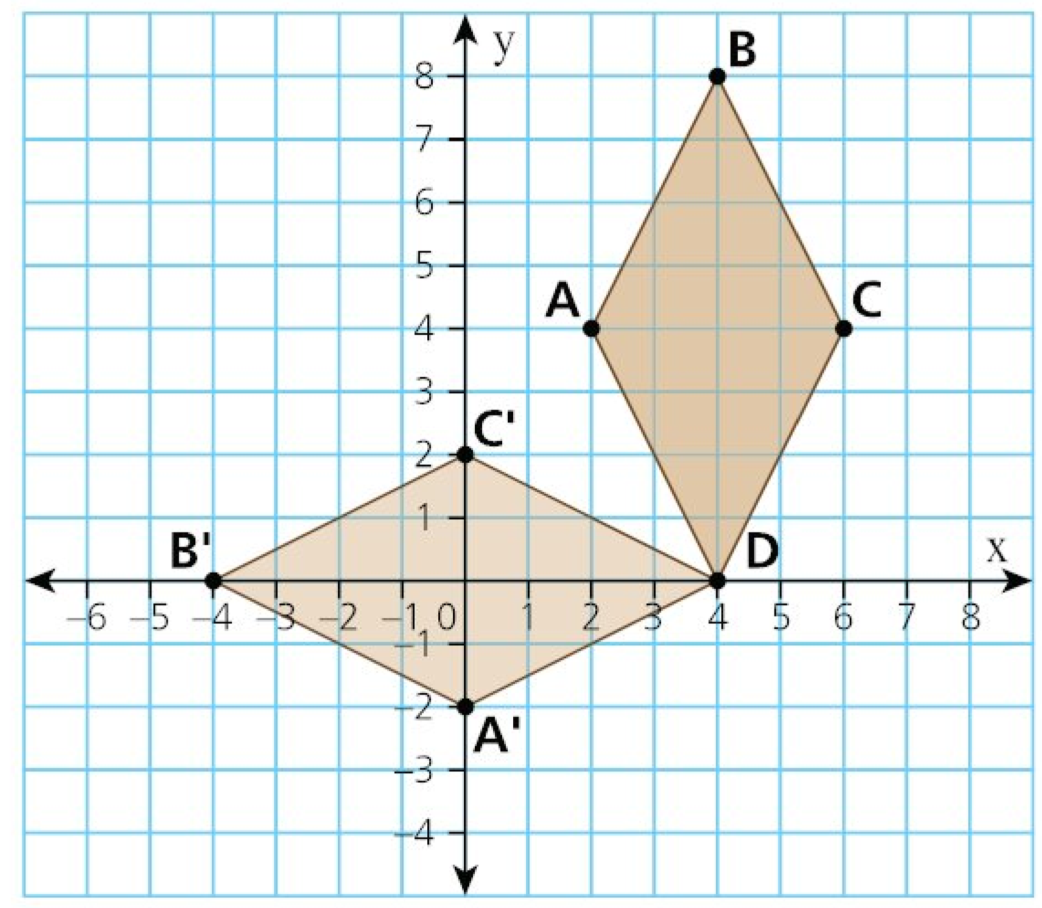


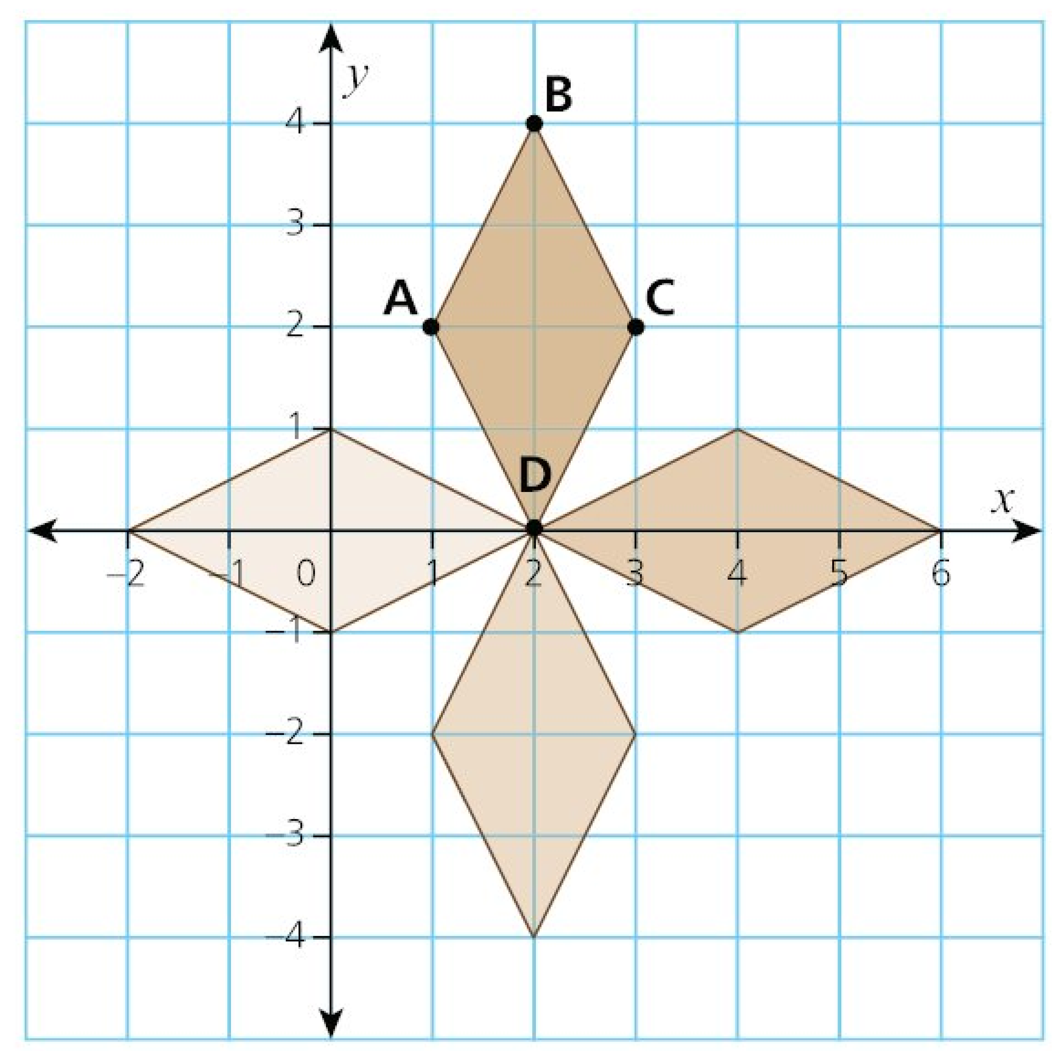
How many ways can you rotate a figure?

ACTIVITY: Just plotting around

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Communication skills: Make inferences and draw conclusions; Understand and use mathematical notation





Describe how these two images relate to one another.

Create a table of values from the figure ABCD, and another from the values in the image A'B'C'D. How are the coordinates in each of your tables related to one another?

Can you write a general rule about rotating images 90°? **Justify** your response with examples of your own.

Investigate rotations of 180° and 270°, and describe and justify any patterns you notice.

Hint

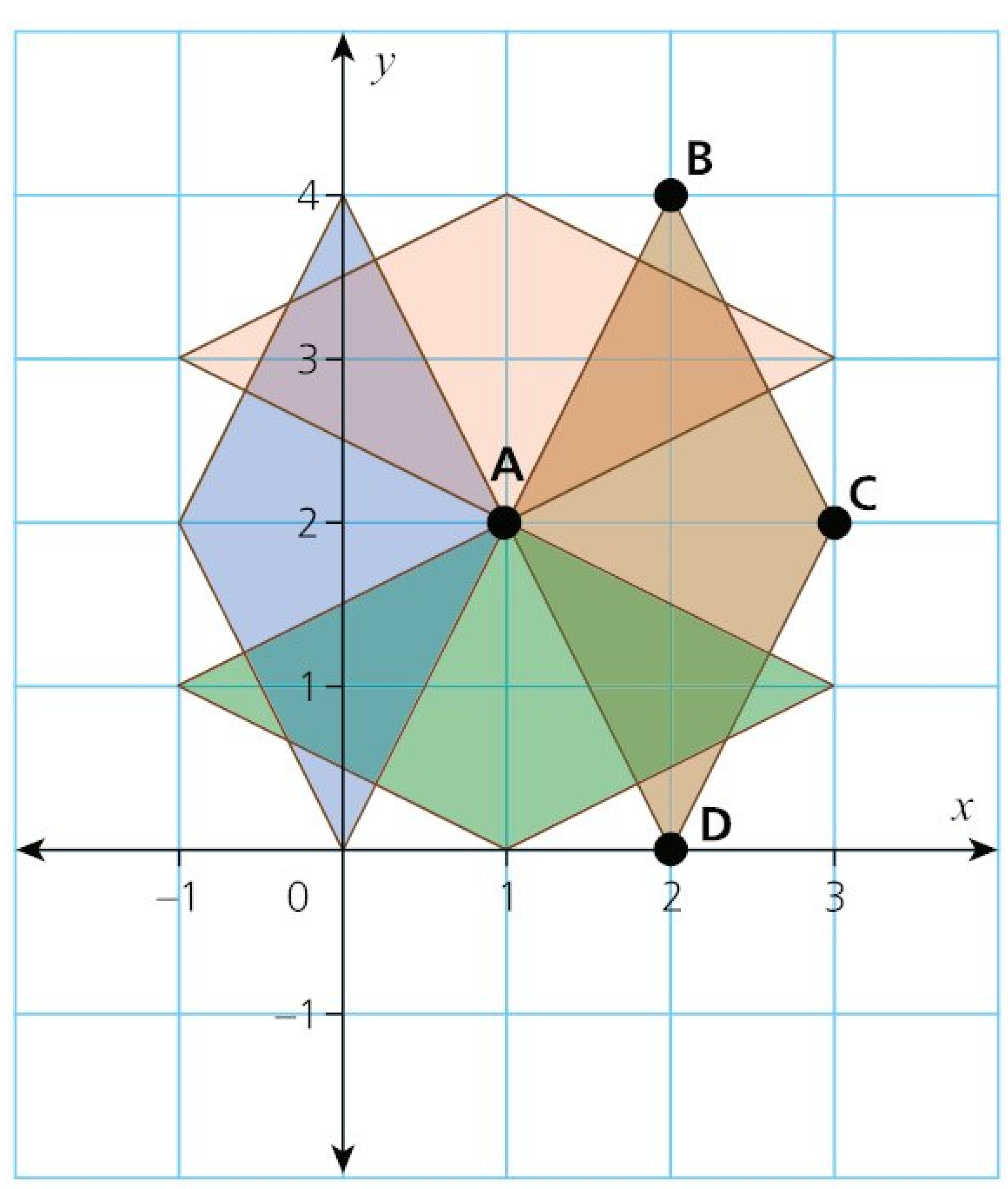
What problem-solving techniques do you know that can help you get started? Would a table or a diagram help?

Assessment opportunities

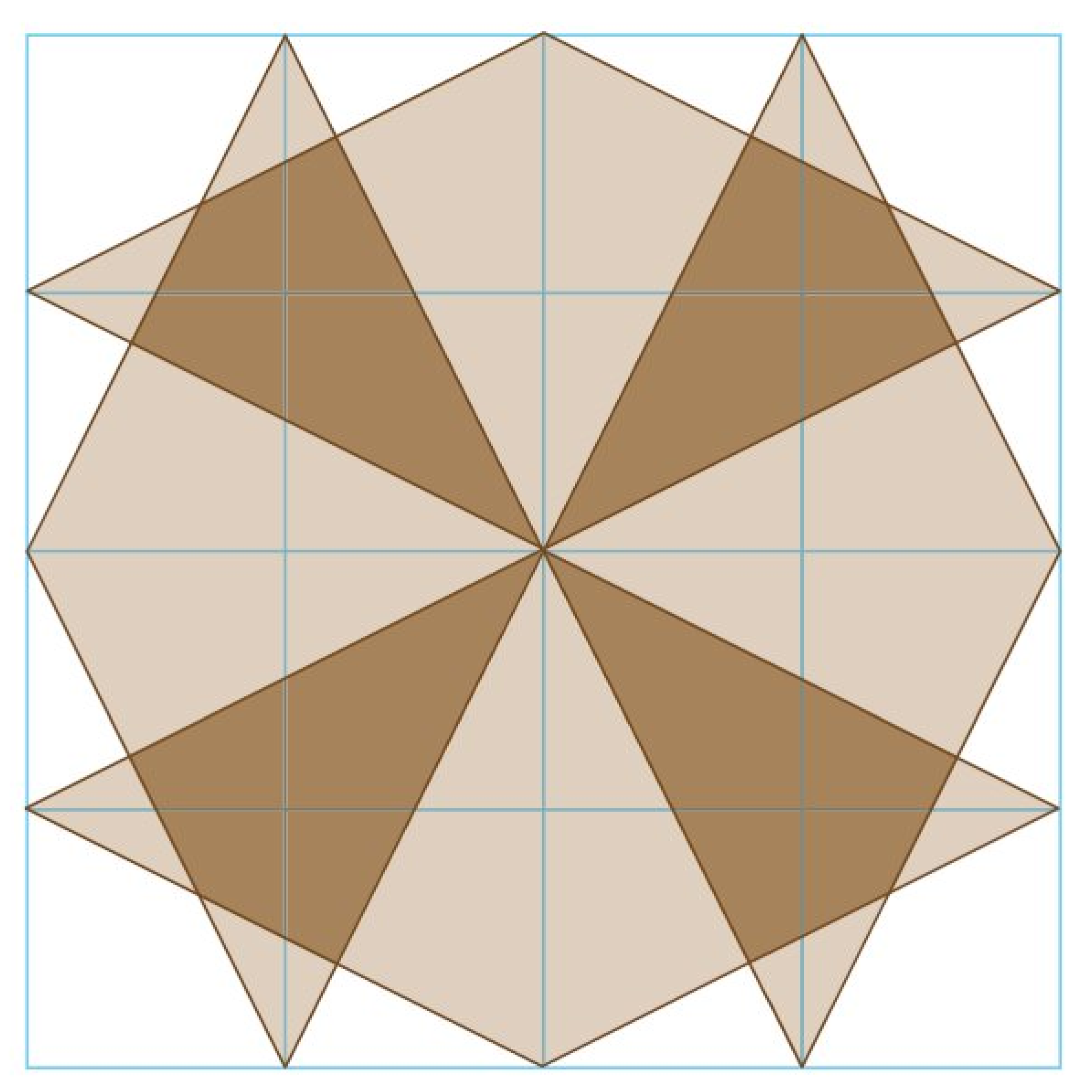
- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

Notice again, that in the activity the figure and its image are **congruent**.

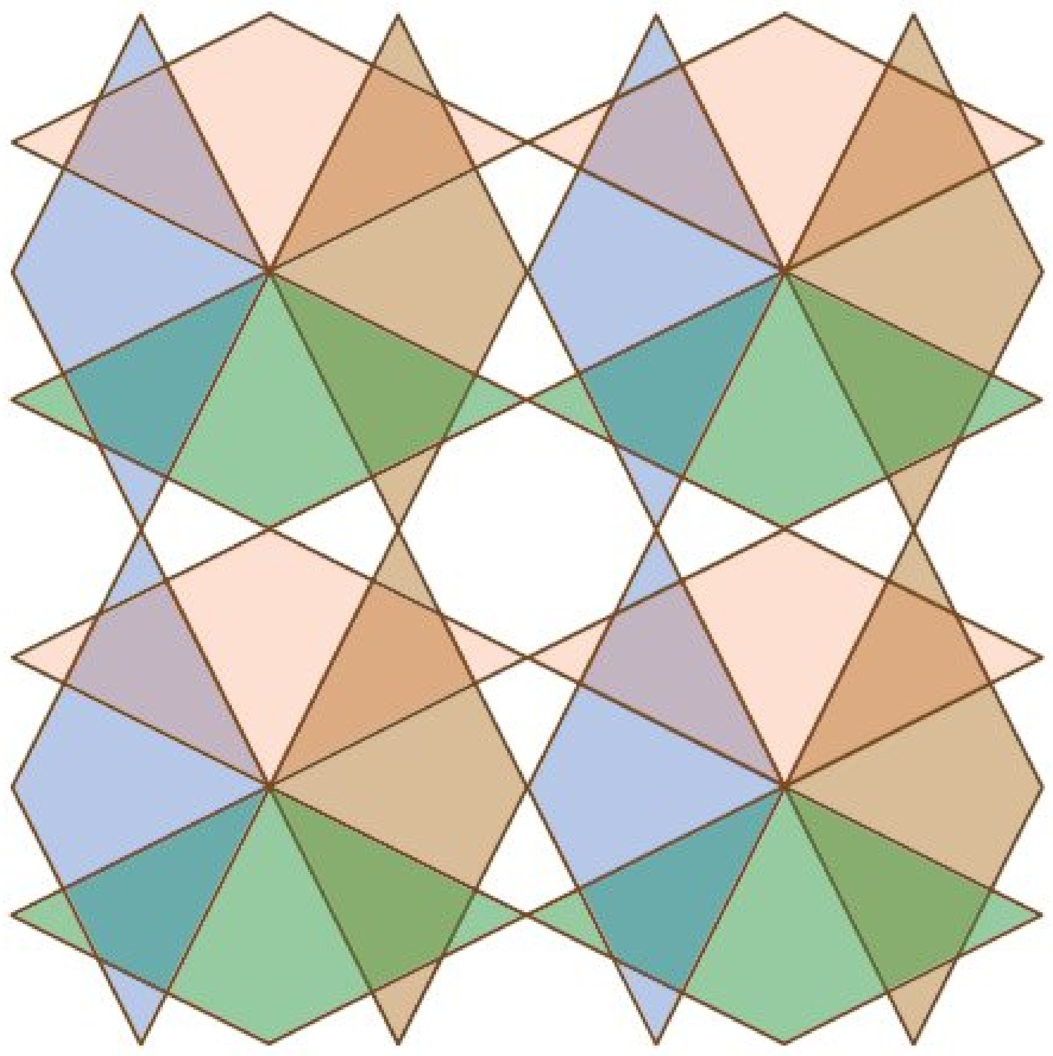
In the activity's lower Cartesian plane, the image is rotated around one single point D (2, 0). This is called the **centre of rotation**, and can be any point inside or outside of the image. The same image could have been rotated about point A instead, which would have looked like this.



When drawn as a single colour , it can be a beautiful figure.



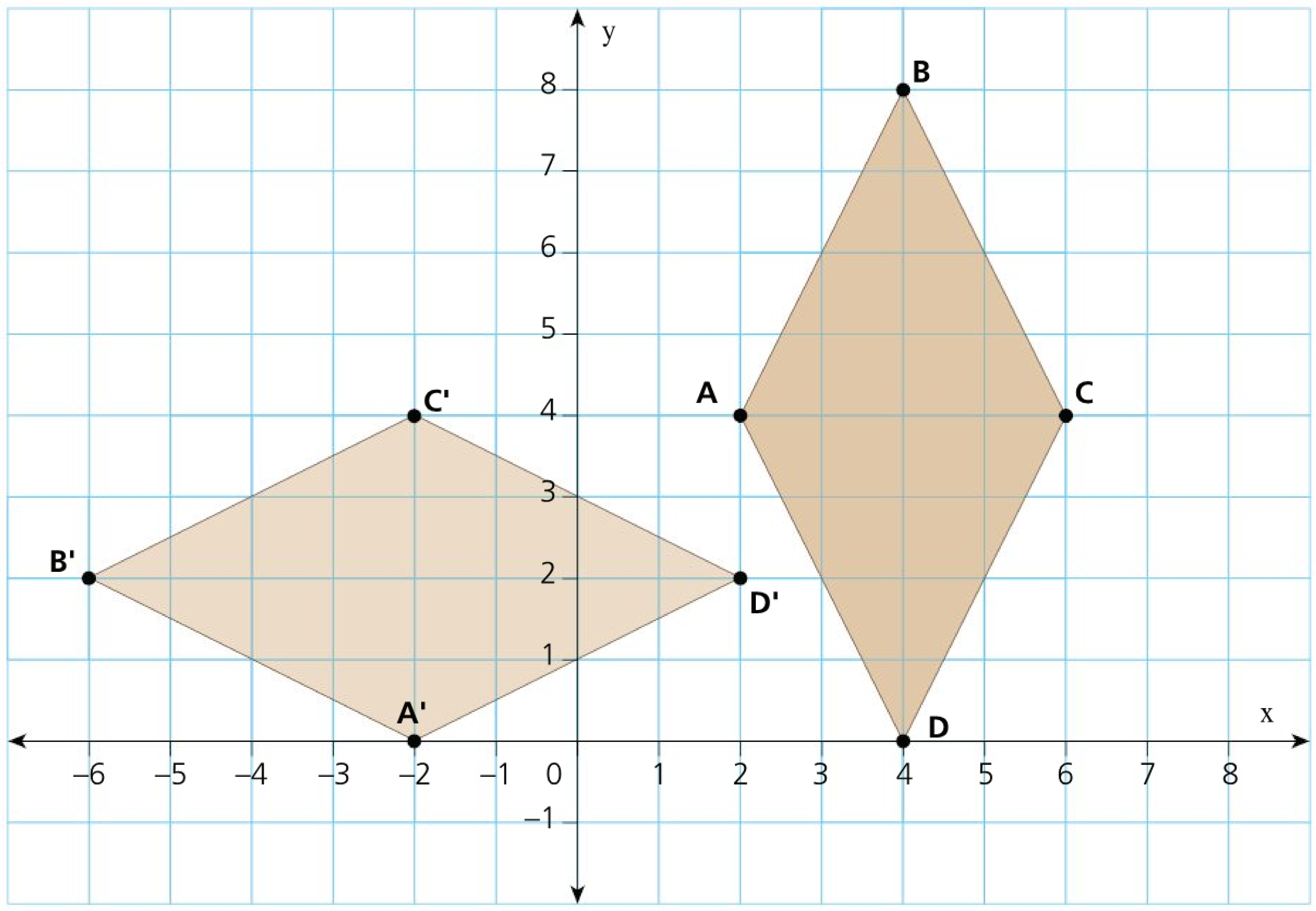
And can even be made into a quilt pattern!



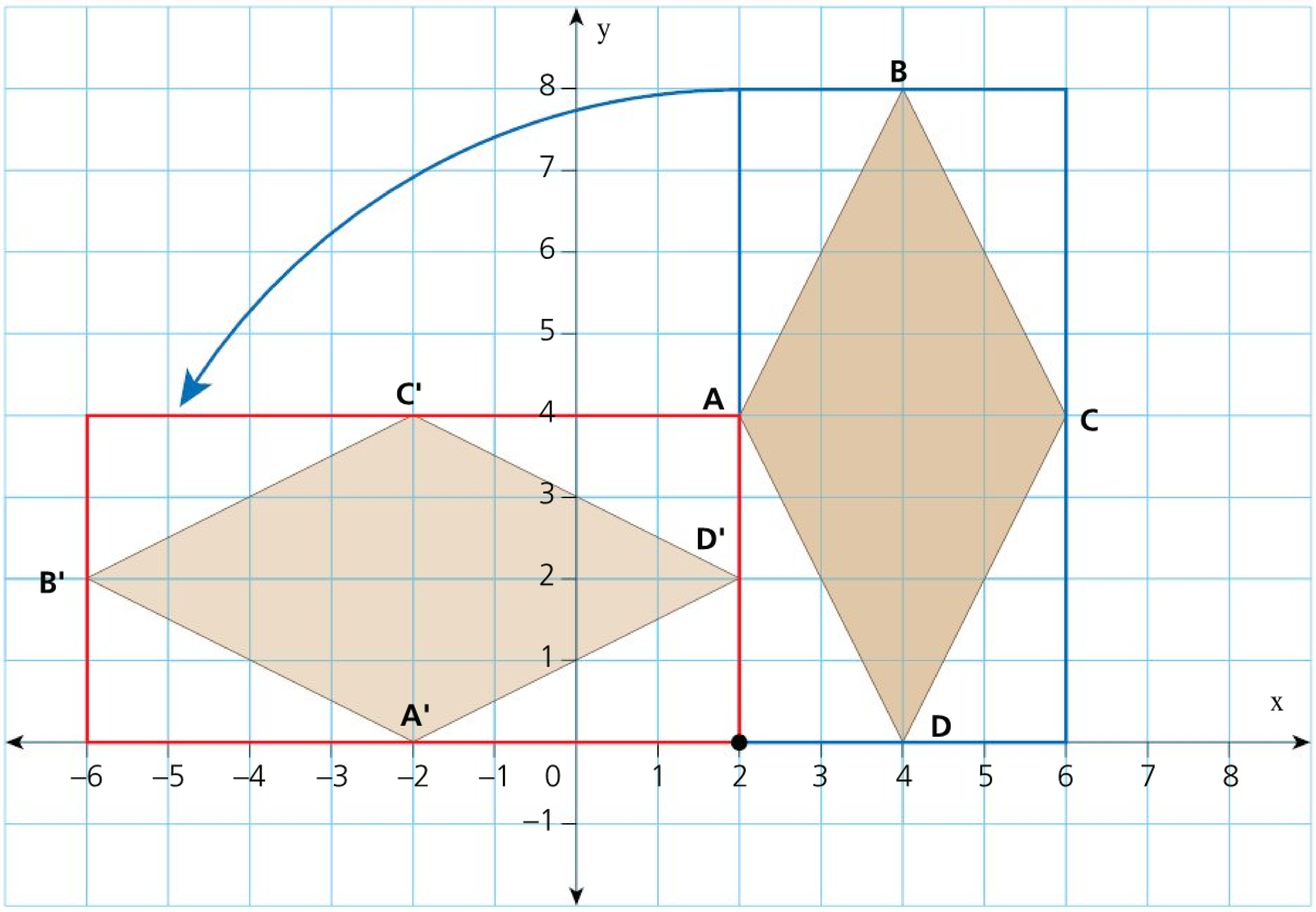
! Take action

- ! Draw a large Cartesian plane on card or paper. From another sheet, cut out a shape of your own design. Place the shape somewhere on the plane and insert a thumbtack (pushpin) at one of the vertices. Demonstrate to a partner the effects of various types of rotations (90° and 180° for example). Then show how to move the thumbtack to change the centre of rotation. Can you use your model to explain the difference between a rotation of 180° and a reflection in the x-axis? Video yourself explaining the transformation and consider posting your recording online for others to experience your visualization.

The centre of rotation is not always a point on the figure. Sometimes it is a point on the plane. For instance, this image has been rotated about the point (2, 0).



To see this more clearly, it sometimes helps to draw a box from the centre of rotation around the figure.



THINK-PUZZLE-EXPLORE

One of the first ever handheld video games was Tetris for the Nintendo Gameboy. Search '**Tetris free online**' and play a few rounds of it.

What do you think you know about Tetris?

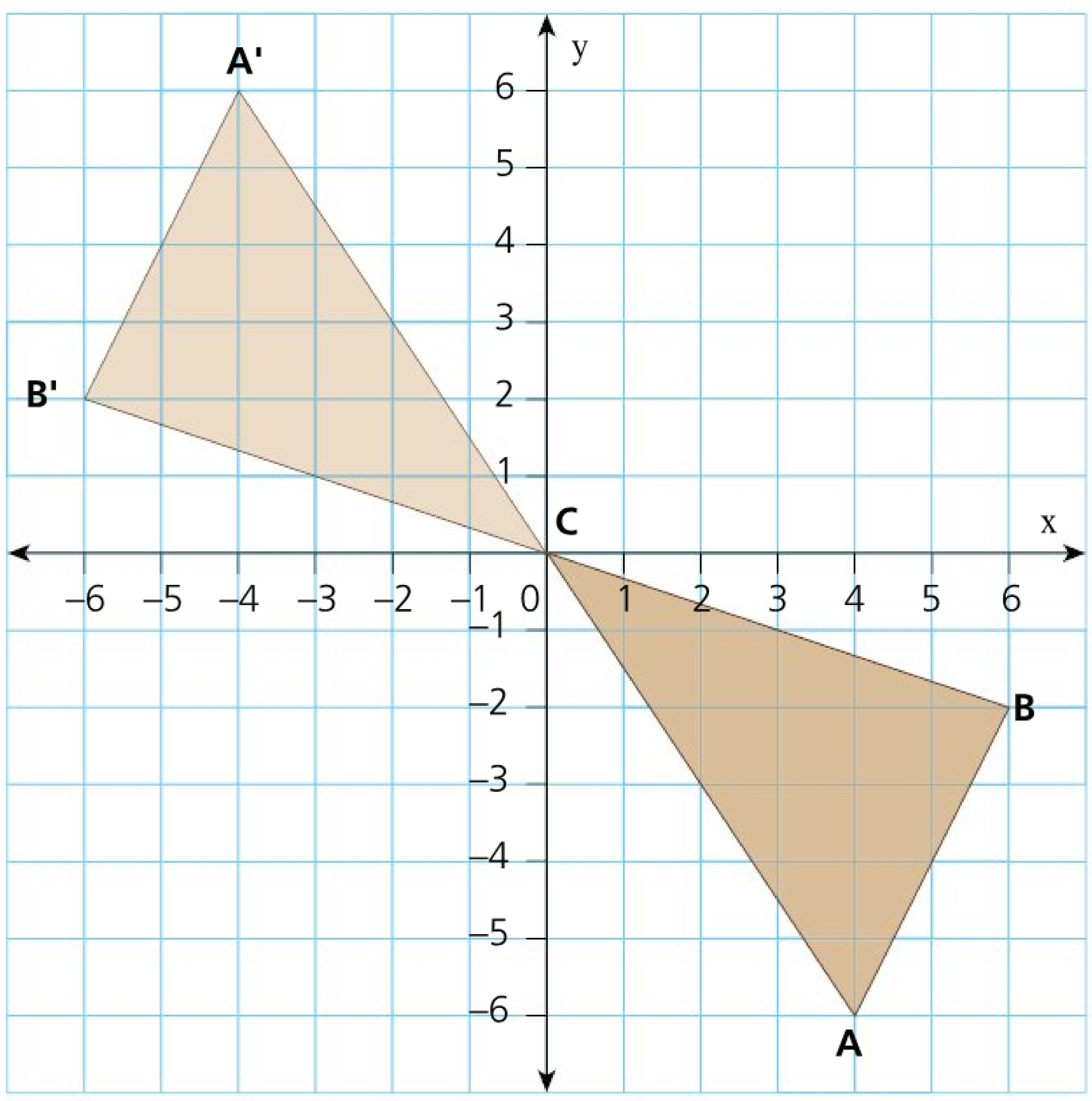
What questions do you have about the connection between mathematical transformations (movements of figures) and Tetris?

Explore other, perhaps more sophisticated, video games or apps that use transformations. What are they? Which transformations are used? How? Do they significantly impact the outcome of the game?

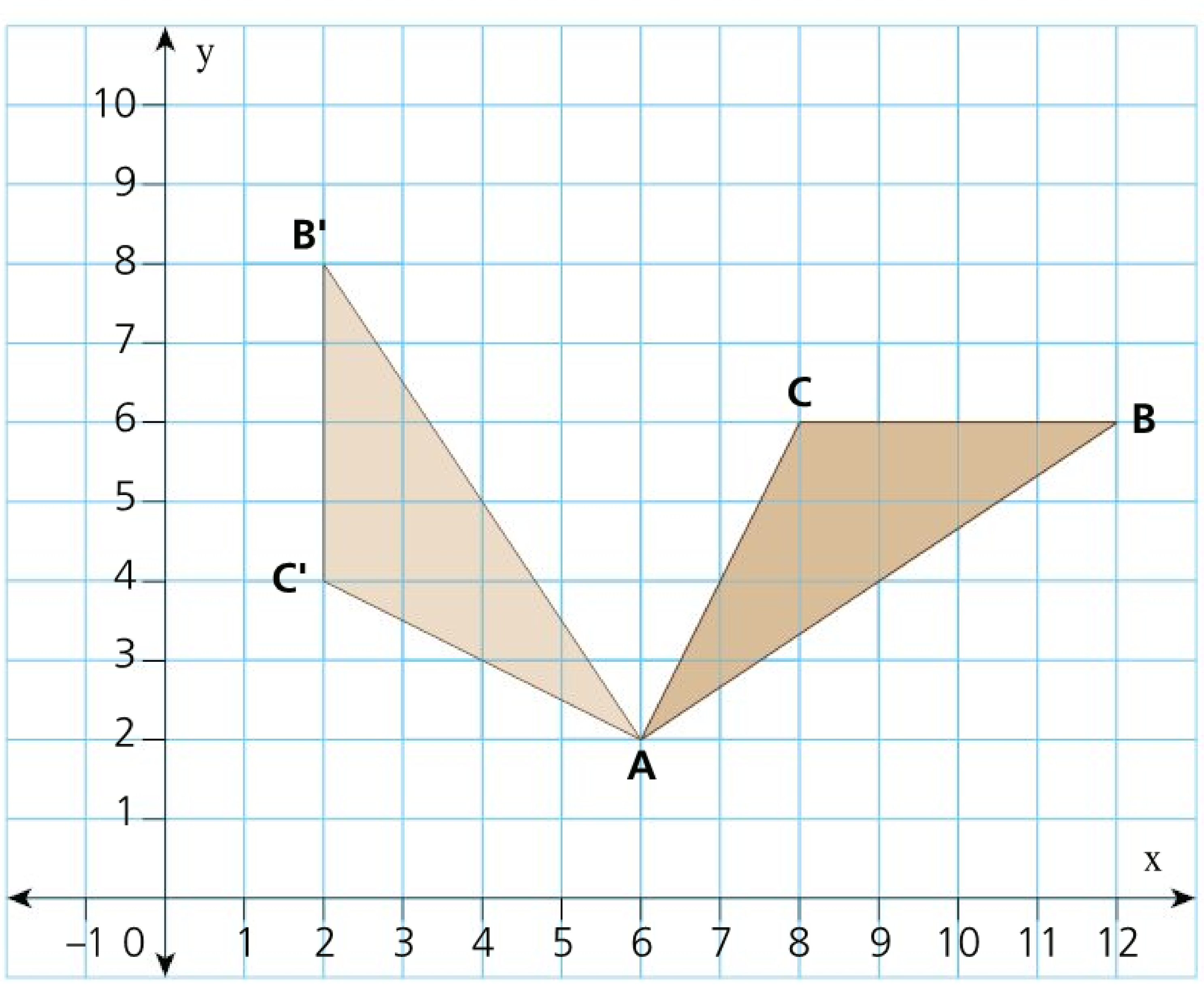
PRACTICE EXERCISE

Describe each transformation by stating the angle of counter-clockwise (anticlockwise) rotation and the coordinates of the centre of rotation.

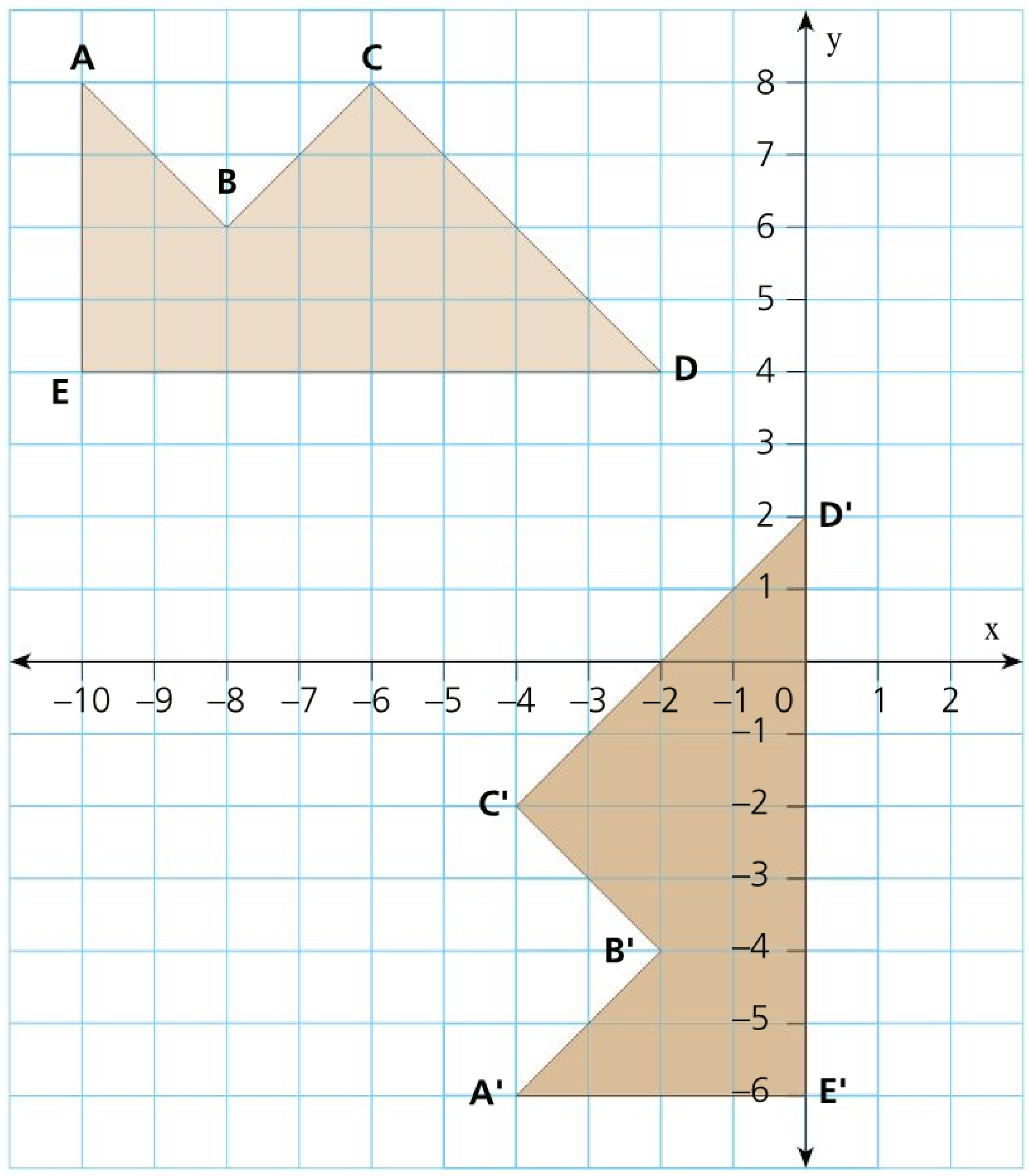
1



2



3



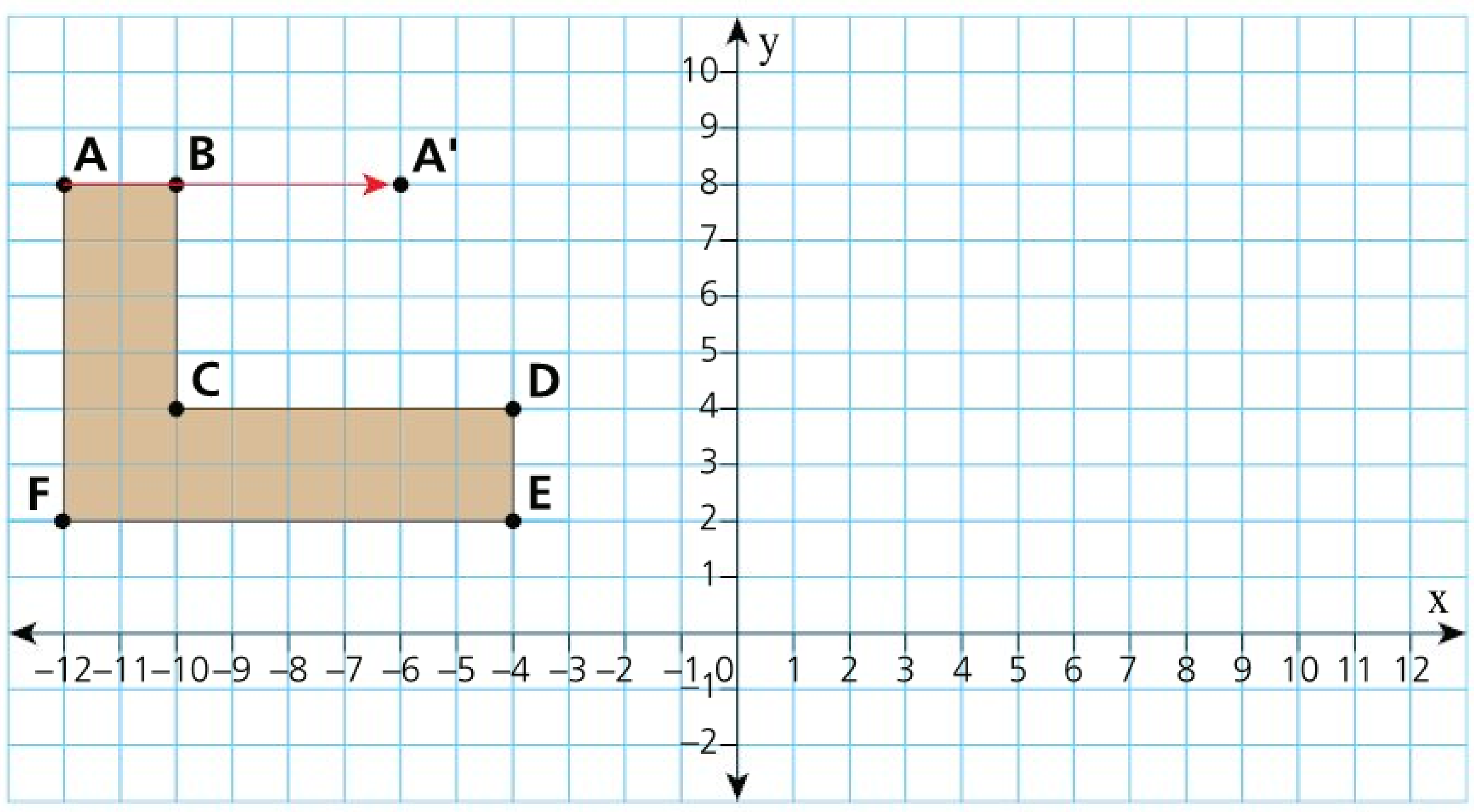
How do two-dimensional figures 'move'?

ACTIVITY: On the move

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Communication skills: Make inferences and draw conclusions; Understand and use mathematical notation

Move each point six units to the right. Point A has been done for you.



The movement of an entire figure horizontally like this (or indeed vertically), while preserving congruence (keeping the shape and size intact), is called **translation**.

Create a table of values for the original figure and another for its image.

What do you notice about the x-values? How does what you've observed relate to your graph? Can you predict the x-values if the translation were two units to the **left** instead?

Investigate a vertical translation of your choosing, and describe and justify any patterns you notice.

Assessment opportunities

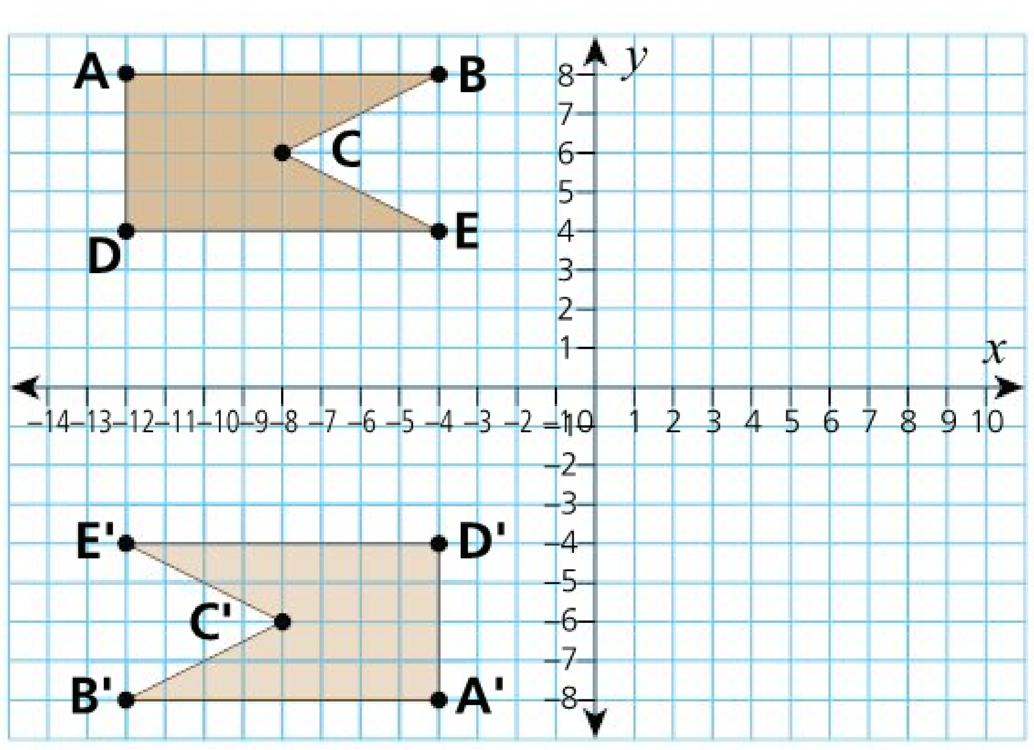
- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

THINK-PAIR-SHARE

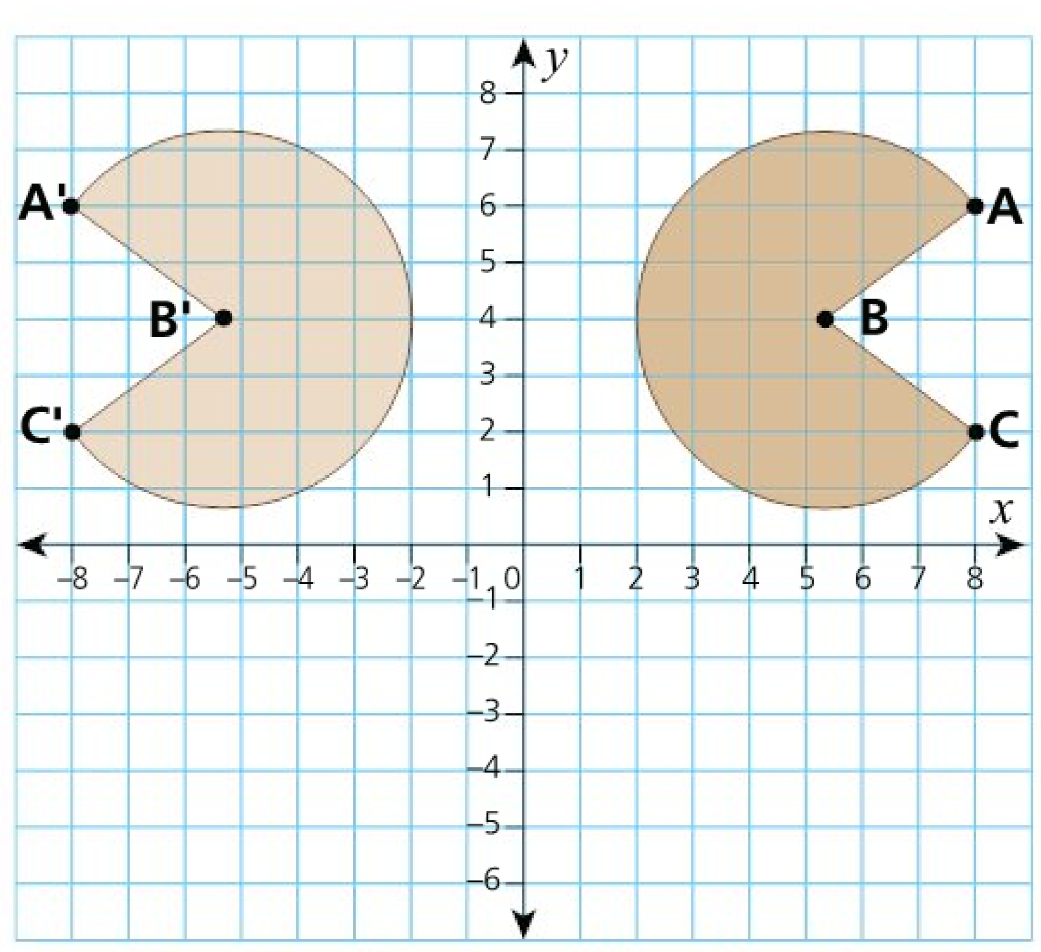
Look at the transformations below. What transformations do you see here? Is there only one transformation at any one time?

Discuss your answers with a partner and share the rest of the class.

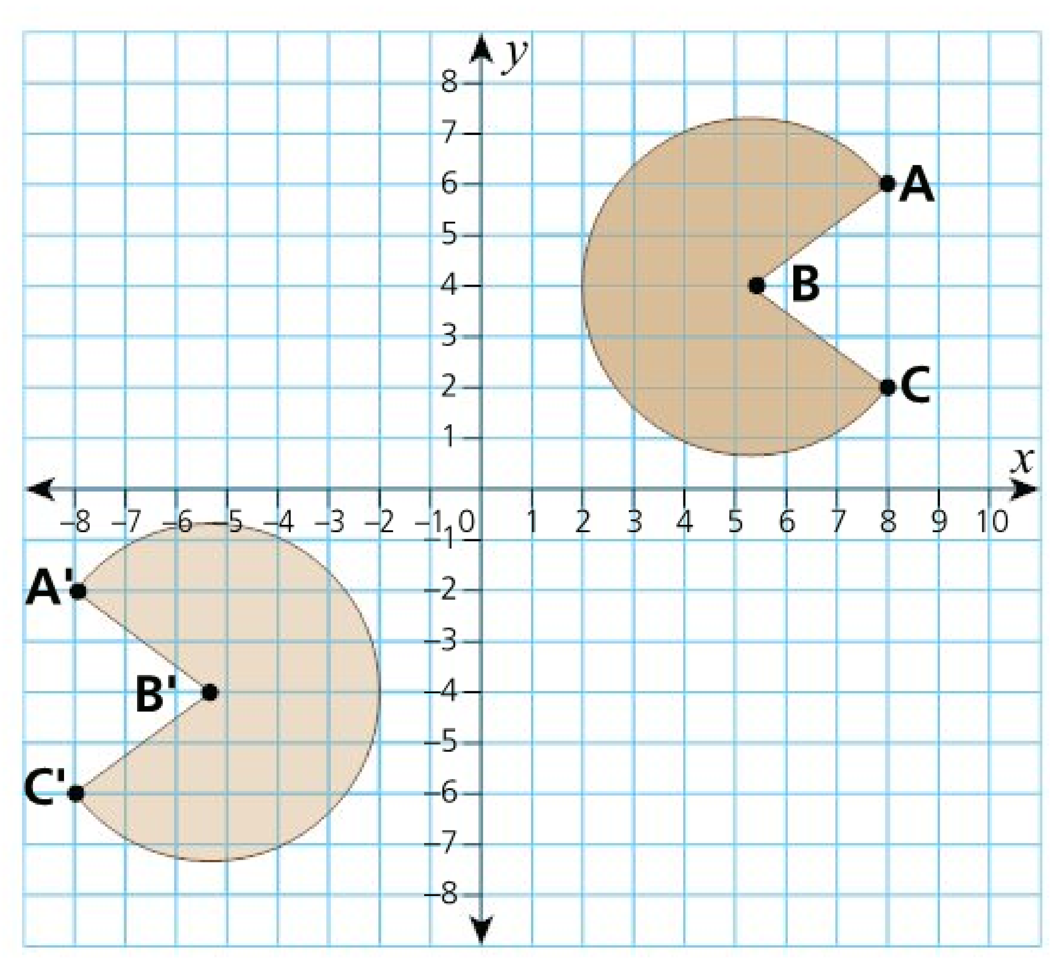
1



2



3



MEET A MATHEMATICIAN: M.C. ESCHER (1898–1972)

Learner Profile: Open-minded

> 'In Escher's mind, mathematics was what he encountered in schoolwork—symbols, formulas, and textbook problems to solve using prescribed techniques. It didn't occur to him that formulating his own questions and trying to answer them in his own way was doing mathematics.'
>
> *Doris Schattschneider*

Maurits Cornelius Escher, whose self portrait can be seen at the top of page 116, was a Dutch graphic artist who conducted mathematical research to fuel his art. Though he failed his high school exams and did not enjoy the traditional approach to mathematics, Escher became obsessed with the 'regular division of the Plane'. He would create tiles and perform rotations, reflections or translations on them to create the illusion of infinity. His art quickly moved from 'landscapes' to what he called 'mindscapes', unique plays on perspective that came from his mind as a combination of inspiration and well-planned mathematical patterning. Escher's mathematical research was extensive and even inspired other mathematicians of his time like Roger Penrose and H.S.M. Coxeter, and of later generations, Branko Grünbaum and Geoffrey Shephard.

Like those of us who enjoy the MYP approach, Escher began his work with Inquiry questions, the two principal ones being:

1 What are the possible shapes for a tile that can produce a regular division of the plane; that is, a tile that can fill the plane with its congruent images such that every tile is surrounded in the same manner?

2 Moreover, in what ways are the edges of such a tile related to each other by isometries?

■ There are lots of logos that use this impossible triangle shape as an inspiration. Search online to see the logos for Google Drive, Commerzbank and NGSS.

■ Look at this work inspired by *Drawing Hands* by M.C. Escher. Do you see the rotation?

■ This staircase was inspired by an Escher work. What transformations do you see?

What qualifies as ‘similar’?

DISCUSS

If someone says their musical taste is similar to yours, what does that mean to you?

Up until now, we have been studying various ways of recreating images of a figure on a plane through reflection, rotation and translation. All of these transformations have a critical feature in common: the sizes, shapes and angles of the figures did **not** change. We refer to these types of transformation as **isometric**.

Links to: Physical education

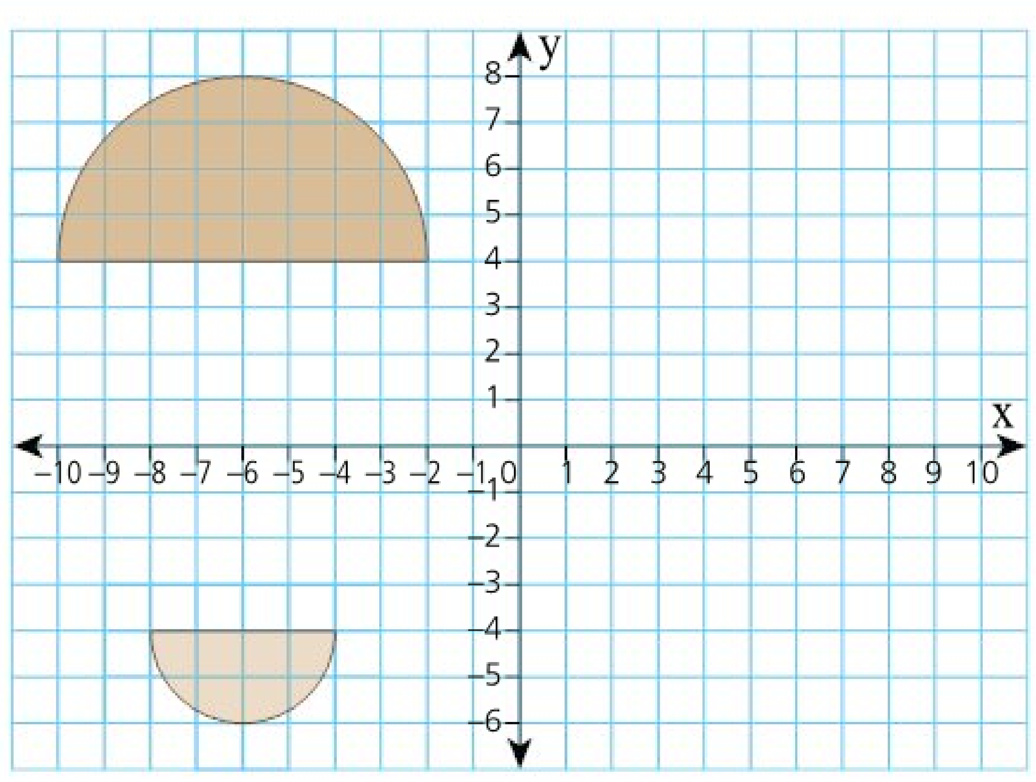
Isometric exercise is a branch of strength training that is done in static positions. These exercises are often used to help overcome injuries as there is very little stress on the joints. There is no visible movement in the joints, and the muscles do not change length during the exercise. Many isometric training exercises are nevertheless very challenging, such as the bridge plank or the Pilates hundreds.

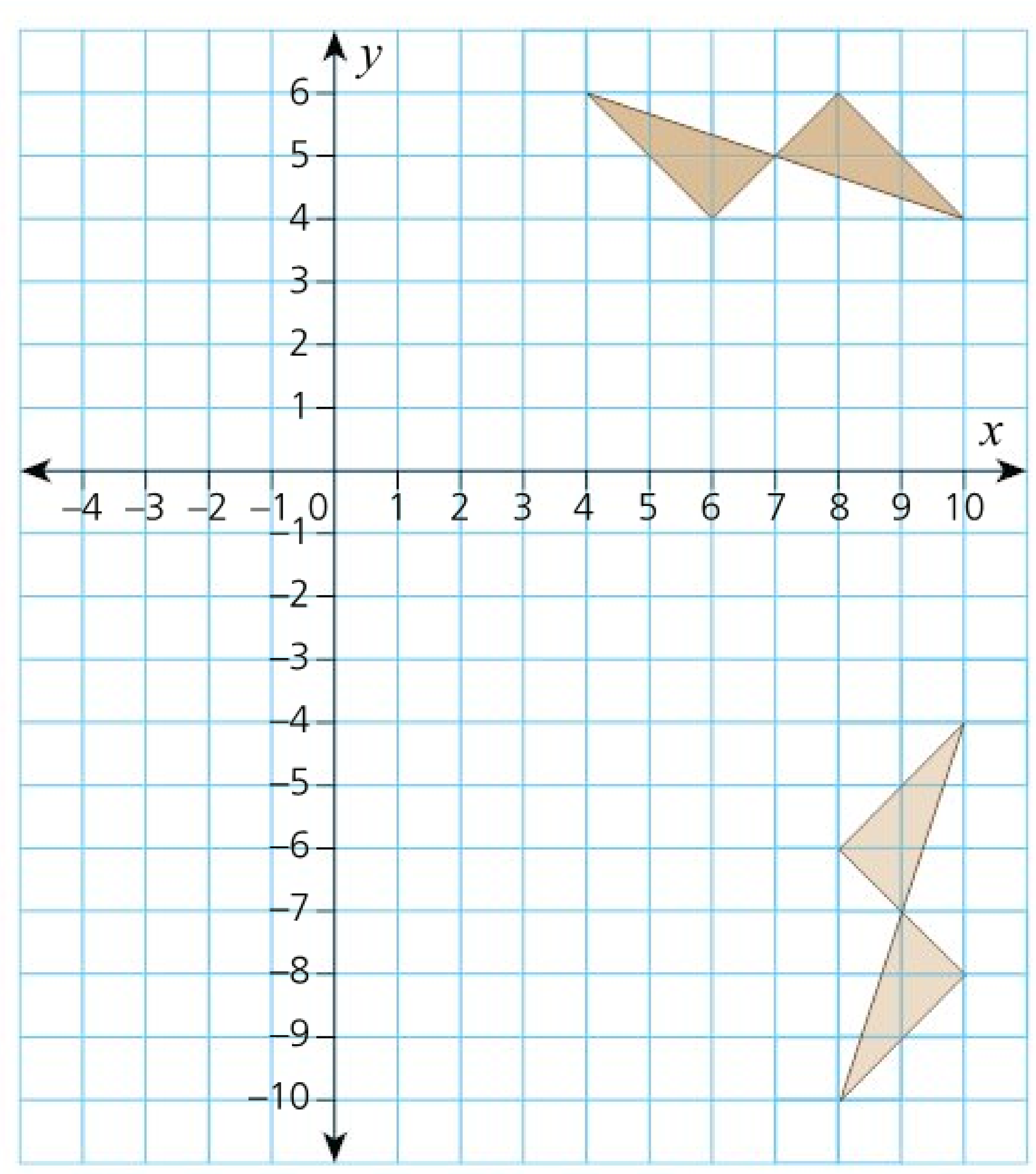
So what would a transformation look like that is not isometric? How would you describe the transformation below?

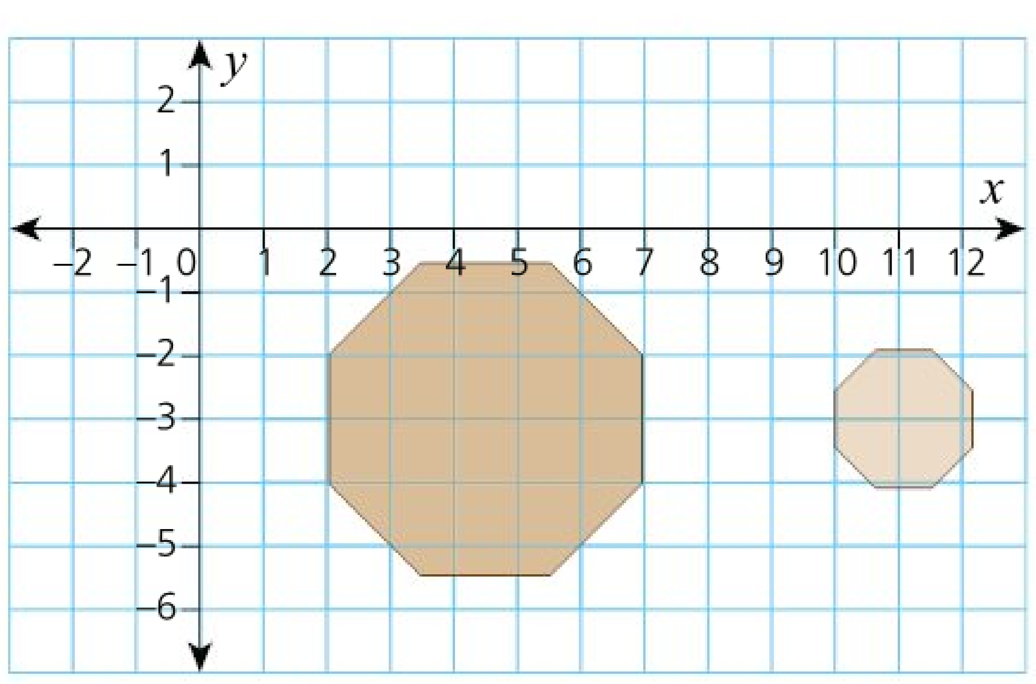
You’ll notice that the second image is just an **enlargement** of the first image. In the book *Alice in Wonderland*, Alice changes size every time she eats or drinks something. Her proportions remain the same, but everything about her – her feet, her hands, her head – changes size. These two images are not congruent, but they can be called **similar**.

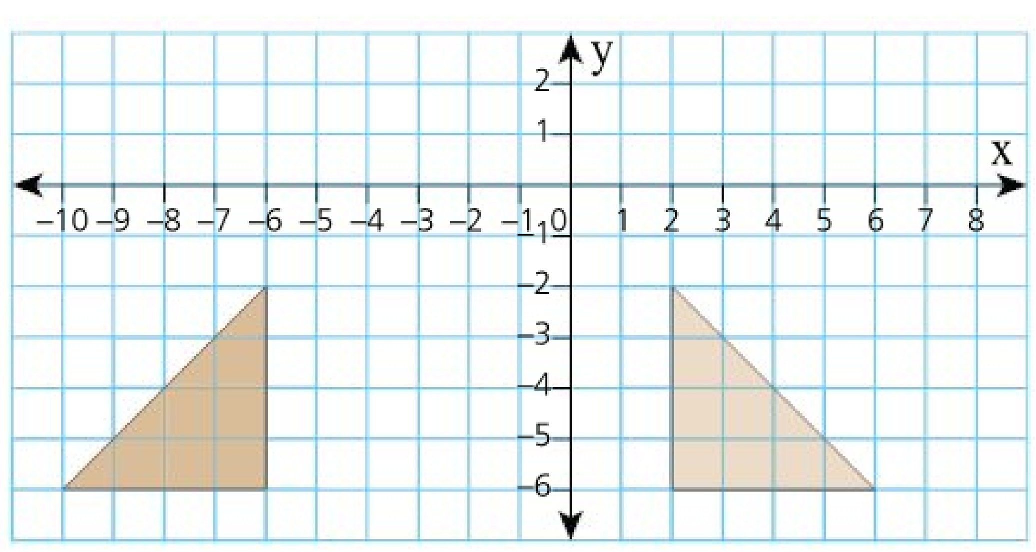
THINK-PAIR-SHARE

Each picture shows a darkly shaded figure and its lighter image after **two** transformations. Which pairs of figures and images are congruent and which are similar?









How do we enlarge a shape?

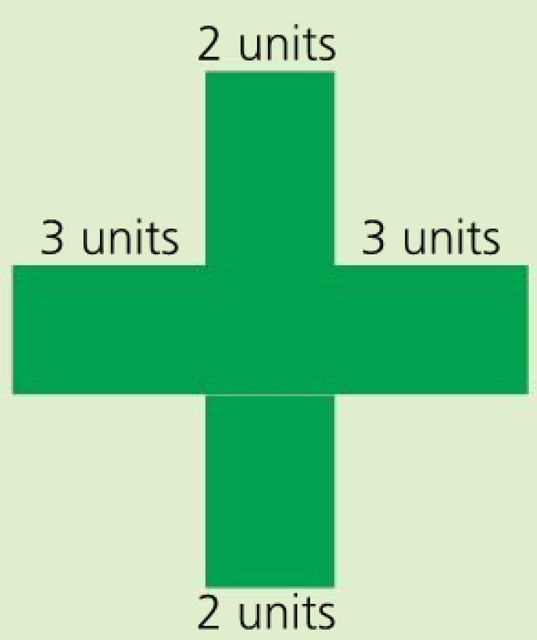
You have no doubt seen figures become enlarged by clicking on an on-screen image. To do this by hand, you need simply to multiply each component of the image by the same number – the **scale factor**.

Example

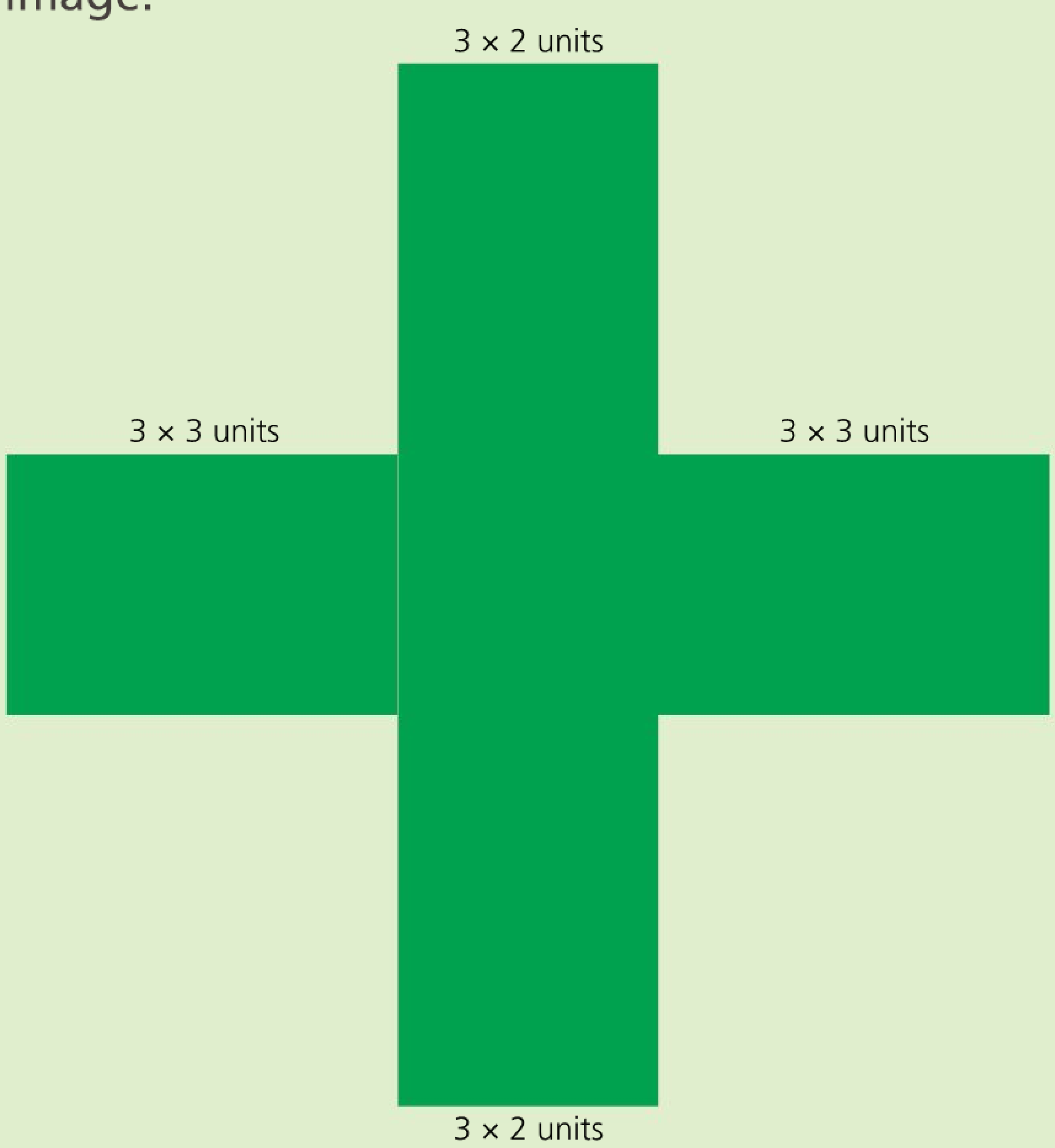
Enlarge this green cross by a factor of 3.

Solution

First, we measure and label all the edges.



Then we multiply each by the scale factor (3) to get the dimensions for the enlarged image.



HOW DO WE 'REDUCE' A SHAPE?

In making an image smaller than its original, we are effectively 'enlarging' it to a fraction of its size. Thus, our scale factor becomes a fraction.

Inukshuks were stone structures built by Inuit people in the arctic regions of Canada and Alaska as signposts to make the way safer for anyone who followed. Their rich symbolism about balance, interdependence and teamwork (see **www.weegates.com/inukshukcorp/InukshukStory.htm**) and their unselfish purpose made them a proud symbol for Canadian people. The inukshuk became the Olympic emblem in the 2010 Vancouver Winter games.

Example

Enlarge the inukshuk in the Vancouver 2010 logo above by a scale factor of $\frac{1}{2}$. Remember, the first step is to measure the dimensions of the shape.

Solution

EXTENSION

What would the scale factor be if you were asked to reduce something by 20%? To enlarge by 20%?

How would you go about enlarging something that does not have straight edges?

SUMMATIVE ASSESSMENT

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

Turning mathematics into fine art

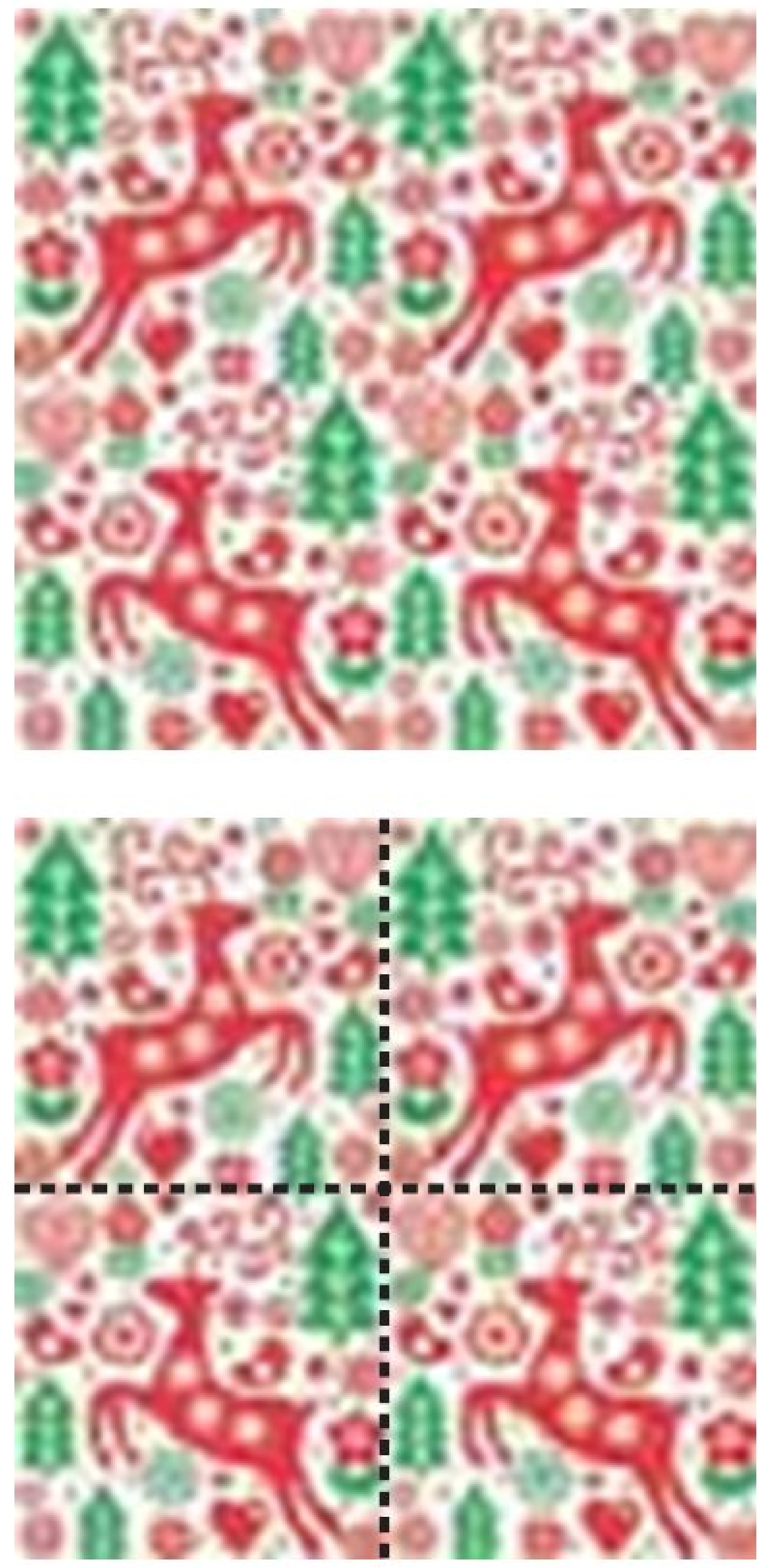
Throughout history, artists across many cultures have used transformations to create harmonious works of art. Islamic artists feel that geometric designs express the perfection of God. Here you can see some examples of such pieces from Europe, the Middle East and Asia.

Your task is to create a personal work of art using transformations. Even the most basic of shapes, transformed and then repeated, can create an intricate and beautiful pattern that is ready for a print, quilt or mosaic.

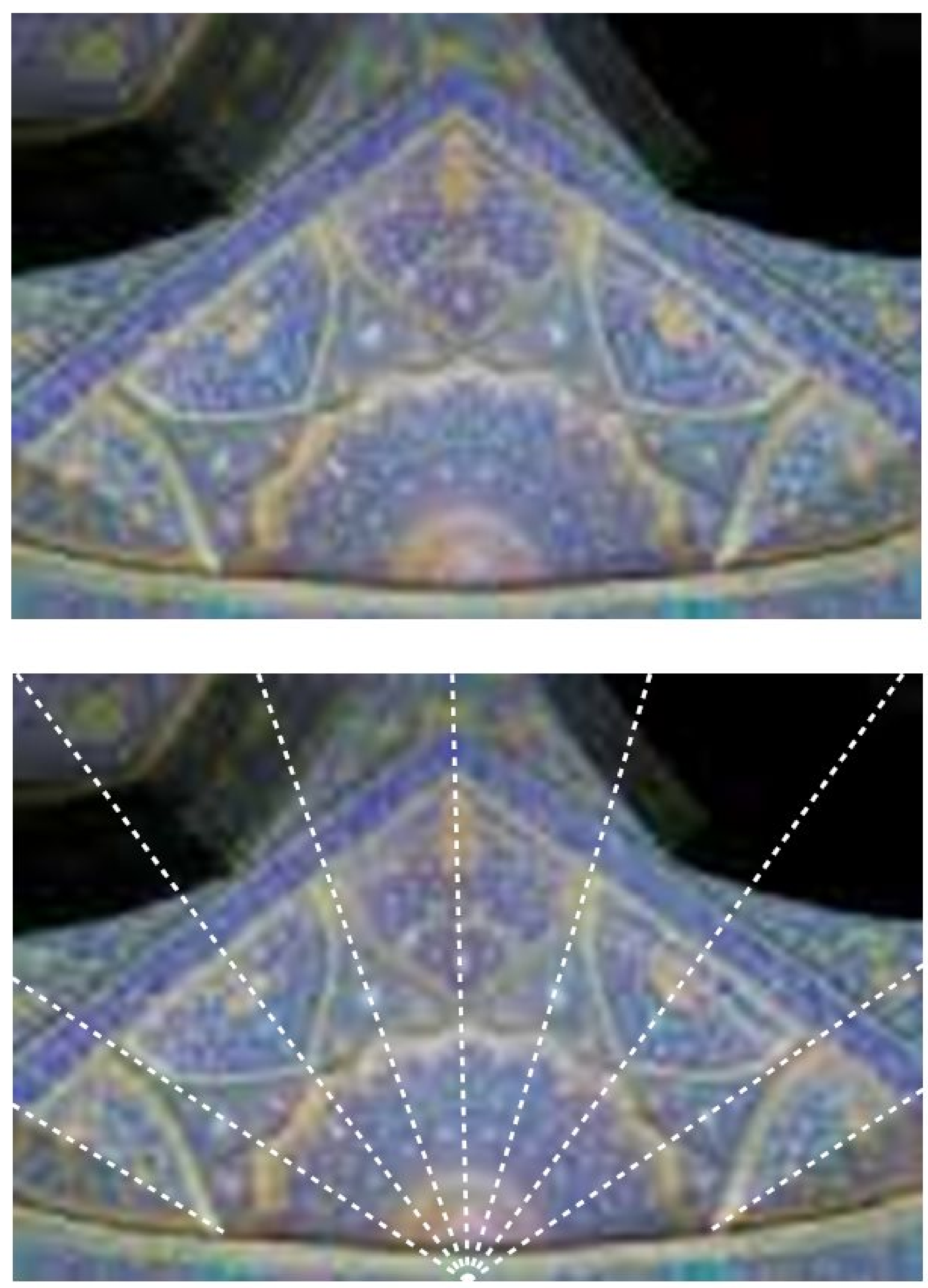
Criterion A checklist

- **Level 1–2: Perform a translation of a basic figure correctly.**
- **Level 3–4: Perform a translation of a basic figure correctly and repeat it into a pattern.**
- **Level 5–6: Perform two or more translations correctly (not necessarily on the same figure, but within the same 'tile') and repeat them into a pattern.**
- **Level 7–8: Perform two or more translations on one or more complex figures which repeat in an infinite pattern.**

What qualifies as a complex pattern? Typically one with multiple figures, or many vertices, or many transformations taking place within the figure itself. Take a look at the examples on the right and on page 126.

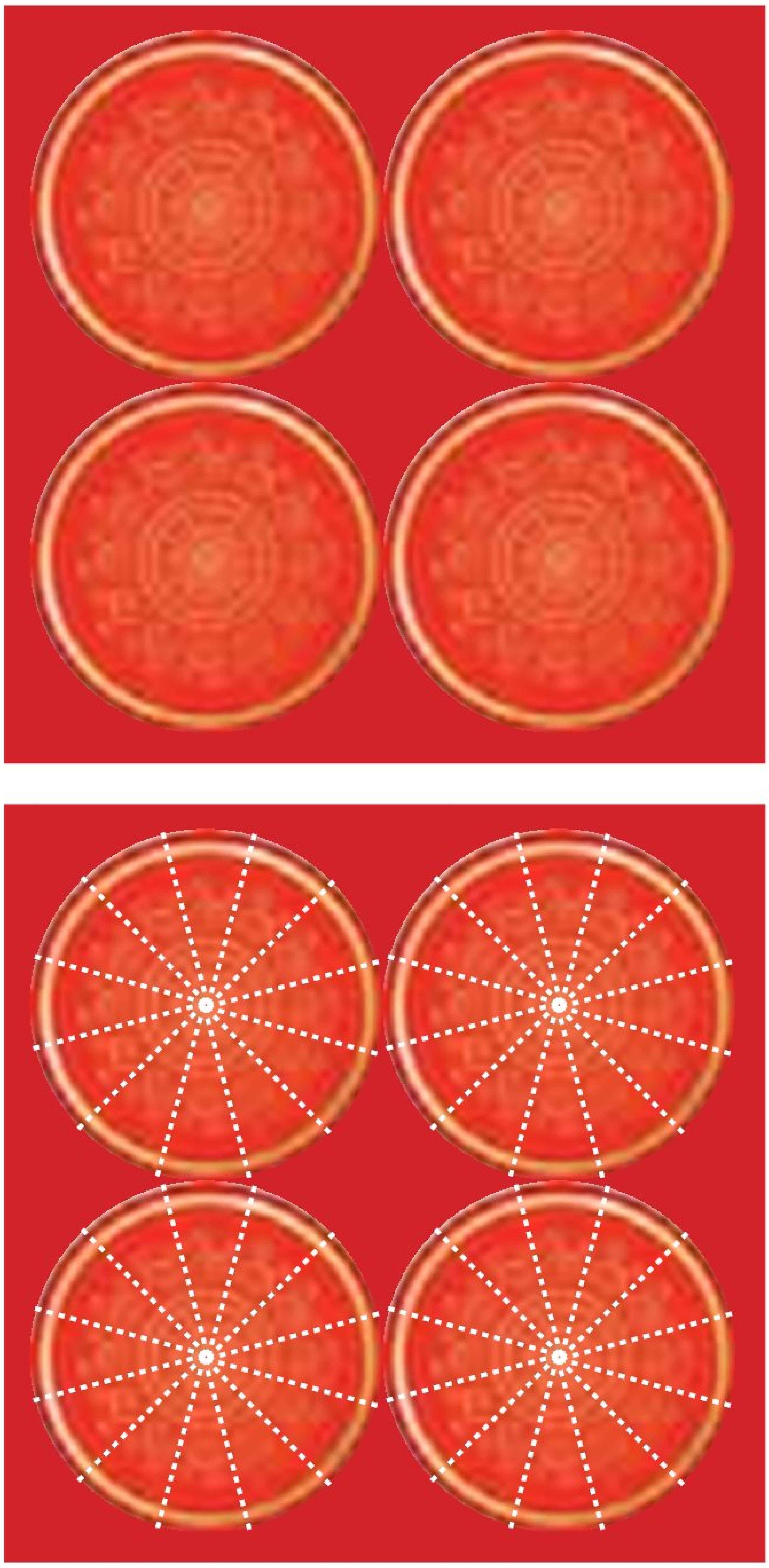


■ Traditional Scandinavian Christmas art. Can you see the reflection and translation?



■ The Shah Mosque, Isfahan, Iran. A series of reflections.

■ Alhambra Palace, Spain. Translations of rotations.



■ Traditional Chinese pattern – 12 rotations of the same image translated vertically and horizontally.

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

Reflection

<table>
<tr><th colspan="7">Use this table to reflect on your own learning in this chapter.</th></tr>
<tr><td>Questions we asked</td><td>Answers we found</td><td colspan="5">Any further questions now?</td></tr>
<tr><td>Factual: How do I turn a table into a graph? How do two-dimensional figures ‘move’?</td><td></td><td colspan="5"></td></tr>
<tr><td>Conceptual: What is mathematical about mirrors? How many ways can you rotate a figure? What qualifies as ‘similar’?</td><td></td><td colspan="5"></td></tr>
<tr><td>Debatable: Where do I stand?</td><td></td><td colspan="5"></td></tr>
<tr><td rowspan="2">Approaches to Learning you used in this chapter:</td><td rowspan="2">Description – what new skills did you learn?</td><td colspan="4">How well did you master the skills?</td></tr>
<tr><td>Novice</td><td>Learner</td><td>Practitioner</td><td>Expert</td></tr>
<tr><td>Communication skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Critical-thinking skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Transfer skills</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Learner Profile attribute(s)</td><td colspan="5">Reflect on the importance of being open-minded for your learning in this chapter.</td></tr>
<tr><td>Open-minded</td><td colspan="5"></td></tr>
</table>

6 How does it all tie together?

Where we are in **space and time** changes what we know, as much as the **form** by which it is **represented**.

CONSIDER THESE QUESTIONS:

Factual: What is binary? Which structures define your hometown?

Conceptual: What is meant by mathematical synonyms? What makes an image 'mathemagical'? How can I beat the system?

Debatable: Is paternity leave a fair benefit? Are primes beautiful?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

IN THIS CHAPTER, WE WILL …

- **Find out** how to expand and factorize algebraic expressions in preparation for next year.
- **Explore** the ideas from previous chapters in new ways, each linked to a related concept.
- **Take action** by creating interesting and innovative displays for our classroom and school, which will contribute to the wider school appreciation of mathematics.

These Approaches to Learning (ATL) skills will be useful …

- Communication skills
- Media literacy skills
- Creative-thinking skills
- Transfer skills

We will reflect on this Learner Profile attribute …

- **Balanced**: We understand the importance of balancing different aspects of our lives – intellectual, physical, emotional – to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

◆ Assessment opportunities in this chapter:

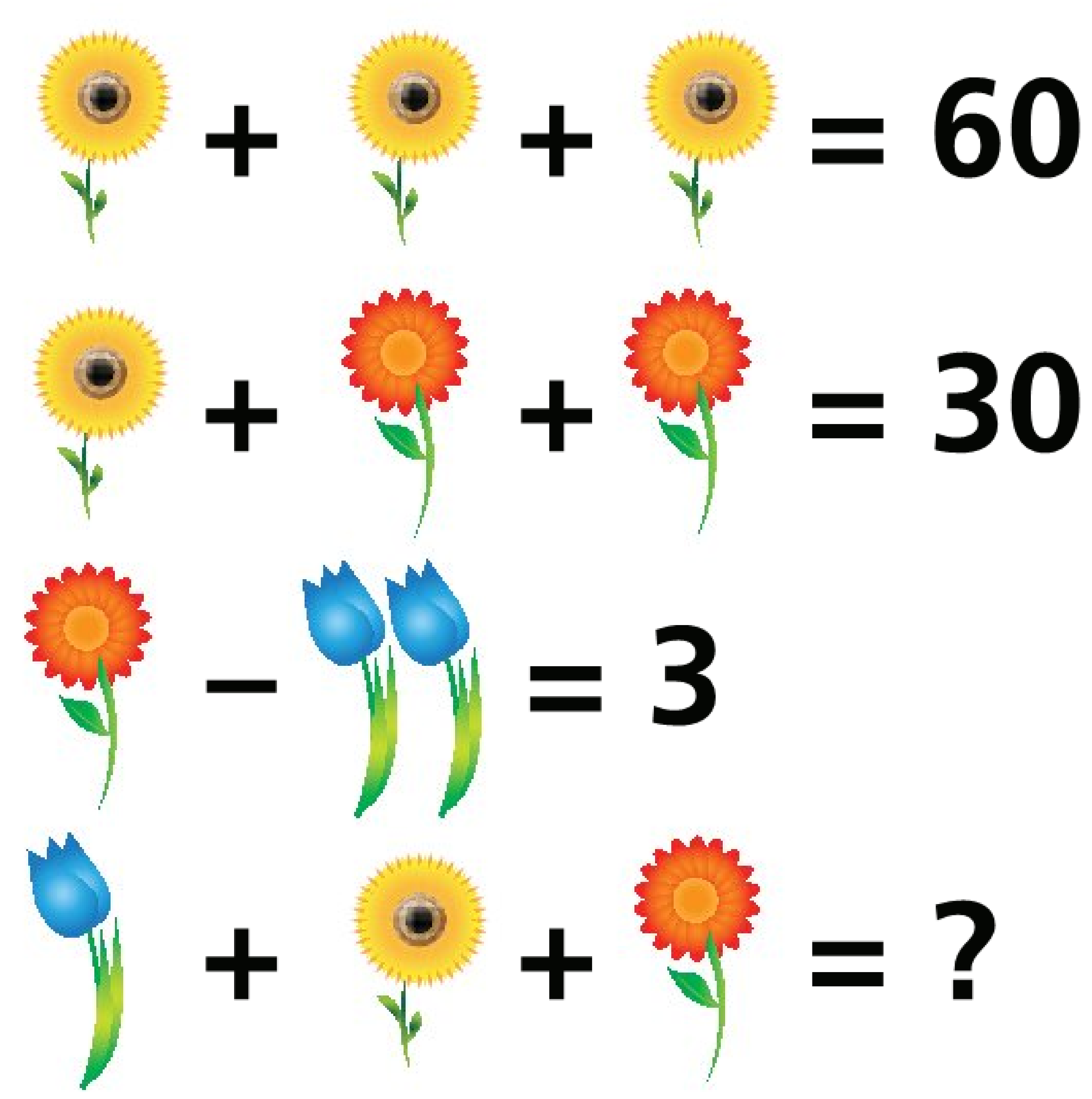
- **Criterion A:** Knowing and understanding
- **Criterion B:** Investigating patterns
- **Criterion C:** Communicating
- **Criterion D:** Applying mathematics in real-life contexts

KEY WORDS

acronym	factor
binary	transmit
expand	

THINK-PAIR-SHARE

Look at this puzzle. Can you solve the final stage? Discuss your working with a partner and share with the rest of the class.



What is meant by mathematical synonyms?

Equivalence: The state of being identically equal or interchangeable, applied to statements, quantities, or expressions.

Can you come up with the smallest possible number to replace x within the expression below, so that it will still work out to a positive number?

$3x - 7$

Remember from last year that algebraic expressions can have coefficients, variables and constants? In this case:

- 3 is the coefficient (the number next to the variable)
- x is the variable (the symbol or letter that can be replaced with any number)
- –7 is a constant (a stand-alone number that will never change).

Here, the variable is x and we can replace it with any number we like. Let's use trial and error to see what number will yield the smallest positive result:

$3(5) - 7 \quad = 15 - 7 \quad = 8$

$3(4) - 7 \quad = 12 - 7 \quad = 5$

$3(3) - 7 \quad = 9 - 7 \quad = 2$

$3(2) - 7 \quad = 6 - 7 \quad = -1$

By substituting different numbers into the variable, we were able to determine that $x = 3$ is the lowest number that will still give us a positive result.

Last year, we learned about simplifying. So we understand that two fractions can have an equal value, but one is in simplest (and most easy to understand) form. We simplified fractions like this:

$$\frac{8}{16} = \frac{1}{2}$$

Then, we simplified algebraic expressions by collecting like terms. Again, both expressions are equal, but one is shorter, simpler and easier to understand:

2 + 3 = 5

Finally, when substituting, there was another opportunity to simplify:

2  – 4, where  = 8

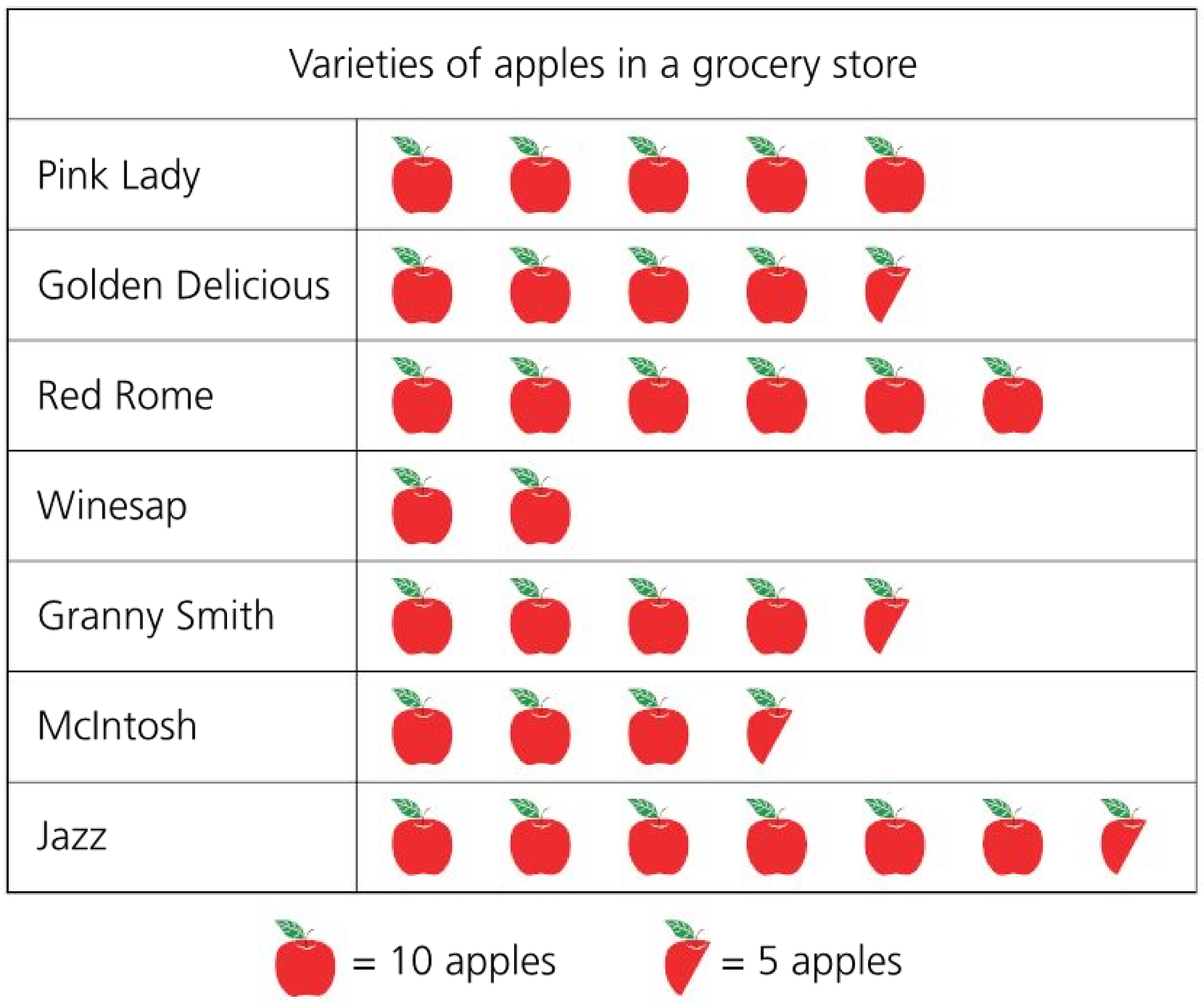
= 2(8) – 4 (substitute)

= 16 – 4 (simplify)

= 12

DISCUSS

How are pictographs like substitution? Refer to the example below to help move your discussion forward.



PRACTICE EXERCISE

1 Substitute '1 Chihuahua is equal to the number 3' into this expression and simplify!

2  + 1

PRACTICE EXERCISE

Simplify each expression. In each case there is something important to remember – what is it?

1. 4 + 3 ^{2} – 7 – 2 + 6 ^{2}
2. × × × ×
3. 3 (7)

DISCUSS

Can we simplify an equation? Why or why not?

Links to: Languages

In many languages and cultures, there are commonly used expressions or 'sayings' which offer advice or have underlying meanings. One example of two equivalent expressions is 'I heard through the grapevine', and 'a little birdie told me'. These both have the same meaning – that you heard a rumour and are either spreading it (shame on you!) or seeking confirmation (fact checking before forming an opinion is important). Can you think of other expressions in English or in any other language that have the same meaning?

An equation is a collection of two expressions that are **equal**. In an equation, we can simplify one expression (one side of the equals sign) or the other, but that's as far as we can take it. Still, this makes solving an equation much easier:

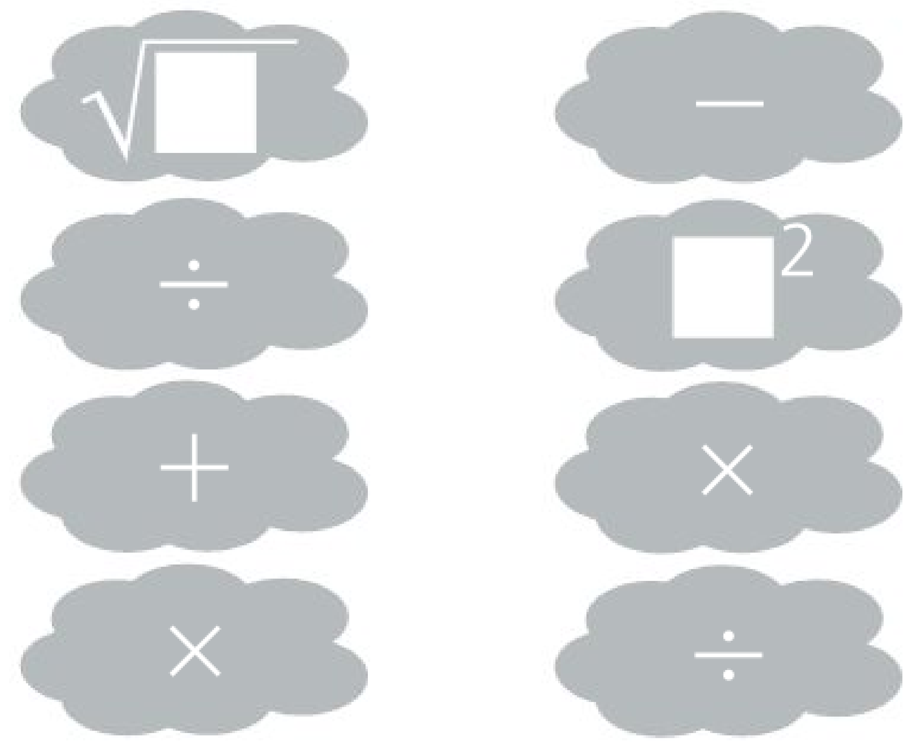
$15a + 3a - 8a = -5$

$10a = -5$

Where do we go from here? Let's review inverse operations from *Mathematics for the IB MYP 1*.

WHAT MAKES YOU SAY THAT?

Match the operation with its inverse by drawing lines between each operation and the exact opposite operation (the inverse).



So going back to $10a = -5$, to solve we will need to divide by 10:

$$\frac{\cancel{10}a}{\cancel{10}} = -\frac{5}{10}$$

$$a = -\frac{1}{2}$$

Let's look at another example.

Simplify, then solve:

$8a - 3 = 5$

Since we have × 8 and – 3, using inverse operations we will + 3 and ÷ 8. Or is it divide by eight and add three? Does it make a difference? Try both and see if you arrive at the same answer.

Not sure which answer to choose? Substitute each back into the original equation to see which one is correct. Also, recall from *Mathematics for the IB MYP 1*:

> *It is very important to identify which operation to do first.* ***Always undo addition or subtraction before multiplication or division.*** *This* ***undoing*** *takes the* ***reverse*** *order of operations compared to those you learned before. (Remember your acronym? Was it BEDMAS, or something else?) Think of this undoing as retracing your steps through your acronym.*

This means that to solve our equation, we need to add first, then divide:

$8a - 3^{\ +3} = 5^{\ +3}$

$8a^{\ \div 8} = 8^{\ \div 8}$

$a = 1$

Let's substitute $a = 1$ into the original question to check our answer:

$8(1) - 3 = 5$

Looks good!

Go back to the puzzle on page 129. Describe when and how substitution, simplification and solving were each used to arrive at a solution.

EXPANDING

In Canada, one-dollar coins are called 'loonies' after the common loon, a bird, pictured on them, and two-dollar coins are referred to as 'toonies', because Canadians like to rhyme (and due to their two-dollar value). At an amusement park, a father gives each of his five children a combination of loonies and toonies to spend however they like. Let's write an expression for the value of money each child was given.

Let   represent the number of loonies given.

Let   represent the number of toonies given.

Each child has money to the value of   + 2 .

People are often confused by this type of expression. This does not translate to 'one loonie and two toonies' – the quantity of each coin is already represented by   and . The coefficients come from the **value** of each of these coins. The loonie is a dollar, so its coefficient is 1. The toonie is worth two dollars, so its coefficient is 2. If each child had, say, three loonies and one toonie (in other words,   = 3 and   = 1), that would give them:

$3 + 2(1) = 5$ Canadian dollars in total

But we don't know how many of each they received, so let's go back to the expression   + 2 . If the father gave the same amount to each of five children, we can find out how much he spent by multiplying this entire expression by 5:

5(+ 2)

This is a much cleaner way of writing:

(+ 2) + (+ 2) + (+ 2) + (+ 2) + (+ 2)

which is the 'expanded form'. However, what if we want to know exactly how many loonies and toonies the father started out with? We could collect like terms from the messy expanded expression above:

  + 2 +   + 2 +   + 2 +   + 2 +   + 2

=   +   +   +   +   + 2 + 2 + 2 + 2 + 2

= 5 + 10

Alternatively, we could look at the tidy version and take a short cut, sometimes referred to as the **rainbow method** because of the arrows above the term outside the brackets:

5(+ 2)

Multiply along each arrow:

5 ×   + 5 × 2

= 5 + 10

Example

Use the rainbow method to expand this expression.

$9f(5a - 2b + 6c^2 + 12d)$

Solution

$9f(5a - 2b + 6c^2 + 12d)$

$= 45af - 18bf + 54c^2f + 108df$

We can have two expressions that are equal to one another but take on different forms, such as:

$2(x + 4)$ and $2x + 8$

In this case, the expressions are equivalent or identical for any value assigned to x.

We can also have an equation showing the manipulation of the number being equal to a different manipulation, for example:

$2x + 4 = 3x - 1$

Here the expressions are equal to each other for one specific value of x. In this case, it is true only if $x = 5$. If x is anything else, this equation is not true.

An expression with a single term is referred to as a monomial. What do you think binomial and polynomial mean? Use the examples below to help you:

monomial	$4a$
binomial	$4a + 7$
polynomial	$4a + 7$ or $4a + 7 - 5$ or $4a + 7 - 5 - 2b$ (and so on ...)

Either way, this act of taking out the brackets is known as **expanding**. It is a key step if you want to simplify a polynomial that is being multiplied by something outside the brackets.

Remember that algebra can be used to express relationships.

Imagine you and a friend both won the lottery and it made you twice as happy as you were before. You both started out at different levels of 'happy', but both were doubled.

🙂 + 😁	Before
2(🙂 + 😁)	Because you won the lottery
2🙂 + 2😁	Now

Now imagine one of you is a morning person, awake and excited to start the day – while the other is still in bed pulling the covers over their heads. Someone pulls back the curtains and opens the window to let the sun come in. This quadruples everyone's feelings, whether good or bad.

🙂 – ☹️	Before
4(🙂 – ☹️)	Because the sun came up
4🙂 – 4☹️	Now, happy is even happier and sad is even sadder!

FACTORIZING

Just as all the operations we have studied (+, −, ×, ÷, squared, square root) have inverses, expanding also has an inverse. It is called factorizing. Actually, we were doing this even before we learned about algebra!

- Expanded form: 2 + 2 + 2 + 2
- Factored form: 4(2) or 4 × 2

ACTIVITY: Fascinating factors

ATL

- Critical thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions

Expanded form	Factored form
$4x + 4$	$4(x + 1)$
9 + 9 − 9	9(+ − 1)
$12x + 3y$	$3(4x + y)$
$5x + 7x^2$	$x(5 + 7x)$
$10c + 10$	

Each row contains two equal expressions – one in factored form, and one in expanded form. Do you see a relationship between the expressions in the 'expanded form' column and their associated expressions in the 'factored form' column? How could you go from factored to expanded, or from expanded to factored? Test your theory on the expression in the final row.

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion B: Investigating patterns.

As we have already seen, to go from factored form to expanded form, we multiply the term outside the brackets with each term inside the brackets. This is useful for simplifying expressions and solving equations. To reverse this, and go from expanded form to factored form, we factorize. Since expanding meant multiplying, it is only natural that factorizing should involve dividing. This is useful in writing concise expressions and in future courses when working with algebraic fractions.

- **Step 1**: Find the highest common factor.
- **Step 2**: Divide each term by the highest common factor and write the quotients inside a pair of brackets. Write the common factor on the outside.
- **Step 3**: Check your answer by expanding it out again – mentally, if not on paper.

Example

Factorize these expressions:

1 $6a + 9$

2 $16x + 76x^2$

3 $25\ ^2 - 100\ ^2$

Solution

Step 1: First find the common factors in each pair.

1	$6a$	9	**common factor = 3**
2	$16x$	$76x^2$	**common factor = 4x**
3	$25\ ^2$	$-100\ ^2$	**common factor = 25**

Step 2: Having found the common factor in each case, remove it from each pair. This is known as 'taking out' the common factor.

1	$6a = 3(2a)$	$9 = 3(3)$
2	$16x = 4x(4)$	$76x^2 = 4x(19x)$
3	$25\ ^2 = 25\ (\ )$	$-100\ ^2 = 25\ (-4\ )$

So, the answers are:

1 $3(2a + 3)$

$3(2a + 3)$

Common factor

$6a \div 3$ $9 \div 3$

(Divide each term by the common factor)

2 $4x(4 + 19x)$

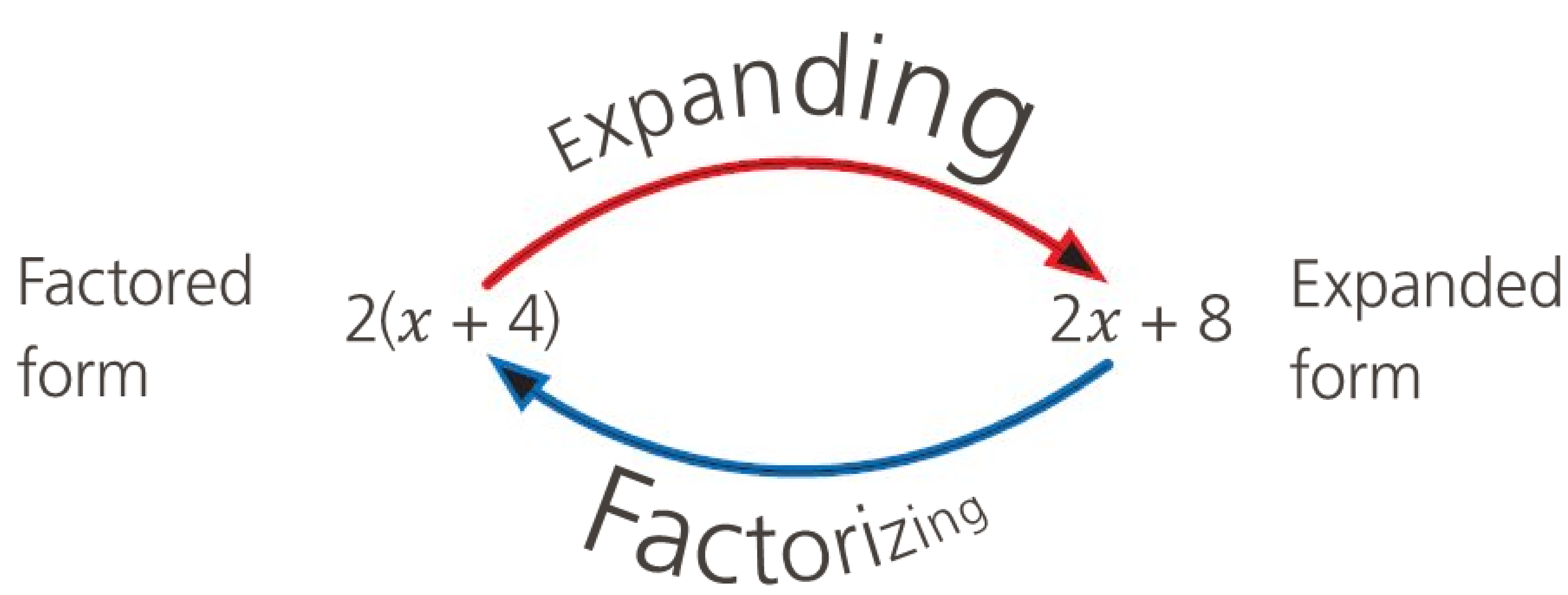
3 $25\ (\ -4\ )$

Step 3: How can we check if our answer is correct? Since factored form is the inverse of expanded form, we need to expand our answer and see if it will match the question.

1 $3(2a + 3) = 6a + 9$ So our factored form is equal to the expanded form.

2 $4x(4 + 19x) = 16x + 76x^2$

3 $25\ (\ -4\ ) = 25\ ^2 - 100\ ^2$

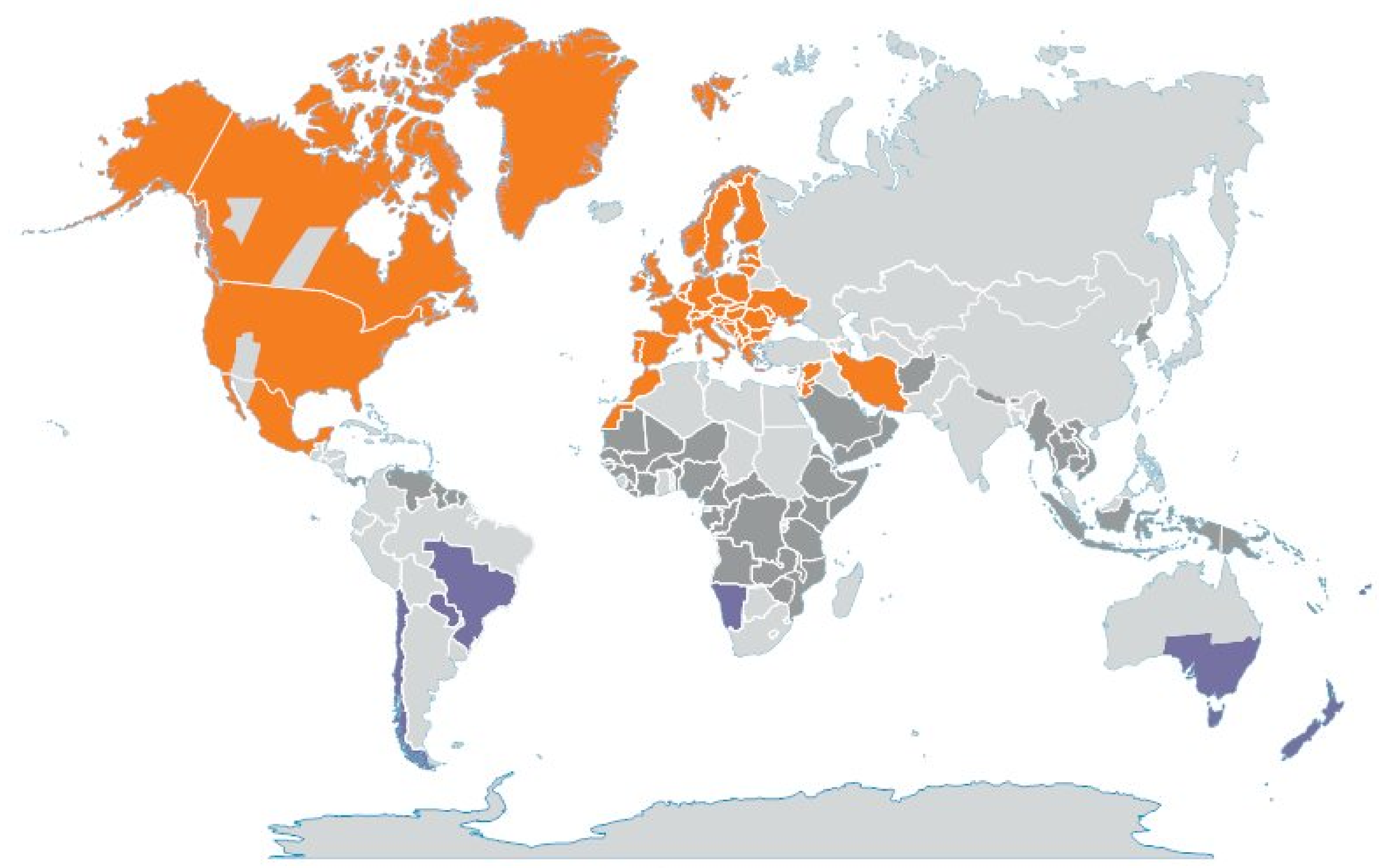


What is binary?

Change: A variation in size, amount or behaviour.

Time is a funny thing. It changes depending on where you are because of something called **time zones**. It changes in some places twice a year because of a system called daylight-saving time. This is where all clocks in that region are set forward an hour in spring and set back an hour in autumn, or fall. You saw a graph detailing the change in times over the year in Chapter 2, page 51.

This map shows daylight savings across the globe. It changes depending on your orientation in space, but also time, as some regions formerly used it but don't anymore.



Don't forget that some regions use the 24-hour clock, while others do not. To change from a 24-hour clock to a 12-hour clock (using am and pm), you must subtract 12 from the hours total. How do you change a time into 24-hour clock? What are the advantages and disadvantages of each system?

Links to: Individuals and societies (History)

Interestingly, time has a base 60 – 60 minutes in an hour and 60 seconds in a minute – which is very different from the base 10 system we use in most of the rest of our lives. In fact, a decimal system for time was introduced in France during the French Revolution. The day was divided into 10 hours or a 100 minutes. Each minute was changed to 1000 seconds, with a proposed week of 10 days too. It clearly didn't last.

Another base system for numbers is the binary system. Binary is a way of representing numbers as a combination of powers of 2. It uses the fact that we know:

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$ and so on

So if I wanted to represent 26 in binary, I would analyse how many of these 'powers of 2' combinations I would need.

We know that:

$26 = 16 + 8 + 2$

which is the same as:

$26 = 2^4 + 2^3 + 2^1$

Hint

Start by taking out the largest power of 2 that you can find and move down through the powers until you have exhausted all the possibilities.

We could express this in a table showing possible powers of 2. We would enter a number 1 in a cell to show that we would include that power of 2 in the sum and 0 would mean it is not included. This is the table for the number 26.

Power of 2	2^5	2^4	2^3	2^2	2^1	
	0	1	1	0	1	= 26

So does the number 26 changed into binary look like this: 1101?

No! The system we have devised only shows even numbers – there would be no way to show an odd number. To include all natural numbers, we can make use of the fact that any number to the power of 0 is equal to 1 ($2^0 = 1$). So, we need another column in the table!

Power of 2	2^5	2^4	2^3	2^2	2^1	2^0	
	0	1	1	0	1	0	= 26

The correct binary equivalent of the number 26 is 11010.

Example

Change the number 17 into binary.

Solution

17 = 16 (the highest power of 2 we can find in the number) + 1 (this is all that is left)

Power of 2	2^5	2^4	2^3	2^2	2^1	2^0	
	0	1	0	0	0	1	= 17

So the number 17 in binary is 10001.

PRACTICE EXERCISE

1 Change these numbers into binary form.

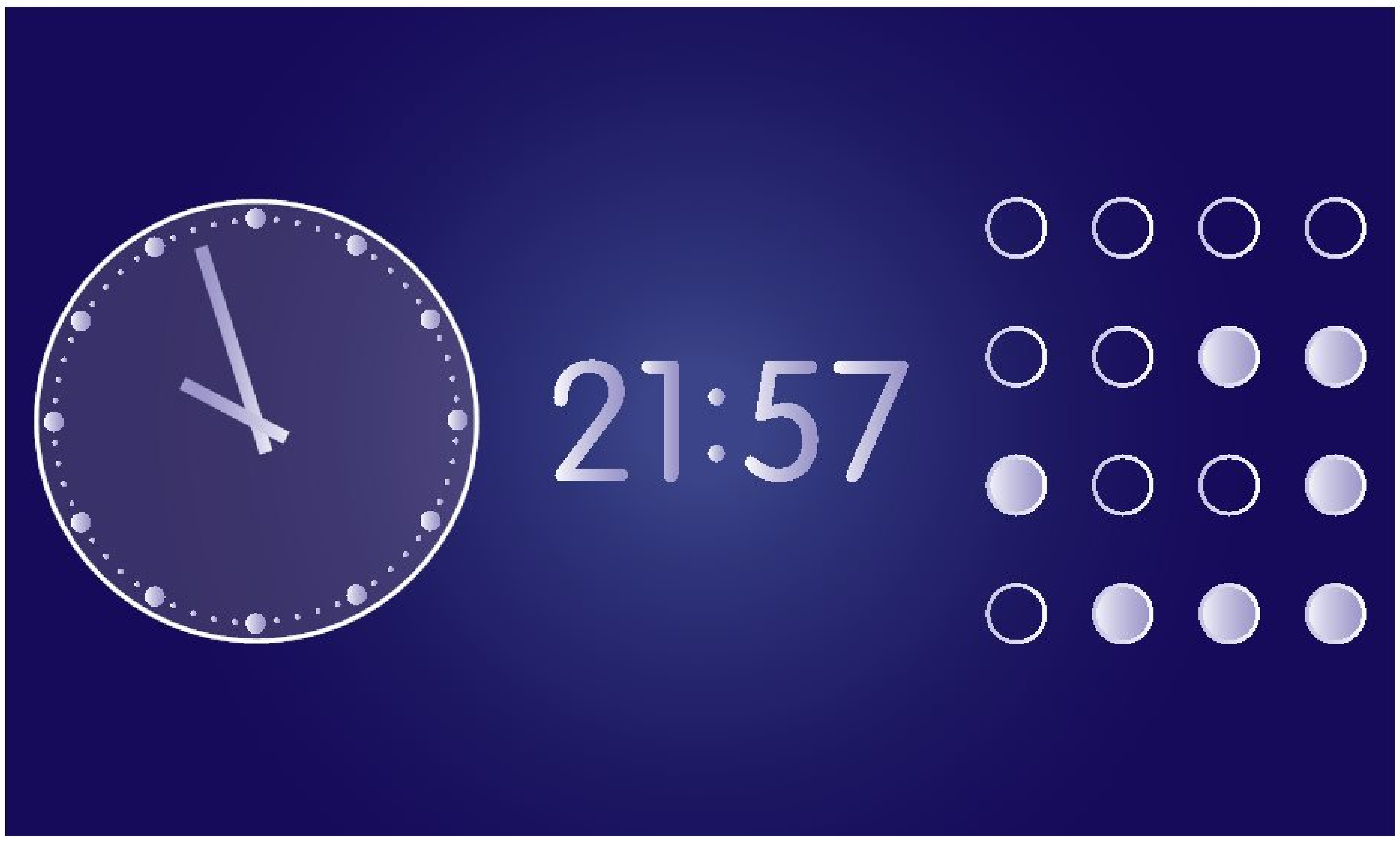
2 Change these numbers out of binary form.

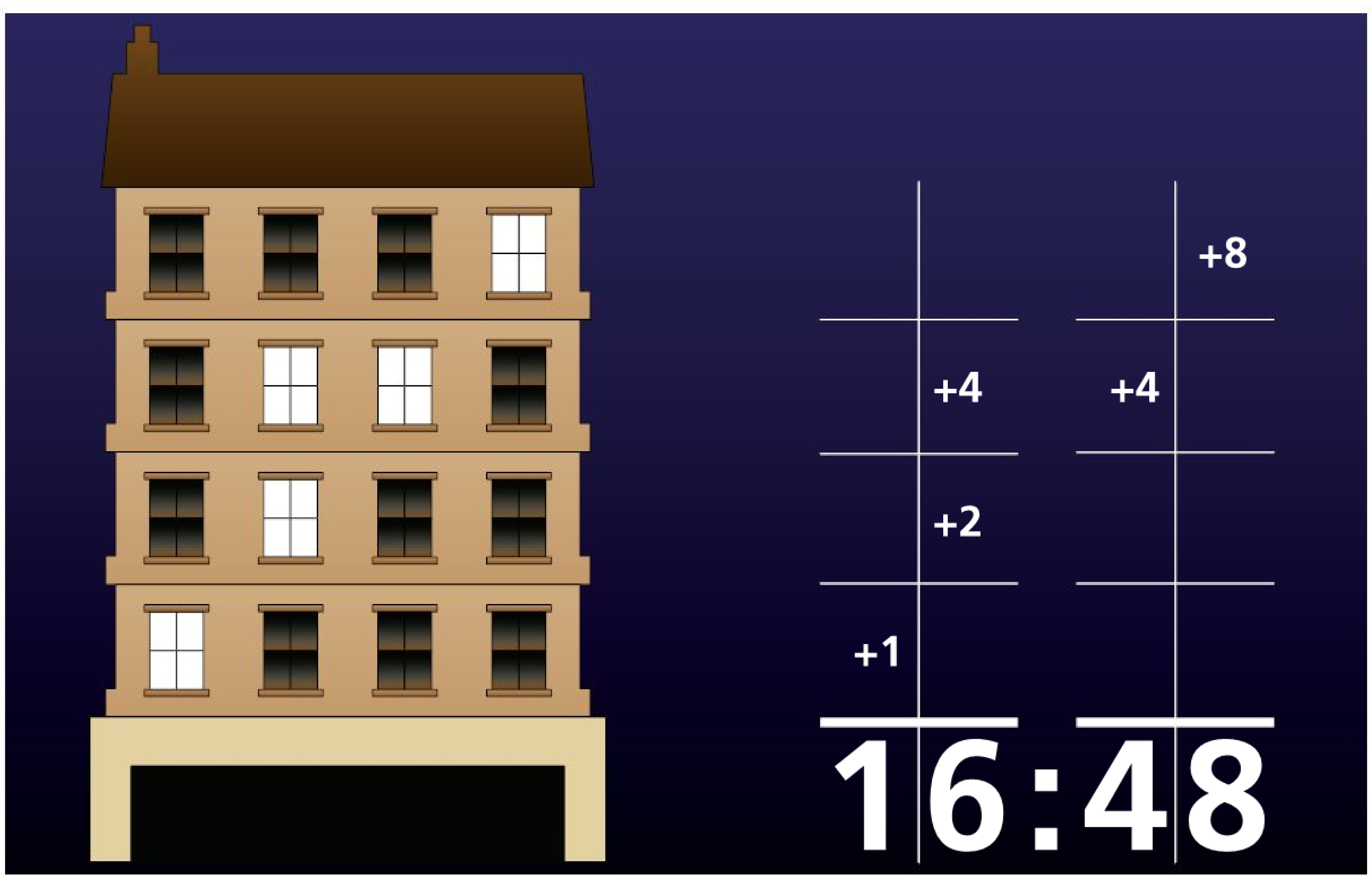
1		2	
a	9	a	001
b	7	b	101
c	4	c	111
d	19	d	1111
e	15	e	1010
f	21	f	1101
g	49	g	1001
h	40	h	10101
i	100	i	1110110

Kickstarter is an example of a crowdfunding website. Inventors put up their ideas, hoping that people will fund a little part of their project and, in return, the investors often get a final product as a result.

This Kickstarter project (**www.kickstarter.com/projects/1050329560/the-city-clock-paris-design**) shows a small clock which uses the idea of binary and the design features of an apartment block in Paris to create a new product. It was a popular campaign and was fully funded.

But how does it work? There are four columns of lights. The first two columns show the hours and the second two columns shows the minutes, like this.





PRACTICE EXERCISE

Now that you are familiar with the 24-hour clock you should be able to change these clock displays into 24-hour (and 12-hour) time.

1

2

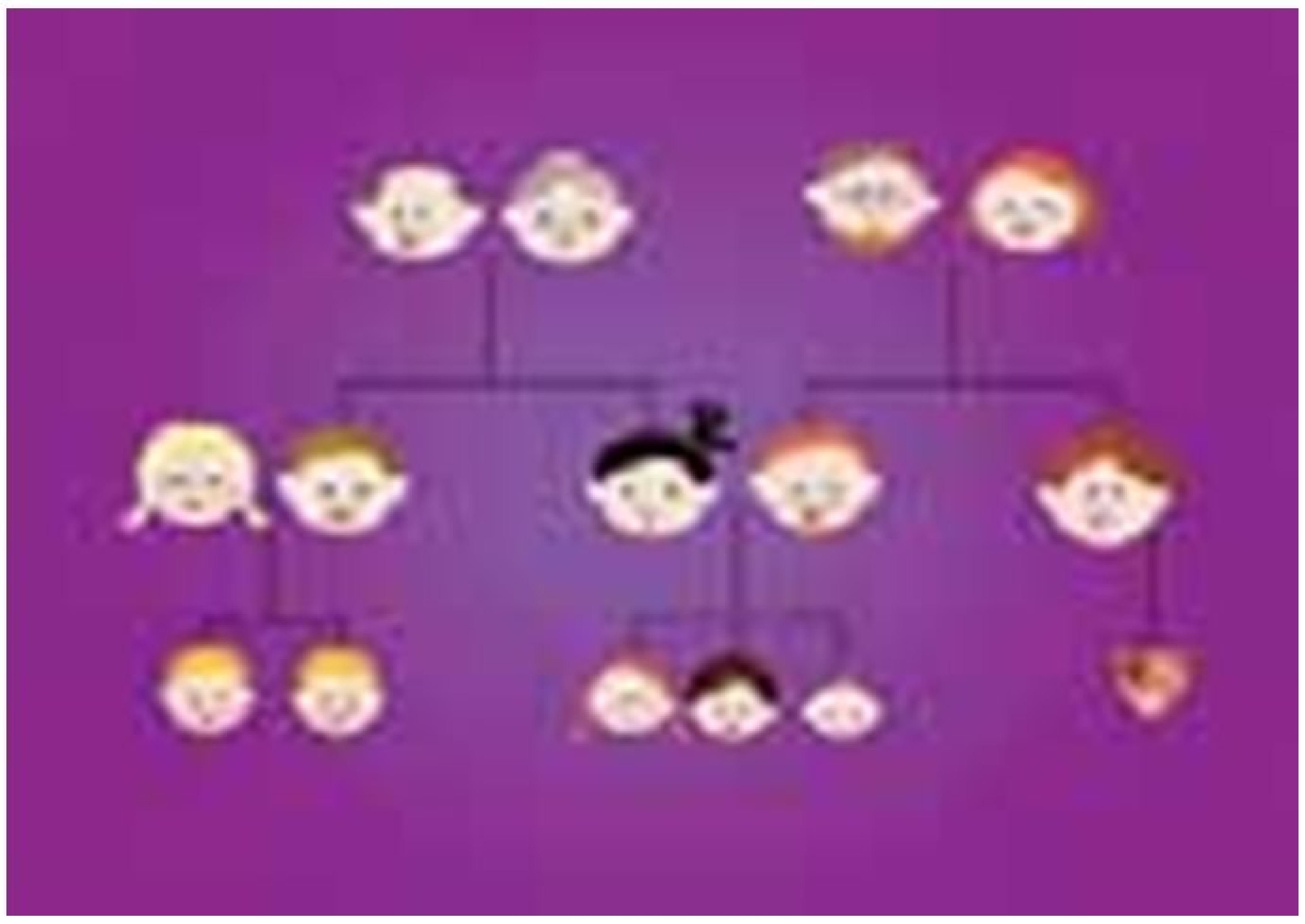
3

4

How do we generalize patterns in numbers?

Generalization: A general statement made on the basis of specific examples.

If we take the sequence of numbers 2, 4, 8, 16, 32, … we can see that each number is double the last one. This is a general statement based on the specific examples.



If we lay out the sequence like this:

$2 = 2^1$

$4 = 2^2$

$8 = 2^3$

$16 = 2^4$

$32 = 2^5$

we can see another general pattern emerge in the powers or exponents.

But look at this:

$2 = 2^1$

$4 = 2^2 \quad = 4^1$

$8 = 2^3$

$16 = 2^4 \quad = 4^2$

$32 = 2^5$

$64 = 2^6 \quad = 4^3$

What new pattern can you see now? Justify your new pattern by testing for higher numbers.

With the numbers in the sequence from 2 to 1024, investigate if there are other similar patterns but for larger numbers. Take the sequence even higher, if you would like to.

Is paternity leave a fair benefit?

Justification: Valid reasons or evidence used to support a statement.

The concept of shared parental leave (SPL) is new to some countries, and already common to others. It enables two parents to divide the time away from work that is offered to care for their newborn or newly adopted child. What do you think a government might be trying to achieve in introducing SPL? Listen to this podcast to hear what the sentiments are about SPL in the UK:

www.bbc.co.uk/programmes/b075thgl

According to the podcast, what percentage of men was recorded as wanting to take parental leave from work? Did the media do enough fact-checking before airing the story? How do you know?

In the podcast, the producer says, 'We heard experts ruminating about the reasons for why men wouldn't be taking it up'. What sorts of justifications might people have come up with to account for such a small number? Does it sound like anyone brought up the **real** reason the percentage was so low?

What was wrong with the sampling method? Estimate the actual percentage that should have been reported.

If you were to conduct such a survey, how would you choose a sample to best represent the population of new fathers? Be sure to account for adequate sample size. Justify your response.

According to the podcast, why is this survey harmful? Can you think of any additional reasons?

Do you think SPL would be harmful or beneficial to your community? Or does it already exist? If so, is it being well-used? Look up the name of your local representative and write a letter to them expressing your opinion. Prepare arguments to justify your position, researching the actual numbers from proper sampling methods as well as comparing success rates in other countries. Some things to consider: How might SPL change a child's upbringing? How might it change or reinforce society's stereotypes about men, women and same-sex partners? How might childless employees feel about the benefit? What percentage of men are eligible for the benefit? Are there cultural sensitivities to consider? Be sure to include qualitative and quantitative justifications in your letter.

Try this search term: **[country name] paternity leave statistics**.

Here are a few articles to help you get started:

- **www.slate.com/blogs/xx_factor/2013/04/03/paternity_leave_in_iceland_helps_mom_succeed_at_work_and_dad_succeed_at.html**
- **www.businessinsider.com/countries-with-best-parental-leave-2016-8?r=UK&IR=T/#sweden-3**
- **www.thebump.com/a/paternity-leave-for-men**
- **http://money.cnn.com/2016/02/17/pf/working-parents-paid-leave/?iid=EL**
- **www.dol.gov/asp/policy-development/PaternityBrief.pdf**
- **http://money.cnn.com/2016/06/16/pf/parental-leave-fathers/index.html**

What makes an image 'mathemagical'?

Measurement: A method of determining quantity, capacity or dimension using a defined unit.

Activity: Mystic roses

■ ATL

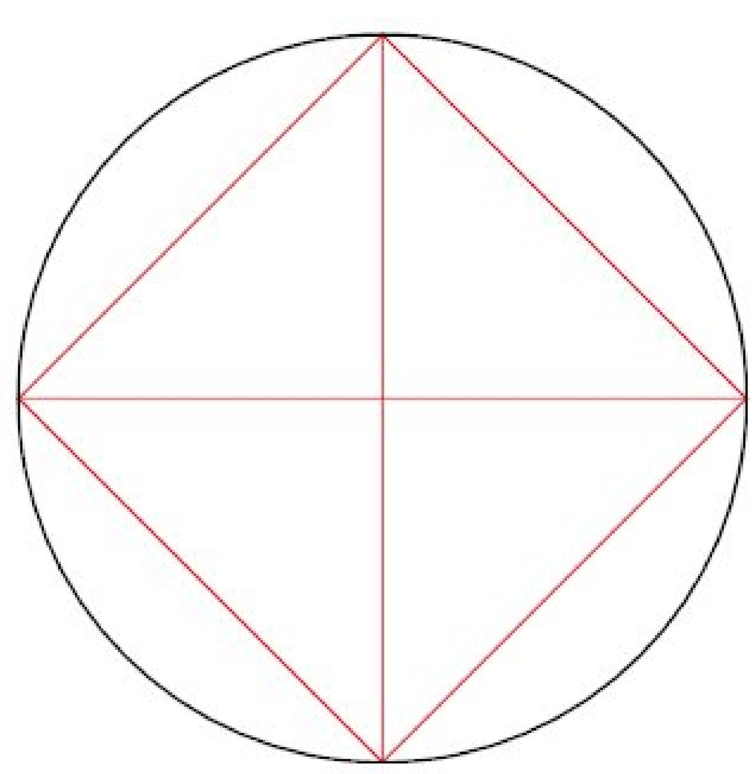
- Creative thinking: Generating novel ideas and considering new perspectives

In this activity, you will demonstrate that you have mastered some important manual skills. You will show that you can:

- **construct a circle using a pencil and compass**
- **find and mark points on a circle, at equal distances**
- **connect these points with a pencil and a ruler**
- **carry out all these skills, accurately, carefully and in a precise way.**

1 **Construct a circle with an appropriate radius. You will be drawing lines inside the circle, so make sure there is enough space to see any patterns.**

2 **Mark four equally spaced points on the circle's circumference. Connect each of these points to the others.**



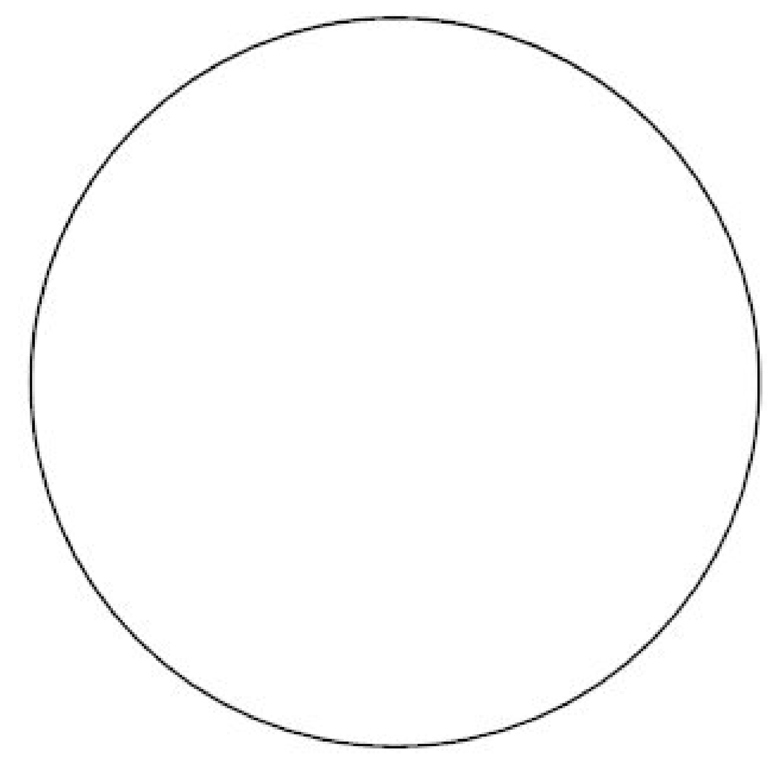
You should see a shape like this one.

How accurate was your shape? Did you have any wobbly lines or were the points not quite perfect? Try it again to master this before we move to more and more points on the line. How many line segments did you draw in total?

3 **Now, draw another circle. Mark eight points on your circle. Connect each of these points to the other. What does this shape look like? How many line segments did you draw?**

4 **Four points and eight points are relatively easy to mark. Now we will try some harder divisions of the circle. Copy and complete a circle with these points shown:**

- **5 points**
- **6 points**
- **7 points**



5 **How did you space those points equally? Did you estimate? Or use a compass or a protractor? Remember that the goal of this activity is to be precise!**

6 **Now, in all three diagrams, connect each of the points to every other. How many connecting line segments did you draw in each diagram? Copy and complete the table.**

Number of points	Number of line segments

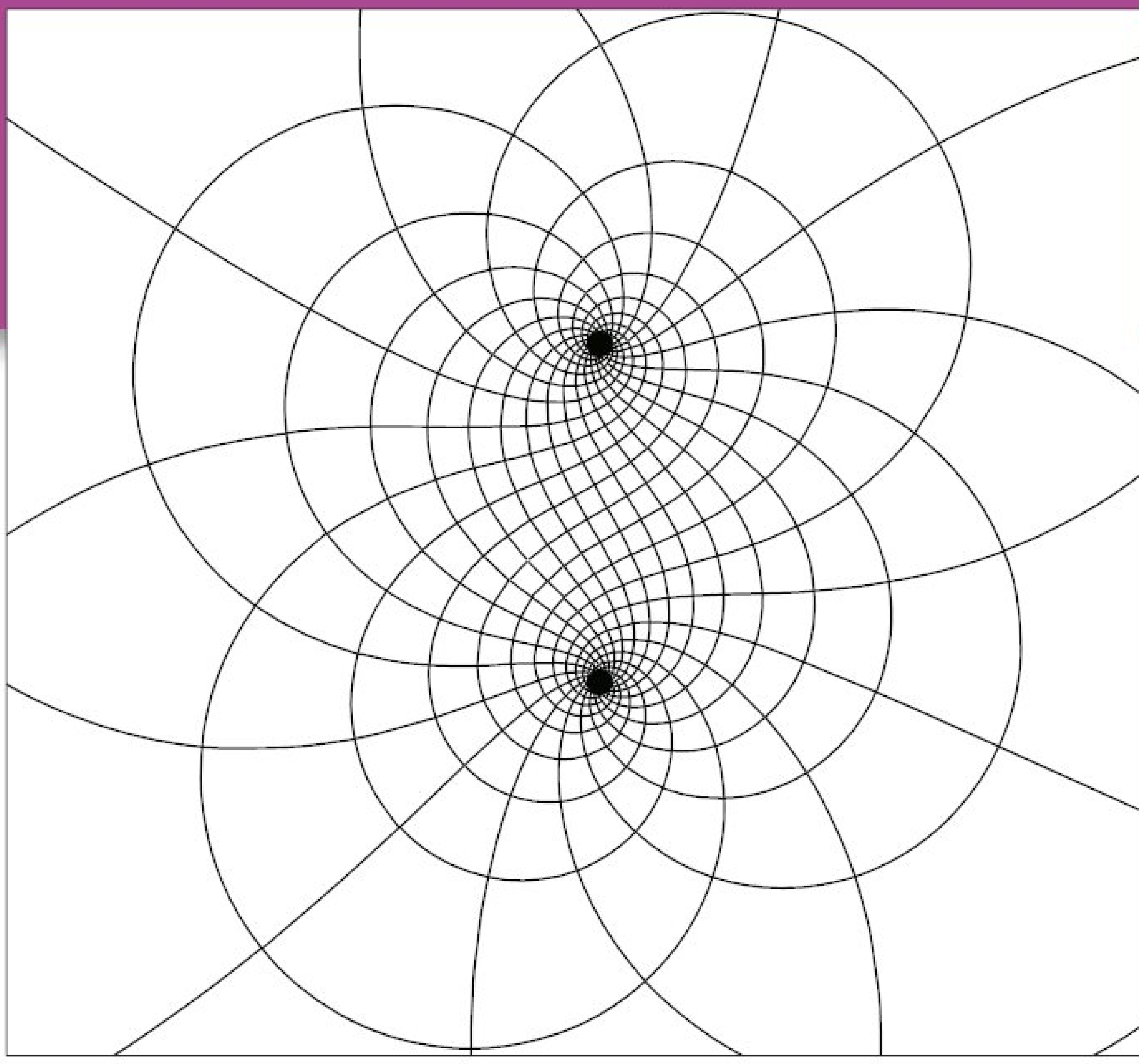
7 **Now it is time for some really challenging mystic roses! You will need to make your circles even larger for these ones. Draw circles with 10 points and 18 points.**

8 **Describe any patterns you see. Can you find a general rule to calculate the number of line segments for any given number of points on a circumference?**

9 **How can we measure any number of similar connections (connecting line segments)? What if, instead of points, we are connecting people and the connections are handshakes, friendships or competitive games in a tournament? Lots of questions make use of this idea and it can help us understand networks better.**

◆ Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns.

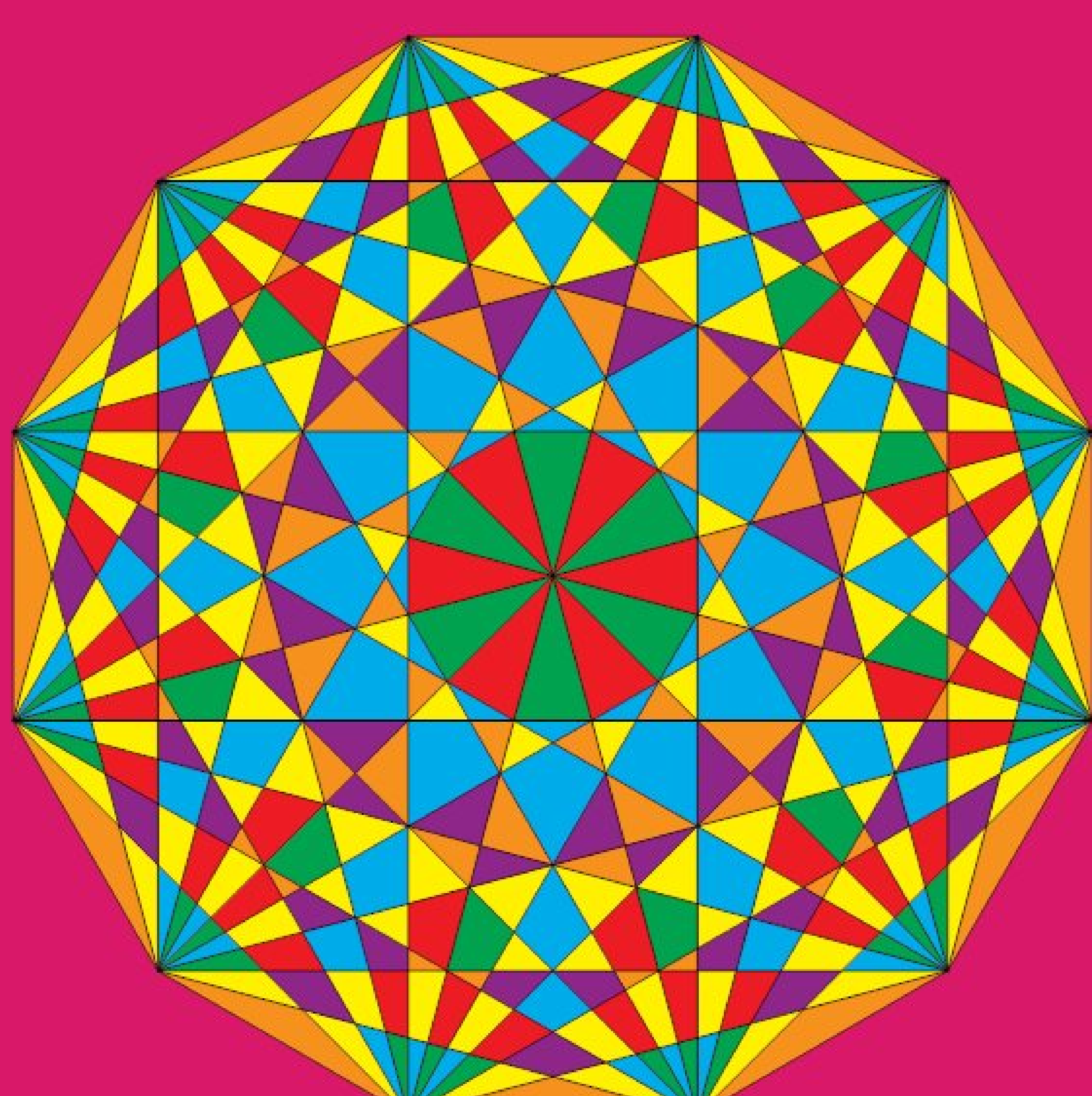


This image comes from a colouring book called *Colouring Adventures in Numberland*.

Now visit **https://nrich.maths.org/6703** to draw these shapes quickly using technology. There are also some great questions to consider there, as well as a final challenge to test that you really understand this idea.

! Take action

- ! Why not combine the ideas of the mystic rose and the popular phenomena of mindfulness and adult colouring books to create your own image? The images from your class could make a great end-of-year display or backdrop to a graduation or assembly. They could be projected on a wall or enlarged to make fun wallpaper.



MEET A MATHEMATICIAN: ALEX BELLOS (1969–PRESENT)

Learner Profile: Balanced

Let's let Alex tell you about himself:

> Hi, I'm Alex Bellos. I write and I talk. Usually, about mathematics. Sometimes, about football and Brazil. My popular science books – *Alex's Adventures in Numberland* and *Alex Through the Looking-Glass* – are both bestsellers and have been translated into more than 20 languages. (In America they have the titles *Here's Looking at Euclid* and *The Grapes of Math*.) I write a maths blog and a puzzle blog for *The Guardian* and frequently speak on maths at conferences, festivals, in schools and in companies. My maths blog won the prize for best science blog 2016 awarded by the Association of British Science Writers. *Football School*, a kids' book I wrote with Ben Lyttleton, was runner-up in the 2017 Blue Peter Book Awards.
>
> My latest books are *Can You Solve My Problems?*, a compendium of almost 200 maths and logic puzzles mixed with historical, biographical and mathematical background, and the mathematical colouring book *Visions of Numberland*, a collaboration with Edmund Harriss.

Source: **www.alexbellos.com/**

Alex's wide range of skills, talents and interests shows how balanced mathematicians can be when they look outside the classroom.

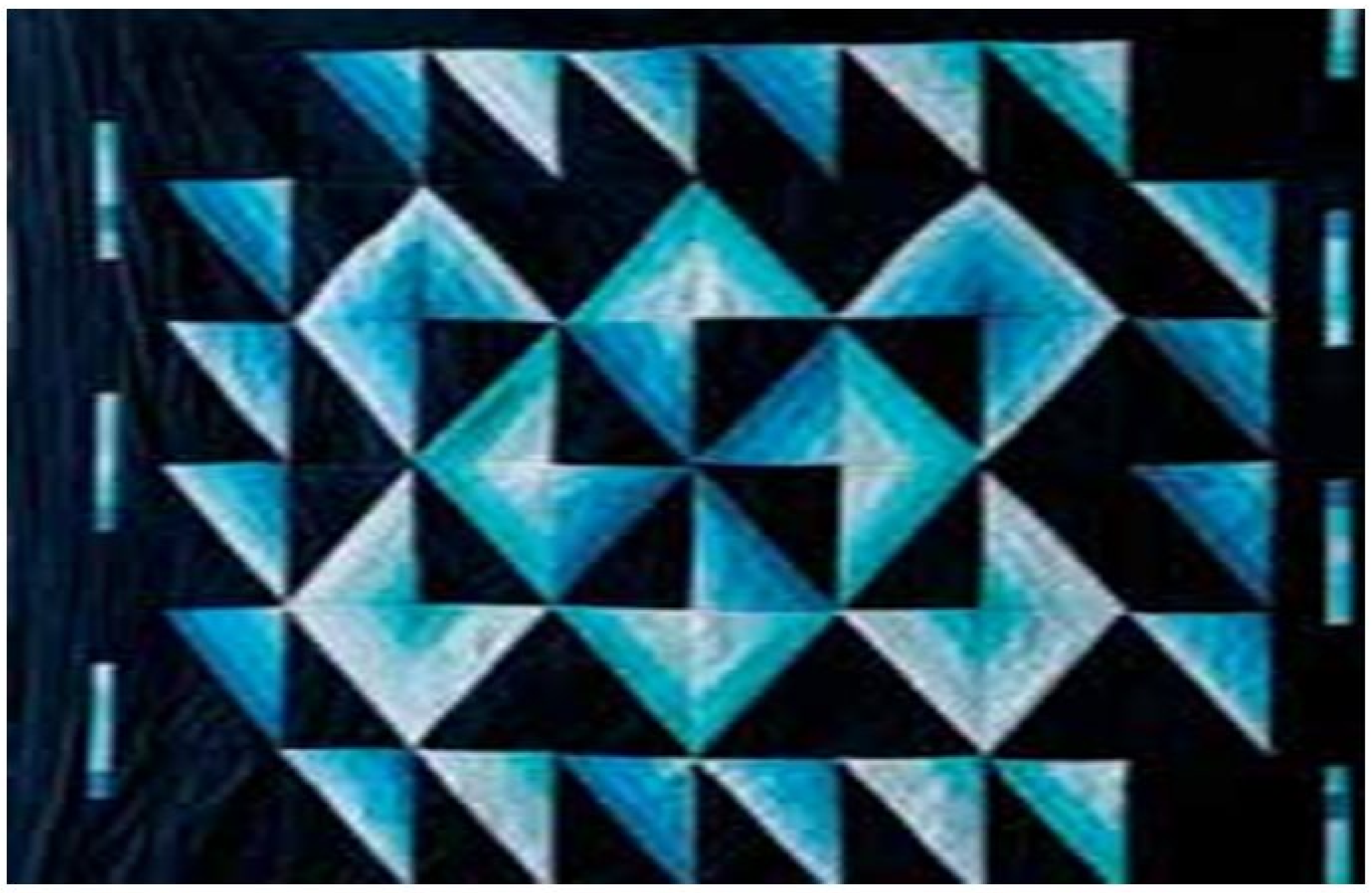
Pattern: A set of numbers or objects that follow a specific order or rule.

Quilting has become much more than a hobby in many places around the world and in some it has even begun to approach an art form!

Quilts are blankets or textiles which are made up of layers of material held together by knots and stitching. In the US, quilts made by the Amish community have become extremely famous as some of the best examples of the art form.

The Amish are a private, secluded community who live by principles of simplicity, humility and peace and choose to live without modern conveniences such as electricity and technology. Their ancestors originally emigrated to North America from Germany and Switzerland over two centuries ago.

Within their strict beliefs of simplicity, the Amish quilters managed to find ways to create intricate and vibrant quilts using, either consciously or unconsciously, many mathematical ideas.



Describe what mathematical ideas you can find in the quilt above. You may choose to write it in sentence form, creating a summary paragraph or you may wish to list each different idea you see in bullet point form.

Are primes beautiful?

Quantity: An amount or number.

ACTIVITY: Meet the Emirps

ATL

- Creative thinking skills: Generating novel ideas and considering new perspectives

A prime number is a number which can only be divided by itself and 1. Is 1 a prime number? What is the only even prime number?

How many primes can you name?

On a 100-square, shade in all the prime numbers.

Look at the number 13. If 7 is the fourth prime number, what prime number is 13?

13 is also an emirp! An emirp is any prime number whose reversal (the number read backwards) is also a prime. Look at your 100-square again. Did you shade in 31?

So, 13 and 31 are emirps! Can you find other numbers in the 100-square which follow this rule? Meet as many emirps as you can!

How can you check if a number is an emirp? Is 9787 an emirp? Is 9788?

Assessment opportunities

- In this activity you have practised skills that can be assessed using Criterion A: Knowing and understanding.

SOME NUMBERS ARE INTERESTING; SOME ARE NOT

'Some numbers are interesting, some are boring. You jump with joy when you find 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100 because they are such beautiful squares, and they will probably break up as such. Even among squares, some are more interesting than others – for example 9, 16, 36 and 64.

Other numbers fail to excite, 13, 17, 19, 23, 29, 31 because they have few possibilities of change, AND they are primes!

The most interesting number for the Babylonians was 60 because it is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60. This was a huge asset when fractions, ratios and percentages were undeveloped.

This gave us 60 minutes per hour, 60 seconds per minute, 6 × 60° to complete a full circle and their second favourite number, 24, is rather useful when you divide it!

Celtic tribes of ancient Europe favoured 20 as a number. The Inuit of the North Pole are said to have described anything over 5 as simply 'a large number' and nothing more.

The arrival of rational numbers and of scientific notation has eliminated such difficulties.'

Source: Sean O'Dubhghaill, Numbers in Time.

DISCUSS

1. Why could square numbers be considered 'beautiful'?
2. What is another name for the divisors of a certain number?
3. What is the author's feeling about primes (prime numbers)?
4. Do you think this author knows about emirps?
5. Who were the Babylonians?
6. Where in space and time did they exist?
7. Are there cultures other than the Inuit who do not have words for large numbers?

People used to think that literacy was a gift from the gods. What does this statement make you think? What does it indicate about a particular place or time? When do you think it might have been thought?

This economics podcast **www.bbc.co.uk/programmes/p050skkr** talks about a mystery stone tablet, which dates from over 5000 years ago. On this tablet was great writing which could not be deciphered (decoded).

What do you think these symbols might represent? What could they be used for?

Listen to the podcast to hear all about **correspondence counting**. This type of counting uses a symbol to represent a quantity. How were the symbols formed? What did they mean to the traders, as goods entered and left the marketplace?

These symbols represent a way to track quantities *without* a common numerical form. This form of representation made it possible to add and subtract without using arithmetic.

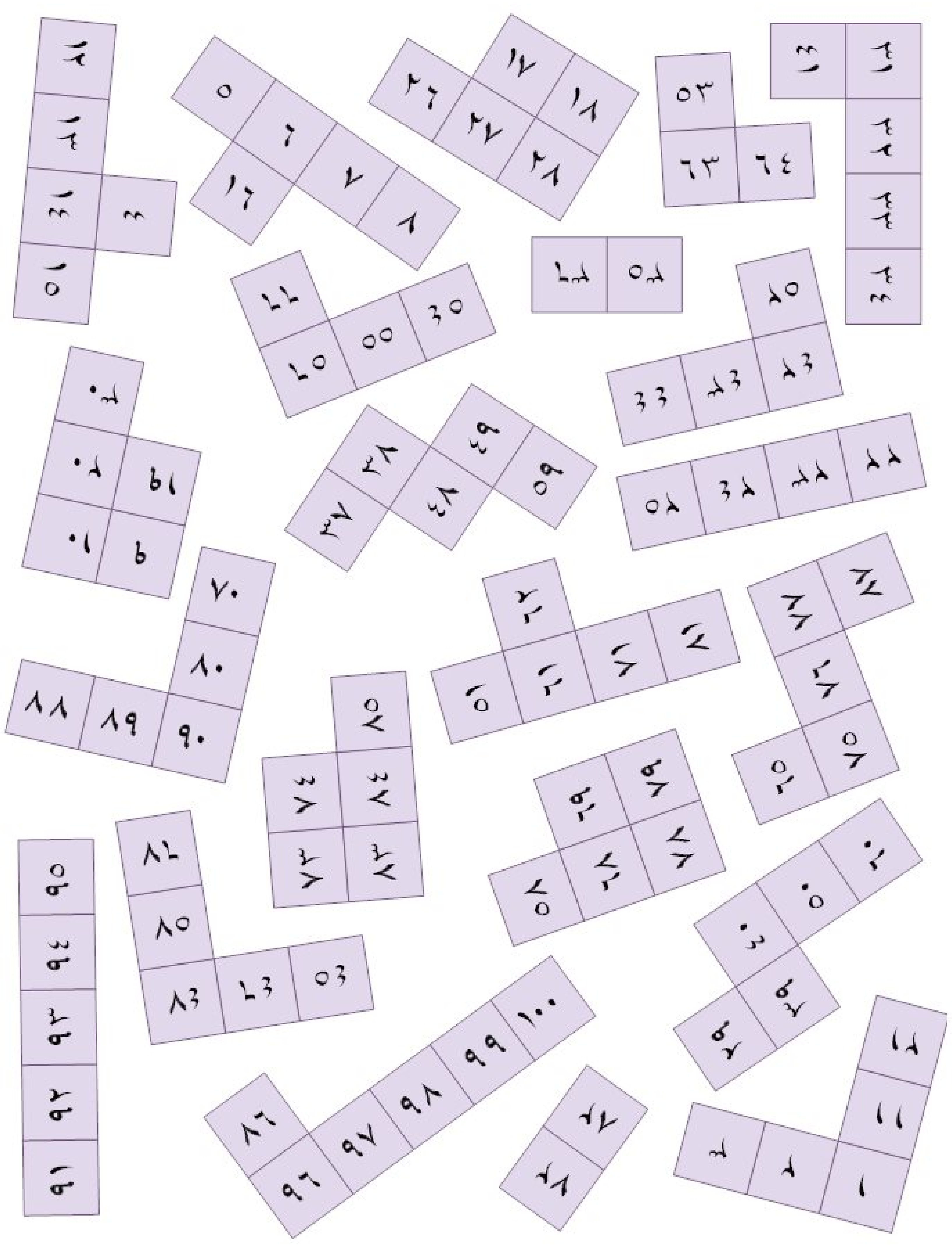
The invention of numbers to represent quantities has been different, depending on the place and time when it occurred.

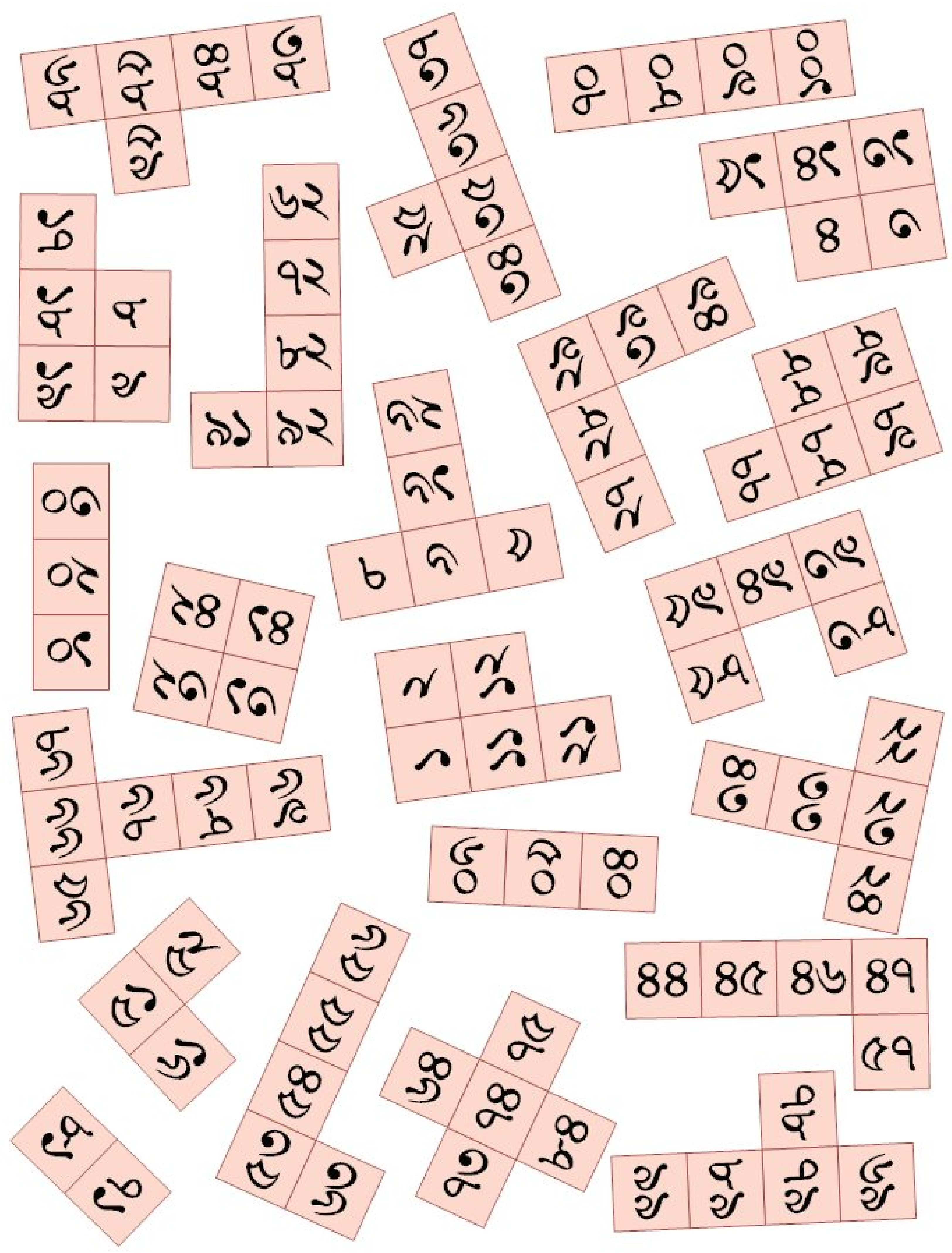
Representation: The manner in which something is presented.

THE DEVELOPMENT OF NUMBERS

Brahmi		𑁒	𑁓	𑁔	𑁕	𑁖	𑁗	𑁘	𑁙	𑁚
Hindu	०	१	२	३	४	५	६	७	८	९
Arabic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Medieval	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Modern	0	1	2	3	4	5	6	7	8	9

Using the information in the table and your logic, can you piece together these 100-square jigsaws made up from two different number systems from across time? Trace the pieces, cut them out and move them around to fit.





How good is your mental mathematics?

Simplification: The process of reducing to a less complicated form.

ACTIVITY: Are you a mathematics star?

ATL

- Critical thinking skills: Use critical-literacy skills to analyse and interpret media communications

Chat shows are a popular type of television show – a host has famous people onto their show and they talk to them about their lives and their work. These stars are often promoting a movie, an album or a book.

One such show is hosted by Chris Moyles and one of the games he likes to play with his guests is to have them perform some mathematical problems quickly. To make it even more appealing to the guests and the audience, the problem is performed in song by a real (sometimes famous) musician.

You can find a selection of the questions from different shows here: **www.sheffieldmaths.co.uk/Chris%20Moyles%20Starters.html**

www.mathematicshed.com/chris-moyles-musical-maths-shed.html

or search **Chris Moyles maths**.

Choose a video and play the mathematics question once. (Make sure your teacher has pre-screened it so they know when to press pause, before the solution is revealed.)

Play the video a second time, again stop at the pause mark. Try to complete the problem in your head and write down the final answer.

Now play the video a third time to check your answer and play it all the way to the end. Did you get the same answer as the celebrity?

Repeat this process for a few more videos. How good are you at getting the same answer as the guest? Which are your best operations and which are your worst? Work through the problem as a class and discuss the various steps.

Can you trust these answers though? Perhaps you are right and the celebrity is wrong!

We have learned before that there is an order to operations. This means that multiplication and division is done before addition and subtraction, for example. We also know that brackets (parentheses) and powers (indices) are done before those.

Go back to the first video. Play it through again but instead of calculating the answer in a series of steps, each done before you move on to the next, this time write out the problem as a whole! Complete the calculation, according to the BEDMAS rules.

Are your answers different? If yes, why is that? If not, explain why they are not different.

Assessment opportunities

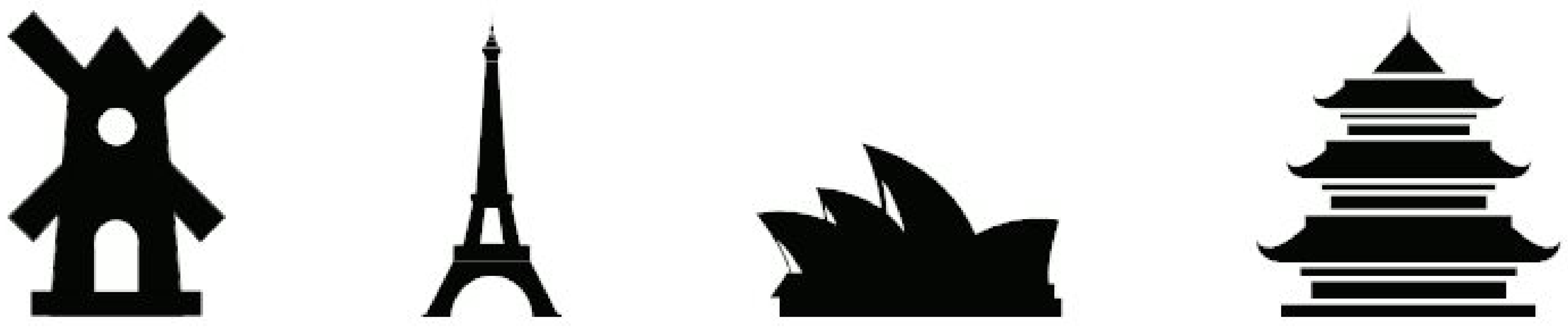
- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Which structures define your hometown?

We understand that space, in a mathematical sense, means the dimensions around us. Our orientation in space and time give us a sense of where, when and who we are. We understand that things change, depending on where and when they are.

But what about the things that never were? We can see built structures all around us – incredible beautiful structures, both big and small.

Sometimes these built structures are defined by where they are, but other times they define their own sense of identity. Which countries and symbols are instantly recognizable because of these built entities?



But what about those structures which were **never** built?

When projects or structures are being planned, architects often submit possible plans and a winner is chosen. The rejected plans never get built; it can be very interesting to see an alternative version of a familiar landmark. The geometry of these spaces can also be intriguing – how an unbuilt highway would have cast shadows or could have blocked out the view.

Space: The frame of geometrical dimensions describing an entity.

Here is an alternative design to London's Tower Bridge.

We can clearly identify lots of the ideas we encountered in Chapter 3.

You can see other unbuilt structures in London on this website: **http://londonist.com/2011/12/unbuilt-london-bridges-to-nowhere-and-mad-masterplans**

Listen to the 99% Invisible Podcast: Unbuilt **http://99percentinvisible.org/episode/unbuilt/**

An entire 'unbuilt' city.

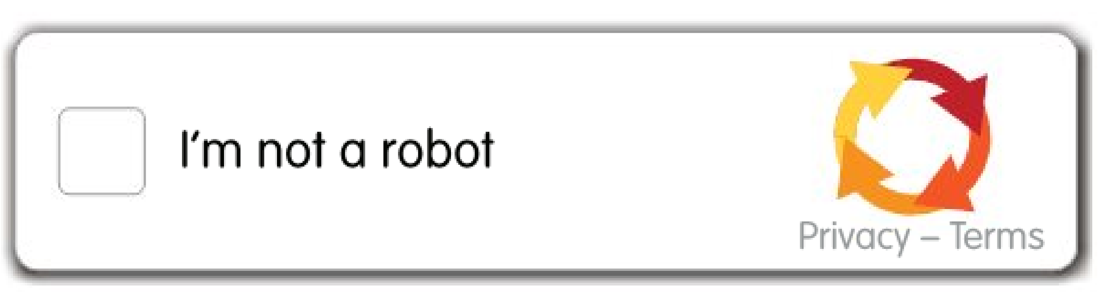
Are there any unbuilt structures near you? How could you find out? What would be different if the unbuilt had been built?

How can I beat the system?

System: A group of interrelated elements.

A recent development in our changing times is the need to verify that online activity is definitely human. Why is this? Well, the use of specially programmed computer 'bots' can affect websites in a negative way. To prevent bots from doing things like buying up all the concert tickets quickly or crashing a website by bombarding it with requests, a useful system has been created where a quick 'human' check is introduced.

These checks involve tasks that are simple for most humans to complete but very difficult for computers to master. They can involve recognizing and repeating a series of floating letters, clicking on all the images of a certain animal in a selection, recognizing numbers from a fuzzy photographic image, or even just checking a box with the computer mouse.



But programmers need lots of images so they can change them regularly and randomly, as computers are excellent at spotting patterns.

Your end-of-year activity is to provide the programmers with a selection of images which show numbers in use in everyday life. Look on posters, clocks, door numbers, cafeteria menus … anywhere you can see a number in use.

How many can you find? Did you find any primes? Powers? Emirps?

CHALLENGE

Can the whole class fill a 100-square of numbers with images? This would make a fantastic mosaic for the mathematics corridor!

Reflection

Use this table to reflect on your own learning in this chapter.					
Questions we asked	Answers we found	Any further questions now?			
Factual: What is binary? Which structures define your hometown?					
Conceptual: What is meant by mathematical synonyms? What makes an image 'mathemagical'? How can I beat the system?					
Debatable: Is paternity leave a fair benefit? Are primes beautiful?					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Communication skills					
Media literacy skills					
Creative-thinking skills					
Creativity and innovation					
Transfer skills					
Learner Profile attribute(s)	Reflect on the importance of being a balanced person for your learning in this chapter.				
Balanced					

Glossary

bi-variate relating to two variables

cartesian plane a surface or grid that uses two axes to describe any point in space

congruent identical in form, with the same lengths and angles as one another

correlation a measure of the relationship between two variables

cross-section the surface revealed when a shape is cut through or cut across

cumulative frequency a table showing a 'running total' of the frequencies of groups

datum A single number, measurement or piece of information

distribution How numbers are spread out from one another

ellipse A regular oval shape; a shape with two semi-axes

frequency How often a result or datum appears in a sample or population

hexagon A six-sided regular shape

integer A positive or negative whole number

interquartile range The difference between the lower (first) and upper (third) quartiles

isometric Without changing the sizes or dimensions of a shape

line of best fit (LOBF) A line that tries to show the truest representation of a relationship for a dataset

mode The most common or frequent number appearing in a set

multiplicand A number that is multiplied by another

negative correlation A relationship in which one variable decreases as the other increases

orientation Position in space, described using either direction or angle

partition A cut or split

population A particular group to be recorded or studied

positive correlation A relationship in which one variable increases as the other one increases

quadrants A division of space into four sections

quotient A division of one number by another

reflection A symmetrical operation whereby every point of an object is translated the equal distance across a mirror line

representation A model or a form

scale factor A number indicating how many times larger the enlargement operation will make each dimension of the shape

set A collection or group based on some common condition

similar Objects that have identical angles but different side lengths

stem-and-leaf diagram A diagram in which data are grouped into classes or groups (stems) and each individual datum is shown as a 'leaf'

strong correlation An indication that two variables have a strong effect on or relationship with one another.

subset A small set fully contained within another set

superimpose To place on top of; to overlay

surface area The combined area of all surfaces of a three-dimensional object

translation A geometric, isomorphic movement of an object

vinculum The line which separates the numerator and denominator

weak correlation An indication that two variables have a weak effect on or relationship with one another

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MYP *by Concept* 2

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Mathematics

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Irina Amlin has been an MYP Mathematics educator and department head, the Chair and frequent presenter of the island-wide Bermuda Math Educators' Learning Network, and has been involved in IB curriculum development and assessment activities. Her primary area of focus is illustrating the relevance of mathematics in the everyday world around us, and guiding students to experience its applications across many disciplines. She is also the author of *Mathematics for the IB MYP 1: by Concept*.

Rita Bateson was, until very recently, the Curriculum Manager for MYP Mathematics and Sciences at the International Baccalaureate® (IB) and continues to be involved in curriculum review. She is an experienced teacher of MYP and DP Mathematics and Sciences, and is Head of Mathematics in her current school. She has taught in many international schools in Europe as well as North America. Her interests include overcoming mathematics anxiety in pupils and STEM education. She is also the author of *MYP by Concept 4&5 Mathematics*.

Series editor: Paul Morris

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